РАДІАЦІЙНА ФІЗИКА
RADIATION PHYSICS

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DETERMINATION OF EMPIRICAL PARAMETERS OF DEPTH-DOSE DISTRIBUTION MODELS FOR CASES WITHOUT ANY CONTROL OF THE ELECTRON BEAM SPECTRUM

Two preconditions are crucial for the calculation of depth-dose distributions in existing semi-empirical models. An available detailed information regarding the electron beam spectrum is one precondition. And the dependence of the model's empirical parameters on the energy and incidence angle of the beam on the irradiated object is the other precondition. However, in industrial applications, the electron beam spectrum is typically not controlled during the radiation sterilization process. The present paper addresses and solves the problem of developing a procedure for using existing semi-empirical models to calculate depth-dose distributions if the electron beam spectrum is not controlled.

The procedure for determining the empirical parameters of the model is suggested. It is based on the selection of a single irradiation mode from a set of values for electron energies and electron beam incidence angles on the layer that make a significant contribution to the depth-dose distribution. An estimation of the electron energy range in the spectrum is carried out based on an approximation of the spectrum by a triangular distribution with parameters corresponding to standard characteristics of the electron depth-dose distribution. The errors in determining the empirical parameters of the model are evaluated using a set of irradiation modes in the electron energy range from 5 to 10 MeV and electron beam incidence angles on the layer from 0° to 60°.

The error of the SEM2U model is determined based on the deviations of the model's predictions from the results of numerical experiments obtained by simulating depth-dose curves in the layer using the Monte Carlo method. Standard characteristics of the radiation sterilization process are used to estimate errors: values of optimal layer thicknesses, dose uniformity indices, and beam energy utilization coefficients.

The method for using semi-empirical models is described for the cases without any control of the electron spectrum. The method consists of processing measurements taken using a dosimetric wedge to obtain the depth-dose curve under normal incidence of the electron beam on the irradiated object (the baseline curve for the models) and selecting a single irradiation mode to determine the empirical parameters of the models. The depth-dose curves in a semi-infinite medium, calculated using the described method, are compared with the results of simulating depth-dose curves using the Monte Carlo method. For these comparisons, an electron beam with a broad Spectrum_8 spectrum is used at various beam incidence angles on the medium. The error of the calculations of depth-dose curves for electron beams with the Spectrum_8 spectrum is evaluated according to the characteristics of the two-sided irradiation process, which is standard in radiation sterilization. The error is estimated based on the deviation of the predictions of the semi-empirical SEM2U model from the results of numerical experiments obtained by simulating depth-dose curves in the layers of optimal thickness using the Monte Carlo method.

It is shown that the procedure for determining the empirical parameters of the model, based on the selection of a single irradiation mode, provides sufficient accuracy for the calculations in the SEM2U model for electron beams with a broad spectrum. The possibilities of the proposed method for using semi-empirical models to calculate depth-dose curves when selecting optimal irradiation modes in radiation sterilization in cases without any control of the spectrum are discussed.

Keywords: electron beam dosimetry, sterilization, two-side irradiation, depth-dose curve, selection of optimal modes, semi-empirical model, Monte Carlo method.

1. Introduction

Optimization of irradiation processes in radiation technologies [1–8] requires minimizing the degree of dose nonuniformity within the volume of the treated object [9–14]. A high dose uniformity ratio (DUR) for electron beams can be achieved using the two-sided irradiation method [3, 4]. However, this requires that the irradiated layer has an optimal thickness h_{opt}. This requirement produces technical difficulties in planning and controlling the irradiation process, which are associated with the need to vary the electron energy when irradiating layers of different thicknesses *h*. One way to overcome these difficulties is to change the angle θ of incidence of the electron beam on the surface of the irradiated object without changing the electron energy *E* [15–21]. In this case, the beam incidence angle should be selected such that the optimal thickness $h_{opt}(\theta,E)$ for two-sided irradiation of the layer is equal to *h*.

The selection of the electron beam incidence angle on the surface of the irradiated object to optimize the two-sided irradiation process can be carried out using a Monte Carlo simulation of depth-dose curves [10, 11]. Unfortunately, this approach is not well-suited for finding optimal irradiation parameters in the practical operations of radiation sterilization centers. In [17, 21], semi-empirical models of the depth-dose curve were developed for a limited range of electron beam incidence angles on the layer. The empirical model parameters were determined by fitting data calculated using the semi-empirical model to data obtained via Monte Carlo simulation. For example, for the ISE model [21], the empirical parameter of the model was determined using data on optimal layer thicknesses [17] at various angles of incidence of the electron beam on the layer. It should be noted that, unfortunately, the use of single-parameter semi-empirical models can provide a good description of only one of the technological indicators of the irradiation process.

In [18], two-parameter semi-empirical models were presented. The models can jointly describe the optimal layer thickness and the dose heterogeneity index DUR during two-sided irradiation, which is necessary for making the decision on the feasibility of using irradiation parameters for the radiation sterilization process. These models were developed based on the distribution of transferred energy within the volume of the material, which is initiated by the normal incidence of a point radiation beam on the surface of a semi-infinite medium (the Dose-Map object). Semi-empirical models were developed for the depth-dose curve from the radiation entering the medium at an angle θ, based on the following assumption. The dose was assumed to be distributed uniformly (SEM2U model) or according to a Gaussian distribution (SEM2N model) in the cross-sections of the Dose-Map object at all depths within the object. The SEM2U and SEM2N semi-empirical models were verified using results obtained from the Monte Carlo simulations of depth-dose curves for various angles of incidence of the electron beam on the object.

The development and implementation of procedures and methods for determining the empirical parameters of the model is a critical task for the practical application of semi-empirical models. These parameters are determined by fitting the results of calculations using the model to available reliable data (e.g., measurement results or Monte Carlo simulations).

