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NUCLEAR STRUCTURE AND SYMMETRY EVOLUTION IN $^{190,192}_{84}$Po: INSIGHTS FROM IBM-1, VMI, AND GVMI

In this study, three nuclear models – the Interacting Boson Model-1 (IBM-1), the Variable Moment of Inertia (VMI), and its generalized form (Generalized Variable Moment of Inertia - GVMI) – were employed to investigate the low-lying collective states of the even-even polonium isotopes ^{190}Po and ^{192}Po. The models were used to compute positive-parity energy levels $E(L)$, transition energies $E\gamma$, reduced electric quadrupole transition probabilities B(E2), and electric quadrupole moments Q_L, based on available experimental data and ideal level schemes. Characteristic energy ratios $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$, and $E(8_1^+)/E(2_1^+)$ were evaluated to assess the dynamical symmetries of the nuclei and compared with theoretical limits of the SU(5), SU(3), and O(6) symmetry groups. Results indicate that ^{190}Po exhibits dominant O(6) γ-soft symmetry, while ^{192}Po displays transitional behavior between SU(5) and O(6). Comparative analysis revealed good agreement between experimental data and theoretical predictions, particularly from IBM-1 and GVMI, confirming their reliability in modeling structural evolution in medium-heavy nuclei. In addition, theoretical energy levels were extrapolated for high-spin states not yet experimentally observed. For ^{190}Po, IBM-1 predicts the 16_1^+ state at 4.06 MeV, the 18_1^+ state at 4.96 MeV, and the 20_1^+ state at 6.01 MeV. For ^{192}Po, the predicted levels include 12_1^+ at 3.12 MeV, 14_1^+ at 4.11 MeV, and 16_1^+ at 5.20 MeV. These predictions extend the known spectra up to spin 20^+ and offer a theoretical framework for future experimental validation. These findings contribute to a deeper understanding of collective excitations and symmetry dynamics in the polonium isotopic chain.

Keywords: IBM-1, VMI, GVMI, ^{190}Po-^{192}Po-isotopes, energy levels, dynamical symmetry.

1. Introduction

The structure of atomic nuclei remains a central topic in nuclear physics, particularly in the study of transitional and heavy nuclei where phenomena such as shape coexistence, collective motion, and structural evolution become prominent [1]. In the polonium-lead region, even-even isotopes often display competing nuclear shapes due to the interplay between spherical and deformed configurations, resulting in rich manifestations of shape coexistence [2, 3]. Recent macroscopic–microscopic calculations using Nilsson - Strutinsky methods confirm a gradual shape transition in polonium isotopes with increasing neutron number, reinforcing the presence of coexisting nuclear structures in even-even $^{190}_{84}$Po and $^{192}_{84}$Po [4]. These transitions are strongly influenced by configuration mixing between regular and intruder states, as demonstrated in particle-core coupling models [5].

Among these isotopes, $^{190}_{84}$Po and $^{192}_{84}$Po are particularly suitable for investigation due to their proximity to the doubly magic nucleus ^{208}Pb ($Z = 82$, $N = 126$), which serves as a robust reference for probing collective excitations near shell closures [6]. Spectroscopic studies reveal a notable departure from spherical symmetry in light polonium isotopes, suggesting enhanced collectivity at lower neutron numbers [7]. Additionally, theoretical predictions based on relativistic mean field models indicate a systematic evolution from spherical to prolate shapes in this mass region, paralleling trends observed in adjacent mercury ($_{80}$Hg) and lead ($_{82}$Pb) isotopes [8].

Even-even isotopes such as $^{190}_{84}\mathrm{Po}_{106}$ and $^{192}_{84}\mathrm{Po}_{108}$ are highly amenable to theoretical modeling, as their ground-state bands exhibit regular 0^+, 2^+, 4^+, … level sequences typical of quadrupole collective motion. This regularity facilitates the application of algebraic models and symmetry-based approaches [9]. Coulomb excitation measurements and configuration mixing studies have provided crucial data on B(E2) values and level structures, illuminating the delicate interplay between shape coexistence and collectivity [10]. Meanwhile, high-spin spectroscopy in nearby isotopes has yielded insight into moments of inertia and deformation properties in this transitional region [11]. Equivalent nuclear structure studies have also been performed on xenon ($_{54}$Xe) isotopes [12], providing additional information about deformation behavior in medium-mass nuclei. There is also an existing body of theoretical work regarding the energy level schemes of the polonium isotopes [13], consistent with collective trends, affirming the applicability of the model.

