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МАКРОСКОПІЧНІ НАБЛИЖЕННЯ ДО ОБЕРТАННЯ НЕЙТРОННИХ ЗІРОК

Макроскопічна модель нейтронної зірки (НЗ) як ідеальної рідкої краплини поширюється на обертальні системи з малою швидкістю обертання ω у наближенні ефективної поверхні (ЕП). Враховано градієнтні члени енергії густини НЗ $\mathcal{E}(\rho)$ рівняння стану разом з об'ємними компонентами в головному наближенні за лептодермічним параметром $a/R \ll 1$, де a є товщиною шару ЕП та R – середній радіус НЗ. Для розрахунку адіабатичного моменту інерції (МІ) використовується вираз макроскопічного кутового моменту за малої частоти ω у рамках метрики Керра у зовнішніх координатах Бойера - Лінгвіста та внутрішніх координатах Хогана. Отримано НЗ МІ $\Theta = \tilde{\Theta}/\left(1-\mathcal{G}_{t\varphi}\right)$ через статистично усереднений МІ Θ та азимутально-кутову кореляцію $\mathcal{G}_{t\varphi}$ як суму об'ємного та поверхневого компонентів. Показано, що МІ Θ суттєво залежить від ефективного радіуса R внаслідок впливу сильної гравітації та поверхневих ефектів. Знайдено значні додаткові обмеження на радіус R завдяки кореляції $\mathcal{G}_{t\varphi}$ та поверхневим внескам. З цими внесками адіабатична умова при сильній гравітації виконується для багатьох добре відомих НЗ.

Ключові слова: ядерна астрофізика, густина енергії, нейтронна зірка, метрика Керра, моменти інерції.

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MACROSCOPIC APPROACHES TO ROTATING NEUTRON STARS[a]

The macroscopic model for a neutron star (NS) as a perfect liquid drop at equilibrium is extended to rotating systems with a small frequency ω within the effective-surface (ES) approach. The gradient surface terms of the NS energy density $\mathcal{E}(\rho)$ in the Equation of State are taken into account along with the volume components at the leading order over the leptodermic parameter, $a/R \ll 1$, where a is the ES crust thickness and R is the mean NS radius. The macroscopic NS angular momentum at small frequencies ω is used for calculations of the adiabatic moment of inertia (MI) within the Kerr metric approach in the outer Boyer - Lindquist and inner Hogan coordinate forms. The NS MI $\Theta = \tilde{\Theta}/\left(1-\mathcal{G}_{t\varphi}\right)$, was obtained in terms of the statistically averaged MI, $\tilde{\Theta}$, and its time and azimuthal-angle correlation, $\mathcal{G}_{t\varphi}$, as the sum of volume and surface components. The MI Θ depends dramatically on the effective radius R due to a strong gravitation and surface effects. We found significant additional rotational constraints on the radius R due to the correlation term $\mathcal{G}_{t\varphi}$ and surface contributions. With these contributions, the adiabaticity condition is better fulfilled for a stronger gravity in many well-known NSs.

Keywords: nuclear astrophysics, energy density, neutron star, Kerr metric, moments of inertia.

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