The basic relations for the depth-dose curve in a semi-infinite medium $D_i(x,\theta,E)$ under uniform irradiation by the monoenergetic electron beam of energy E incident on the surface of the object at an angle θ can be represented as the integral transformation of the depth-dose curve in a semi-infinite medium $D(x, 0, E)$ under irradiation of the object by a normally incident electron beam

$$D_i(x,\theta,E) = \int_0^{L_{max}} D(t,0,E)\cdot K_i(t,x,\theta,\alpha_i(\theta,E),Q_i(\theta,E))dt, \quad (1)$$

where $K_i(t,x,\theta,\alpha_i(\theta,E),\ Q_i(\theta,E))$ is the integral transform kernel, $\alpha_i(\theta,E),\ Q_i(\theta,E)$ are empirical parameters of semi-empirical models.

In the ISE model [21] as well as in the SEM2U and SEM2N models [18], if the object was irradiated by a normally incident electron beam, the depth-dose curve $D(x,0,E)$ in a semi-infinite medium was calculated based on a semi-empirical model [22] using the EMID software [23]. In the SEM2U and SEM2N models, when implementing transformation (1), integral transformation kernels were used, which were obtained by approximating the Dose-Map object with power-law functions

$$F(x) = \alpha\cdot L_{max}\left(\frac{x}{L_{max}}\right)^Q,\ Q \ge 0,\ 0 < x \le L_{max}, \quad (2)$$

where L_{max} is the penetration depth of radiation particles in the object [2].

Therefore, these models have two empirical parameters: α, which determines the maximum value of the function $F_{max} = \alpha\cdot L_{max}$, and Q – the exponent of the power function. In Eq. (1), the index $i = 1$ denotes the SEM2U model (uniform distribution), and the index $i = 2$ denotes the SEM2N model (Gaussian distribution).

Procedures for determining the empirical parameters of the models were proposed in [17, 21]. However, the use of these procedures assumes the electron beams to be monoenergetic. Industrial radiation facilities typically have electron beams with a sufficiently broad spectrum. In this case, the depth-dose curve $D(x,\theta)$ can be calculated by averaging the depth-dose curves $D(x,E,\theta)$ in accordance with the electron spectral distribution $S(E)$

$$D_i(x,\theta) = \int_{E_{min}}^{E_{max}}\int_0^{L_{max}} D(t,0,E)\cdot K_i(t,x,\theta,\alpha_i(\theta,E),Q_i(\theta,E))\cdot S(E)dtdE, \quad (3)$$

where E_{min} and E_{max} are the minimum and maximum electron energies in the spectrum, respectively.

The boundaries of the electron energy range in the spectrum (E_{min}, E_{max}) can be estimated as follows. The maximum electron energy $E_{\max}$ can be estimated as the maximum energy that the electron accelerator can provide. The minimum electron energy $E_{\min}$ can be calculated according to standard [4] characteristics of the electron energy, such as the average energy E_{Av} and the most probable energy E_p of the electron source. Using the approximation of the spectrum $S(E)$ by a triangular distribution based on these data, one obtains

$$E_{\min} = 3 \cdot E_{Av} - E_p - E_{\max}. \tag{4}$$

If the spectral distribution $S(E)$ and the functions of the empirical model parameters $\alpha_i(\theta,E), Q_i(\theta,E)$ are known in the range ($E_{\min}$, $E_{\max}$) of electron energies in the spectrum, then Eq. (3) makes it possible to calculate the depth-dose curve $D_i(x,\theta)$ for irradiation with an electron beam of a given spectrum. However, detailed information about the spectrum and the dependence of the model's empirical parameters on the energy E and the angle θ of the electron beam incident on the surface of the object are unknown in practice.

In [17, 21], the empirical parameters of the model were determined based on the use of the optimal layer thickness, whose value was obtained as a solution to the equation

$$D(x,E,\theta) = D(0,E,\theta)/2, \quad h_{\text{opt}}(\theta,E) = 2x. \tag{5}$$

The procedure consisted of the following three steps. First, the value of the optimal layer thickness was determined based on the Monte Carlo simulation of the depth distribution of the electron radiation dose [10, 11]. Second, the value of the optimal layer thickness $h_{\text{mod}}(\theta,E,\alpha_i(\theta,E),Q_i(\theta,E))$ was determined based on the depth distribution of the electron radiation dose $D_i(x,\theta,E,h)$, which was calculated in the model with empirical parameters $\alpha_i(\theta,E),Q_i(\theta,E)$. Third, empirical model parameters were selected from the condition

$$h_{\text{mod}}(\theta,E,\alpha_i(\theta,E),Q_i(\theta,E)) = h_{\text{opt}}(\theta,E). \tag{6}$$

It should be noted that the procedure described above for the determination of the model empirical parameters requires significant time to simulate a set of depth distributions of the electron radiation dose $D(x,E_j,\theta_i,h)$ using the Monte Carlo method for discrete values of electron energies E_j and incidence angles θ_i. Furthermore, for each element in this set, the condition (6) should be verified using a brute-force search, and values for the model's empirical parameters should be selected.

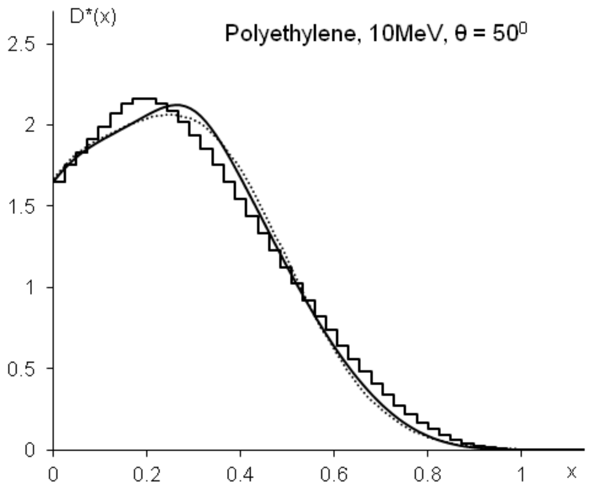
2. Empirical parameters of the depth-dose distribution models for a given angle of incidence of the electron beam

2.1. The method for determining the empirical parameters of the model based on a single irradiation mode

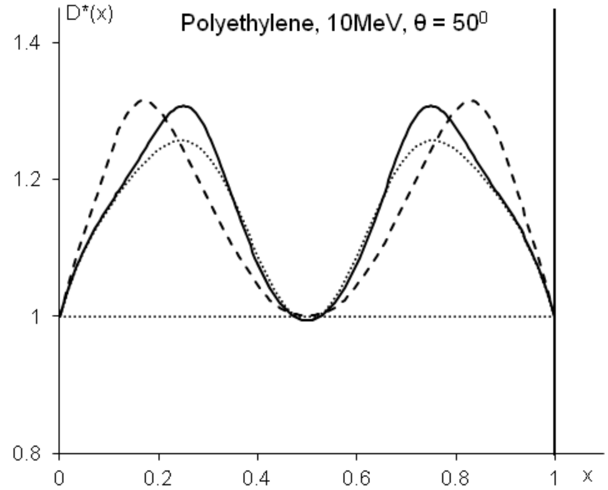
The present paper proposes a less labor-intensive method than that in [17] for determining the empirical parameters α_i, Q_i of the model. The method is based on selecting a single irradiation mode (node) from a set of discrete values of electron energies E_j and beam incidence angles θ_i on the surface of the irradiated object, and determining the empirical parameters of the model based on Eq. (6). In this method, the irradiation mode is selected based on the dependence of the calculated depth-dose distribution results on the empirical parameters of the model. The node is selected for which the changes in the predicted characteristics of the irradiation process during radiation sterilization [4] depend most strongly on the changes in the values of the model empirical parameters.

For the LUE-10 electron accelerator applied for radiation sterilization [6], according to Eq. (4), the estimated limits of the energy range in the electron spectrum are: $E_{\min}$ = 5 MeV, $E_{\max}$ = 10 MeV. Irradiation modes with monoenergetic electron beams having energies of 10.0, 7.0, and 5.0 MeV and beam incidence angles of 30, 40, and 50° are considered. The irradiation mode with electron energy of 10 MeV and beam incidence angle of 50° is selected as the reference for determining the empirical parameters α_i, Q_i of the models. The empirical parameters of the model are determined from the condition of equality of the optimal layer thickness, obtained by simulating the depth-dose distribution using the Monte Carlo method, to the optimal layer thickness, calculated using the semi-empirical model with these parameters.

The results of applying the proposed method to the selected node are illustrated in Fig. 1. Fig. 1, *a* shows the dependence of the electron radiation dose on depth in a semi-infinite medium. The depth x is normalized to the value of the extrapolated average free path length of electrons $\rho(E)$ in polyethylene. Fig. 1, *b* shows the dependence of the electron radiation dose on depth in the layer under two-sided irradiation. The depth x is normalized to the value of the optimal layer thickness $h_{\text{opt}}(\theta,E)$, which is calculated in the semi-empirical model. As can be seen in Fig. 1, *b*, the requirement for the dose value at the center of the layer of optimal thickness to be equal to the dose value at the layer boundary, as defined by Eq. (5), is satisfied. This equation is solved numerically by simply sweeping the values of the empirical model parameters α_i with a step size of 0.01, while keeping $Q_i = 1$ fixed. The empirical parameters determined by this method for the SEM2U model are $\alpha_1 = 0.4$, $Q_1 = 1$, and for the SEM2N model they are $\alpha_2 = 0.2$, $Q_2 = 1$. In Fig. 1, the depth-dose curves calculated using the SEM2U model are shown by the solid curves, while the dotted curves represent the depth-dose curves calculated using the SEM2N model. The histogram and the dashed curve show the results of the Monte Carlo simulations of the depth-dose curves.



a



b

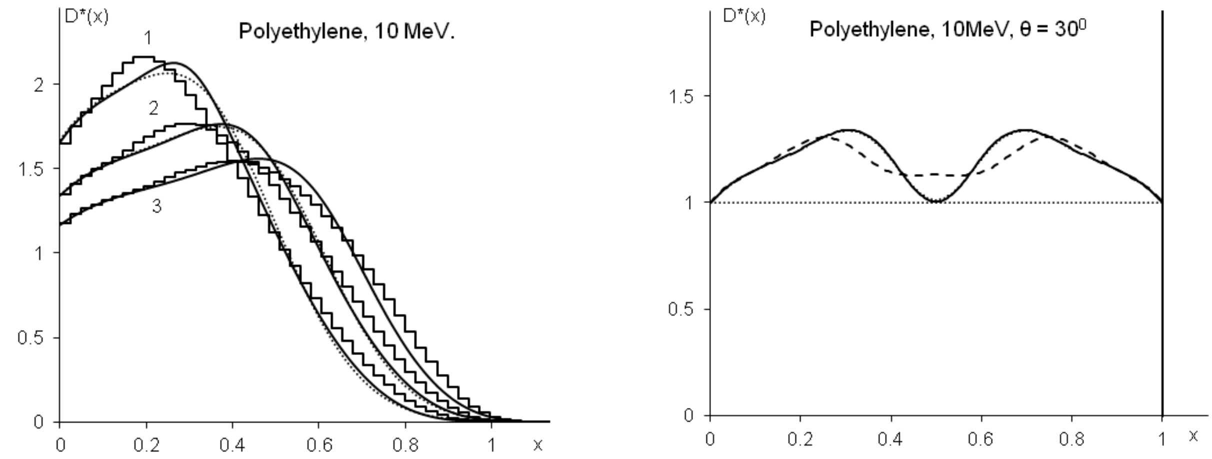
Fig. 1. The depth-dose curves calculated using the SEM2U model are shown by the solid curves; the *dotted curves* represent the depth-dose curves calculated using the SEM2N model. The *histogram* and the *dashed curve* show the results of the Monte Carlo simulations of the depth-dose curves. *a* – Depth-dose curves calculated for a semi-infinite medium. The depth *x* is normalized to the value 4.95 g/cm^2; *b* – depth-dose curves in a layer of thickness $h_{\text{opt}}(E, \theta) = 5.52$ g/cm^2 under two-sided irradiation. Model prediction error is $\Delta h_{\text{opt}} = -0.18$ %.

As shown in Fig. 1, there is a satisfactory agreement between the depth-dose curves for a semi-infinite medium and those for the layer of optimal thickness subjected to two-sided irradiation. Good agreement is observed between the value of the optimal layer thickness obtained by the Monte Carlo method and that calculated using semi-empirical models with the parameters selected according to the condition (6). The relative error for the procedure of selecting empirical model parameters is estimated as the error of the optimal layer thickness value calculated in the model with the selected parameters relative to that obtained based on the Monte Carlo simulation. For the selected irradiation mode, the relative error is $\Delta h_{\text{opt}} = -0.18$ %.

2.2. Comparison of semi-empirical model predictions with numerical experiment results

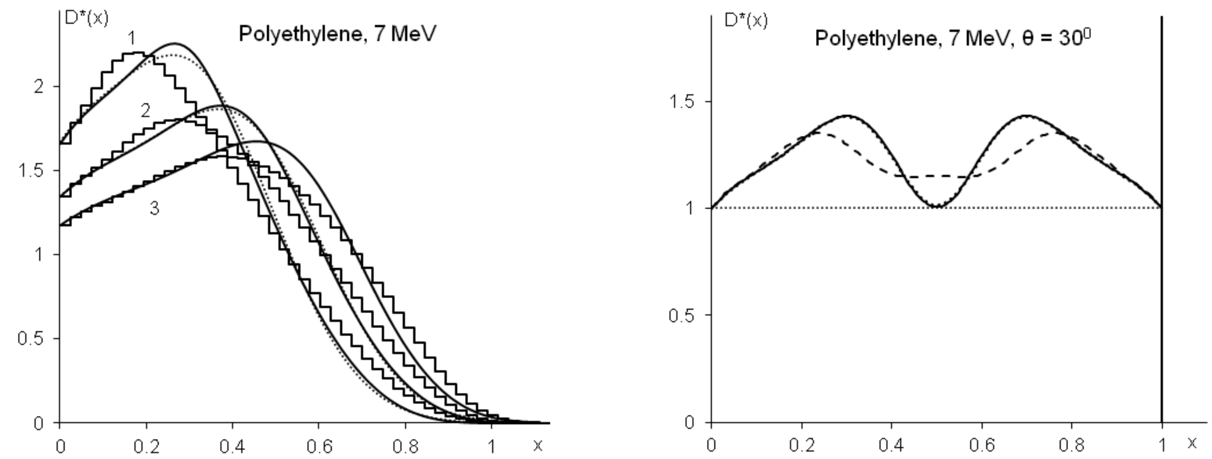
In this Section, the empirical parameters determined by the single-node selection method proposed in the previous section are used to study the error in the predictions of semi-empirical models. To compare the depth-dose curves obtained by the Monte Carlo simulation and those calculated using the selected empirical model parameters ($\alpha_1 = 0.4$, $Q_1 = 1$; $\alpha_2 = 0.2$, $Q_2 = 1$), Figs. 2–4 present the results of calculations of depth-dose curves in a semi-infinite medium and for two-sided irradiation of a polyethylene layer. The depth-dose curves, which are calculated for the optimal layer thicknesses determined in accordance with the model predictions, are presented in Figs. 2, *b*, 3, *b* and 4, *b*. These figures show only the calculation results for irradiation modes in which the relative error of the procedure for selecting empirical model parameters is the largest. The depth-dose curves calculated using the SEM2U model are shown by solid curves, while the dotted curves represent those calculated using the SEM2N model. The histograms and dashed curves show the results of simulating the depth-dose curves using the Monte Carlo method.

As one can see in Fig. 2, the results of the depth-dose curve calculations in the SEM2U model and in the SEM2N model are very close for beam incidence angles less than 50°. This indicates that the influence of object boundaries for these irradiation modes is negligible [18]. However, as it is shown in Fig. 1, the influence of object boundaries is already evident at a beam incidence angle of 50° to the irradiated object, even for the electron energy of 10 MeV. The comparison of the results of depth-dose curve calculations in the SEM2U model (shown as *solid curves*) with the results of depth-dose curve calculations in the SEM2N model (shown as *dotted curves*) makes it possible to conclude that the object boundaries can significantly affect the depth-dose distribution [18].



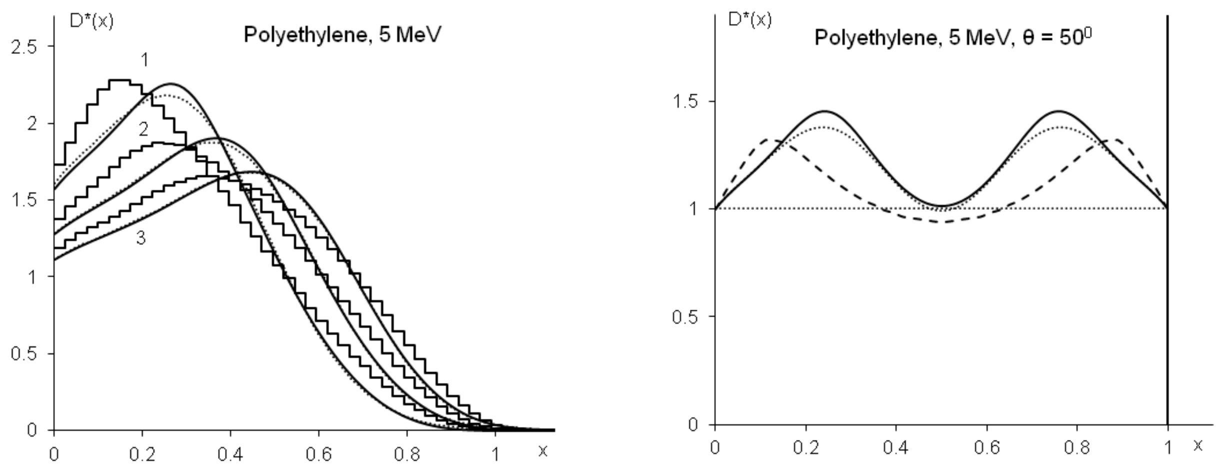
a *b*

Fig. 2. *a* – Depth-dose curves calculated for a semi-infinite medium at different angles θ of incidence of the electron beam on the object. The depth *x* is normalized to the value 4.95 g/cm^2. *1* – θ = 50°, *2* – 40°, *3* – 30°. *b* – Depth-dose curves in a layer of thickness $h_{opt}(E,\theta)$ = 7.58 g/cm^2 under two-sided irradiation. The depth *x* is normalized to the value 7.58 g/cm^2. Model prediction error is Δh_{opt} = −2.5 %.



a *b*

Fig. 3. *a* – Depth-dose curves calculated for a semi-infinite medium at different angles θ of incidence of the electron beam on the medium. The depth *x* is normalized to the value 3.44 g/cm^2. *1* – θ = 50°, *2* – 40°, *3* – 30°. *b* – Depth-dose curves in a layer of thickness $h_{opt}(E,\theta)$ = 5.28 g/cm^2 under two-sided irradiation. The depth *x* is normalized to the value 5.28 g/cm^2. Model prediction error is Δh_{opt} = –3 %.



a *b*

Fig. 4. *a* – Depth-dose curves calculated for a semi-infinite medium at different angles θ of incidence of the electron beam on the medium. The depth *x* is normalized to the value 2.42 g/cm^2. *1* – θ = 50°, *2* – 40°, *3* – 30°. *b* – Depth-dose curves in a layer of thickness $h_{opt}(E,\theta)$ = 2.74g/cm^2 under two-sided irradiation. The depth *x* is normalized to the value 2.74 g/cm^2. Model prediction error is Δh_{opt} = –2.6 %.

This phenomenon is weakening as the angle of incidence of the electron beam on the irradiated object decreases and the electron energy increases. According to the error data for the procedure of selecting the model's empirical parameters, the relative error Δh_{opt} does not exceed 3 % for the irradiation modes shown in Figs. 2–4.

For more detailed estimations of the model errors, the deviations of the model's predictions, under given empirical parameters, from the results of numerical experiments obtained by simulating depth-dose curves in the layer using the Monte Carlo method are considered. The values of the optimal layer thickness h_{opt} [g/cm^2], the dose uniformity indices in the irradiated object K_u [DUR], and the electron beam energy utilization coefficients K_o [%] are compared. These three observables are standard characteristics of the irradiation process during radiation sterilization [4]. The data used for these comparisons are presented in Table 1. The MC columns present irradiation process characteristics calculated based on the Monte Carlo simulation of depth-dose curves in a layer of optimal thickness under specified electron beam parameters. The Model columns present irradiation process characteristics obtained based on predictions from the SEM2U semi-empirical model, under specified model parameters. The M_MC columns present the optimal layer thicknesses, which are calculated based on predictions from the SEM2U semi-empirical model and the irradiation process characteristics, which are calculated for these layer thicknesses based on the Monte Carlo simulations of depth-dose curves. For comparison, the irradiation process characteristics are presented, which are calculated based on the Monte Carlo simulation of the depth-dose curve for the case of normal incidence of the electron beam on the layer of optimal thickness.

Table 1. **Characteristics of the two-sided irradiation during radiation sterilization**

Energy	θ	0°	30°			40°			50°		
		MC	MC	M_MC	Model	MC	M_MC	Model	MC	M_MC	Model
10 MeV	h_{opt}	8.91	7.77	7.58	7.58	6.82	6.66	6.66	5.52	5.52	5.52
	K_u	1.32	1.30	1.30	1.34	1.31	1.31	1.32	1.31	1.31	1.31
	K_o	0.848	0.863	0.842	0.837	0.867	0.847	0.844	0.865	0.865	0.857
7 MeV	h_{opt}	6.18	5.44	5.28	5.28	4.78	4.65	4.65	3.86	3.86	3.86
	K_u	1.40	1.35	1.35	1.43	1.33	1.33	1.41	1.32	1.32	1.38
	K_o	0.822	0.849	0.824	0.808	0.861	0.837	0.817	0.881	0.881	0.835
5 MeV	h_{opt}	4.28	3.79	3.74	3.74	3.34	3.3	3.3	2.67	2.74	2.74
	K_u	1.47	1.38	1.38	1.52	1.35	1.35	1.49	1.32	1.40	1.45
	K_o	0.8	0.835	0.824	0.782	0.856	0.847	0.794	0.871	0.841	0.812

As can be seen from the comparison of the data in Table 1, the relative differences between the values of the optimal layer thicknesses h_{opt} calculated in the semi-empirical model (column "Model") and the values of the optimal layer thicknesses obtained from numerical experiments (column "MC") do not exceed 3 %. Therefore, at the planning stage, for known object thickness h, the semi-empirical model can be used to optimize the irradiation process (to determine the electron energy E and the beam incidence angle θ on the irradiated object) according to the equation $h = h_{opt}(E,\theta)$.

The comparison of the data presented in the columns "MC" and "M_MC" of Table 1 shows that the prediction of the semi-empirical model for selecting the optimal layer thickness $h_{opt} = h_{opt}(E,\theta)$, for given values of electron energy E and beam incidence angles θ on the layer, provides very good agreement between the predicted and optimal regimes.

3. Semi-empirical model for beams with a wide spectrum of electrons

3.1. Basic equations and error estimation for the semi-empirical model

Note that the satisfactory agreement between the predictions (verification) of the semi-empirical model and the results of numerical experiments carried out over a wide range of electron energies E and beam incidence angles θ makes it possible to expect that the proposed procedure for selecting the model's empirical parameters can provide sufficiently accurate calculations for non-monoenergetic electron beams. To verify this expectation, an approximate method for calculating the depth-dose curve $D_i(x,\theta)$ by changing the order of integration in Eq. (3) is considered

$$D_i(x,\theta) = \int_0^{L_{\max}} \int_{E_{\min}}^{E_{\max}} D(t,0,E)\cdot K_i(t,x,\theta,\alpha_i(\theta,E),Q_i(\theta,E))\cdot S(E)dEdt\,. \quad (7)$$

When integrating over energy in (7), the empirical parameters (α_i, Q_i) of the models are assumed to be independent of the electron energy and the beam incidence angle on the irradiated object and are determined by the single-node selection method suggested in the previous section. In this case, expression (7) is significantly simplified and takes the form

$$D_i(x,\theta) = \int_0^{L_{\max}} D(t,0)\cdot K_i(t,x,\theta,\alpha_i,Q_i)dt\,. \quad (8)$$

Here, $D(t,0)$ is the depth-dose curve in a semi-infinite medium under irradiation by a normally incident electron beam. In particular, this quantity can be determined by carrying out the measurements using the dosimetric wedge method [24]. It can also be calculated using the depth-dependent dose profiles in the object $D(t,0,E)$ [17, 18] under irradiation by a normally incident monoenergetic electron beam

$$D(t,0) = \int_{E_{\min}}^{E_{\max}} D(t,0,E)\cdot S(E)dE\,. \quad (9)$$

Eq. (8) is used to verify the assumption that determining the empirical parameters of the model by the single-node selection method provides correct calculations for electron beams with a sufficiently broad spectrum. To verify this assumption, the triangular spectral distribution Spectrum_8 with the following parameters is considered:

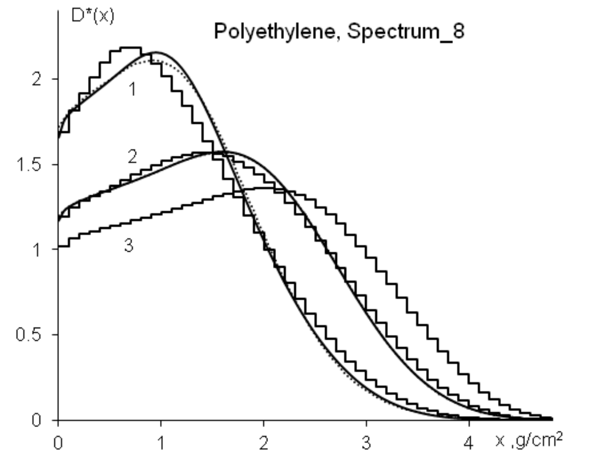
maximum electron energy $E_{\max}$ = 10 MeV;

minimum electron energy $E_{\min}$ = 5 MeV;

most probable electron energy E_p = 9 MeV.

For this distribution, the average electron energy E_{Av} = 8 MeV and the half-width ΔE = 2.5 MeV.

The depth-dose curve $D(t,0)$ in a semi-infinite medium under irradiation by a normally incident beam with this electron spectral distribution is calculated. Fig. 5, *a* shows the results of the Monte Carlo calculation of $D(t,0)$ – see the histogram (3). Numerical values of $D(t,0)$ and values of the empirical model parameters (α_1 = 0.4, Q_1 = 1; α_2 = 0.2, Q_2 = 1) are used to calculate the depth-dose curves $D_i(x,\theta)$ according to Eq. (8). The results of the calculations of the depth-dose curves $D_i(x,\theta)$ for electron beam incidence angles on the object of θ = 20, 30, 40, 50, and 60° are shown in Fig. 5. The depth-dose curves calculated using the SEM2U model are shown by solid curves, while dotted curves represent the depth-dose curves calculated using the SEM2N model. The histograms show the results of simulating the depth-dose curves using the Monte Carlo method.



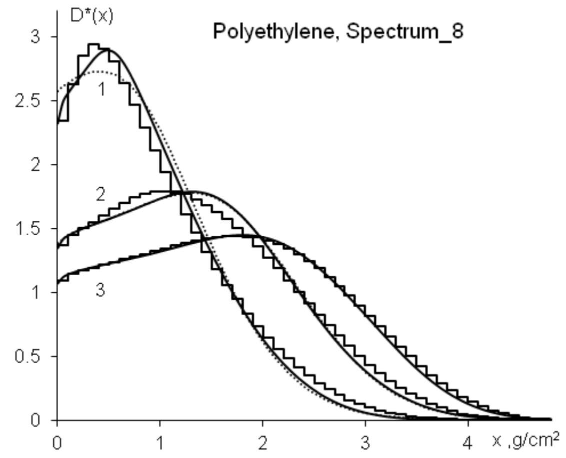


Fig. 5. *a* – Depth-dose curves in polyethylene irradiated by an electron beam with spectrum Spectrum_8 for different angles θ of incidence of the beam on a semi-infinite medium. *1* – θ = 50°, *2* – 30°, *3* – 0°. *b* – Depth-dose curves in polyethylene irradiated by an electron beam with spectrum Spectrum_8 for different angles θ of incidence of the beam on a semi-infinite medium. *1* – θ = 60°, *2* – 40°, *3* – 20°.

One can see in Fig. 5 that the depth-dose curves $D_i(x,\theta)$, calculated using the SEM2U and SEM2N models, agree reasonably well with the results of the Monte Carlo simulations of depth-dose curves (histograms). Note that agreement of the results is observed for a wide range of electron beam incidence angles on the object: θ = 20, 30, 40, 50, 60°.

To estimate the errors in the calculations of depth-dose curves for electron beams with the Spectrum_8 spectrum, the characteristics of the two-sided irradiation, which is the standard one in radiation sterilization, are used [4]. The deviations of the values of the optimal layer thickness h_{opt}, the dose uniformity indices in the layer $K(u)$, and the electron beam energy utilization coefficients K_o are evaluated. The error estimates are based on the deviation of the predictions of the SEM2U semi-empirical model from the results of numerical experiments obtained by simulating depth-dose curves in the layer using the Monte Carlo method. The data used for these estimates are presented in Table 2.

Table 2. **Data for comparing standard characteristics of the two-sided irradiation in radiation sterilization**

Θ	0°	10°		20°		30°		40°		50°		–
	MC	MC	Mod	MC	Mod	MC	Mod	MC	Mod	MC	Mod	–
h_{opt}	7.1	6.99	7.01	6.66	6.66	6.12	6.09	5.34	5.31	4.31	4.36	g/cm^2
K_u	1.33	1.32	1.36	1.31	1.35	1.31	1.35	1.31	1.35	1.31	1.34	–
K_o	0.847	0.848	0.832	0.851	0.831	0.853	0.831	0.859	0.833	0.863	0.843	%

The columns “MC” present the characteristics of the two-sided irradiation, calculated based on the Monte Carlo simulations of depth-dose curves in a layer of optimal thickness at given electron beam incidence angles θ with the Spectrum_8 spectrum. The columns “Mod” present the irradiation characteristics obtained based on SEM2U model predictions for the empirical parameters $\alpha_1 = 0.4$ and $Q_1 = 1$.

As one can see from the comparison of the data in Table 2, the relative differences between the values of the optimal layer thicknesses h_{opt} calculated using the semi-empirical model and those obtained from numerical experiments do not exceed 1 %. The errors in the values of the standard characteristics of the radiation sterilization – the dose uniformity index and the beam energy utilization coefficient – do not exceed 4 and 3 %, respectively. Therefore, the proposed method for calculating the depth-dose curve *Di(x*,θ) can be useful in selecting optimal irradiation modes in radiation sterilization for cases without any spectrum control.

4. Conclusions

In the present paper, a procedure is developed for determining the empirical parameters of the model based on the selection of a single irradiation mode from a set of modes that can make a significant contribution to the depth-dose distribution, for the cases in which the electron spectrum is unknown (uncontrolled). The error of the procedure for determining the empirical parameters of the model is estimated for the sets of irradiation modes with electron energies ranging from 5 to 10 MeV and electron beam incidence angles on the layer ranging from 0 to 60°. It is shown that when using this procedure, the relative error in the predictions of the optimal layer thickness Δh_{opt} does not exceed 3 %. Therefore, the procedure for determining empirical parameters can be *effectively* used to determine the optimal layer thickness for a given two-sided irradiation regime.

The errors of the SEM2U model are estimated based on the deviations of the values predicted using the model from the results of numerical experiments obtained by simulating depth-dose curves in the layer using the Monte Carlo method. It is shown that for the radiation sterilization process, when the thickness of the irradiated object *h* is fixed, this semi-empirical model can be used to determine the optimal irradiation regime (*E*,θ) according to the equality $h = h_{opt}(E,\theta)$. When this irradiation regime is used, good agreement is observed between the characteristics of the predicted and optimal sterilization processes.

The method is proposed for using semi-empirical models in cases in which the electron spectrum is unknown (uncontrolled). The method consists of the following. First, the measurements carried out using a dosimetric wedge are processed to obtain the dose-depth curve under normal incidence of the electron beam on the irradiated object (the baseline curve for the models). Second, the single irradiation mode is selected to determine the empirical parameters of the models. The depth-dose curves $D_i(x,\theta)$ in a semi-infinite medium, calculated using the proposed method, and the results of simulating the depth-dose curves using the Monte Carlo method are compared. For the calculations, an electron beam with a broad model spectrum Spectrum_8 is applied at various beam incidence angles on the layer. Satisfactory agreement of the results is found to be observed for a wide range of electron beam incidence angles from θ = 0° to 60°.

The errors in the calculations of the depth-dose curves in the layers of optimal thickness under two-sided irradiation with electron beams having the Spectrum_8 spectrum for the beam incidence angles θ = 10, 20, 30, 40, and 50° are evaluated. The relative errors of the standard characteristics of the radiation sterilization process – optimal layer thicknesses, dose uniformity indices, and beam energy utilization coefficients – are shown to be smaller than 1, 4, and 3 %, respectively. This confirms that the suggested procedure for determining the empirical parameters of the model provides sufficiently accurate calculations in the SEM2U model for electron beams with a broad spectrum. Thus, the proposed method for calculating the depth-dose curve *Di*(*x*,θ) can be useful in selecting optimal irradiation modes in radiation sterilization for the cases in which the spectrum is not controlled.

The method is proposed for estimating the range of electron energy values in the spectrum based on approximating the spectrum with a triangular distribution whose parameters correspond to the standard characteristics of the electron depth-dose distribution.

The results of calculating depth-dose distributions in the SEM2U model are compared with those in the SEM2N model. The comparison shows that the influence of the object boundary on the depth-dose distribution is weakening with a decrease in the angle of incidence of the electron beam on the irradiated object and with an increase in electron energy.

REFERENCES

1. S. Schiller, U. Heisig, S. Panzer. *Electron Beam Technology* (New York: John Wiley & Sons Inc, 1982) 508 p.
2. International Commission on Radiation Units and Measurements (ICRU). *Radiation Dosimetry: Electron Beams with Energies between 1 and 50 MeV.* ICRU Report 35 (Bethesda, MD, ICRU, 1984) 168 p.
3. R.J. Woods, A.K. Pikaev. *Applied Radiation Chemistry: Radiation Processing* (New York: Wiley, 1994) 535 p.
4. ISO/ASTM Standard 51649. Practice for dosimetry in an e-beam facility for radiation processing at energies between 300 keV and 25 MeV. ASTM Standards 12.02 (West Conshohocken, PA, ASTM International, 2005) 30 p.
5. Yu. Pavlov, P. Bystrov. Software and hardware complex for radiation processing facility control. Radiat. Phys. Chem. 196 (2022) 110110.
6. Z. Zimek, L. Waliś, A.G. Chmielewski. EB industrial facility for radiation sterilization of medical devices. Radiat. Phys. Chem. 42 (1993) 571.
7. R. Pomatsalyuk et al. Real time luminescent dosimetry system for product processing at an electron accelerator. Probl. At. Sci. Technol. 5 (2024) 131.
8. S.V. Denbnovetsky et al. Investigation of electron beam formation in glow-discharge electron guns with an additional electrode. In: Proceedings of the XVIIIth International Symposium on Discharges and Electrical Insulation in Vacuum (ISDEIV). Cat. No. 98CH36073. Eindhoven, The Netherlands, August 17–21, 1998 (Piscataway, NJ, IEEE Service Center, 1998) p. 637.
9. ASTM E2232-21. Standard Guide for Selection and Use of Mathematical Methods for Calculating Absorbed Dose in Radiation Processing Applications (West Conshohocken, PA, ASTM International, 2021) 19 p.
10. F. Salvat, J. Fernández-Varea, J. Sempau (Eds.) PENELOPE-2011: A Code System for Monte Carlo Simulation of Electron and Photon Transport. Workshop Proceedings. Barcelona, Spain, July 4–7, 2011 (Paris: OECD Publishing, 2012) 385 p.
11. V.T. Lazurik et al. *Information System and Software for Quality Control of Radiation Processing* (Warsaw, Institute of Nuclear Chemistry and Technology, 2011) 232 p.
12. V.L. Uvarov et al. Development and validation of an analytical model of isotope production by bremsstrahlung radiation. Appl. Radiat. Isot. 199 (2023) 110890.
13. V.G. Rudychev et al. Optimization of converter and bremsstrahlung characteristics for object irradiation. Radiat. Phys. Chem. 206 (2023) 110815.
14. V.L. Uvarov et al. An analytical approach to optimization of isotope production by bremsstrahlung radiation. Radiat. Phys. Chem. 214 (2024) 111547.
15. V.G. Rudychev, V.T. Lazurik, Y.V. Rudychev. Influence of the electron beams incidence angles on the depth-dose distribution of the irradiated object. Radiat. Phys. Chem. 186 (2021) 109527.
16. M. Rosenstein, H. Eisen, J. Silverman. Electron depth-dose distribution measurements in finite polystyrene slabs. J. Appl. Phys. 43 (1972) 3191.
17. V. Lazurik et at. Implementation of a semi-empirical model for controlling radiation sterilization processes. In: Proceedings of the 4th International Maghreb Meeting of the Conference on Sciences and Techniques of Automatic Control and Computer Engineering (MI-STA 2024). Tripoli, Libya, May 19–21, 2024 (IEEE, 2024) p. 649.
18. I.O. Girka, V.T. Lazurik. Semi-empirical models of electron beam control for radiation sterilization. East Eur. J. Phys. 3 (2025) 422.
19. V. Lazurik, V. Moskvin. Monte Carlo calculation of charge-deposition depth profile in slabs irradiated by electrons. Nucl. Instrum. Methods B 108 (1996) 276.
20. I. Melnyk, A. Pochynok. Theoretical justification of application possibility of different order root-polynomial functions for interpolation and approximation of boundary trajectory of electron beam. Radioelectron. Commun. Syst. 66 (2023) 53.
21. V. Lazurik et al. Control of process parameters of two-sided irradiation of electron beam in radiation technologies. In: Proceedings of the 3rd International Maghreb Meeting of the Conference on Sciences and Techniques of Automatic Control and Computer Engineering (MI-STA 2023) Benghazi, Libya, May 21–23, 2023 (IEEE, 2023) p. 25.
22. T. Tabata, P. Andreo, K. Shinoda. An algorithm for depth-dose curves of electrons fitted to Monte Carlo data. Radiat. Phys. Chem. 53 (1998) 205.
23. V.M. Lazurik, T. Tabata, V.T. Lazurik. A database for electron-material interactions. Radiat. Phys. Chem. 60 (2001) 161.
24. V.T. Lazurik et al. Dosimetry method based on a two-parametric model of electrons beam for radiation processing. Probl. At. Sci. Technol. 6 (2017) 137.

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ВИЗНАЧЕННЯ ЕМПІРИЧНИХ ПАРАМЕТРІВ МОДЕЛЕЙ ГЛИБИННОГО РОЗПОДІЛУ ДОЗИ ДЛЯ ВИПАДКІВ, КОЛИ СПЕКТР ПУЧКА ЕЛЕКТРОНІВ НЕ КОНТРОЛЮЄТЬСЯ

Відомі напівемпіричні моделі дають змогу розрахувати глибинні розподіли дози за наявності детальної інформації про спектр пучка електронів та залежності емпіричних параметрів моделі від енергії й кута падіння пучка на об’єкт опромінення. Однак у технологічних центрах у процесі радіаційної стерилізації спектр пучка електронів, як правило, не контролюється. У цій роботі поставлено й розв’язано задачу розробки процедури використання наявних напівемпіричних моделей для розрахунку глибинних розподілів дози у випадку, коли спектр пучка електронів не контролюється.

Описано процедуру визначення емпіричних параметрів моделі на основі вибору одного режиму опромінення з набору значень енергій електронів та кутів падіння пучка електронів на шар, які дають суттєвий внесок у розрахунок глибинного розподілу дози. Проведено оцінку діапазону енергій електронів у спектрі на основі апроксимації спектра трикутним розподілом з параметрами, що відповідають стандартним характеристикам глибинного розподілу дози електронів. Проведено оцінки похибок визначення емпіричних параметрів моделі з використанням набору режимів опромінення в діапазоні енергій електронів від 5 до 10 МеВ та кутів падіння пучка електронів на шар від 0 до 60°.

Похибку моделі SEM2U визначено на основі відхилень передбачень цієї моделі від результатів числових експериментів, отриманих моделюванням глибинних кривих дози в шарі методом Монте-Карло. Для оцінок похибок використовуються стандартні характеристики процесу радіаційної стерилізації: значення оптимальних товщин шару, показники однорідності дози та коефіцієнти використання енергії пучка.

Описано метод використання напівемпіричних моделей для випадку, коли спектр електронів не контролюється. Метод полягає в обробці вимірювань, виконаних з використанням дозиметричного клина, для отримання залежності дози від глибини при нормальному падінні пучка електронів на об’єкт опромінення (базова залежність моделей) та виборі одного режиму опромінення для визначення емпіричних параметрів моделей. Проведено порівняння глибинних кривих дози в напівнескінченному середовищі, розрахованих описаним методом, з результатами моделювання глибинних кривих дози методом Монте-Карло. Для цих порівнянь використовується пучок електронів із широким спектром Spectrum_8 при різних кутах падіння пучка на середовище. Проведено оцінку похибки розрахунків глибинних кривих дози для пучків електронів зі спектром Spectrum_8 згідно з характеристиками процесу двостороннього опромінення, стандартними при проведенні радіаційної стерилізації. Оцінку похибки проведено за величиною відхилення передбачень напівемпіричної моделі SEM2U від результатів числових експериментів, отриманих моделюванням глибинних кривих дози в шарах оптимальної товщини методом Монте-Карло.

Показано, що процедура визначення емпіричних параметрів моделі на основі вибору одного режиму опромінення дає змогу достатньо коректно проводити розрахунки в моделі SEM2U для пучків електронів із широким спектром. Обговорюються можливості запропонованого методу використання напівемпіричних моделей для розрахунків глибинних кривих дози при виборі оптимальних режимів опромінення в радіаційній стерилізації у випадку, коли спектр не контролюється.

Ключові слова: дозиметрія електронного пучка, стерилізація, двостороннє опромінення, крива глибинної дози, вибір оптимальних режимів, напівемпірична модель, метод Монте-Карло.

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