To capture these features, various nuclear models of increasing complexity are employed. The Interacting Boson Model-1 (IBM-1) presents an algebraic framework where valence nucleon pairs are treated as $s_{l=0}$ and $d_{l=2}$ bosons [14]. The model effectively describes vibrational SU(5), γ-soft O(6), and rotational SU(3) nuclei using group-theoretical structures [15]. Its success in explaining low-lying spectra, B(E2) transitions, and shape phase transitions has made it a standard in nuclear structure theory [16, 17].

In addition to IBM-1, rotational models such as the Variable Moment of Inertia (VMI) [18] and Generalized Variable Moment of Inertia (GVMI) [19] provide valuable insights into the angular momentum-dependent behavior of nuclear systems. These models are particularly effective in describing the gradual evolution of collective motion and the variation of the moment of inertia with spin. Their application to polonium isotopes enhances our understanding of structural transitions and deviations from the idealized rigid rotor behavior, especially in transitional nuclei where both vibrational and rotational features coexist.

This study aims to:

– Determine the angular momentum and parity of some experimentally uncertain or previously unassigned energy levels in ^{190}Po and ^{192}Po using the IBM-1, VMI, and GVMI nuclear models, with a focus on structural indicators such as nuclear softness, collectivity, and deformation trends.

– Evaluate and compare the predictive accuracy of each model by examining energy spectra, ground-state-band energy ratios (e.g., $E\,(4_1^+)\,/E\,(2_1^+)$, $E\,(6_1^+)\,/E\,(2_1^+)$), and electric quadrupole moments against experimental data.

By applying these three complementary theoretical frameworks, this study aims to deepen the understanding of collective structural behavior in neutron-deficient even-even polonium isotopes. The results not only evaluate the predictive capabilities of the IBM-1, VMI, and GVMI models but also contribute to the broader effort to map symmetry evolution and deformation trends across the lead-polonium region of the nuclear chart.

2. Theoretical framework

2.1. IBM-1

The IBM-1, introduced by Arima and Iachello in the 1970s, provides a robust theoretical framework for describing the collective excitations in even-even atomic nuclei. It is especially effective in modeling the low-lying energy spectra of medium and heavy nuclei, where nucleons outside closed shells form correlated pairs. These pairs are treated as bosons, reducing the complex many-fermion nuclear system to a more tractable bosonic system governed by symmetry principles [20].

IBM-1 introduces two types of bosons:

– The s-boson with angular momentum $L = 0$, representing monopole (spherical) correlations.

– The d-boson with angular momentum $L = 2$, representing quadrupole (deformed) correlations.

These bosons are formed by coupling pairs of identical valence nucleons – either proton-proton or neutron-neutron pairs – outside a closed-shell core. The total number of bosons N is conserved and equals half the number of valence nucleons, expressed as:

$$N = n_{\mathrm{s}} + n_{\mathrm{d}}, \tag{1}$$

where n_{s} and n_{d} denote the number of s- and d-bosons, respectively, each counted from the nearest closed shell. This boson number N defines the dimensionality of the model space and serves as a key parameter for describing nuclear collectivity; higher values of N typically correspond to enhanced deformation and stronger collective behavior [21].

IBM-1 is based on the algebraic structure of the unitary group U(6), which operates in a six-dimensional boson space (one s-boson state and five d-boson magnetic substates). The dynamical behavior of a nucleus within this model is classified according to subgroup reduction chains, each corresponding to a characteristic symmetry limit [22]:

– vibrational limit SU(5):

$$\mathrm{U}(6) \supset \mathrm{SU}(5) \supset \mathrm{O}(5) \supset \mathrm{O}(3) \supset \mathrm{O}(2), \tag{2}$$

– rotational limit SU(3):

$$\mathrm{U}(6) \supset \mathrm{SU}(3) \supset \mathrm{O}(3) \supset \mathrm{O}(2), \tag{3}$$

– γ-soft limit O(6):

$$\mathrm{U}(6) \supset \mathrm{O}(6) \supset \mathrm{O}(5) \supset \mathrm{O}(3) \supset \mathrm{O}(2). \tag{4}$$

These symmetry chains define distinct energy spectra and selection rules for electromagnetic transitions [23]. Real nuclei often exhibit transitional behavior between these limits, which can be explored by adjusting the interaction strengths in the model Hamiltonian.

2.1.1. IBM-1 Hamiltonian

The energy spectrum in IBM-1 is governed by a Hamiltonian that includes boson number operators and interaction terms representing pairing, angular momentum, and multipole correlations. A widely used form is [19, 20]:

$$\hat{H} = \varepsilon n_{\mathrm{d}} + a_0\left(\hat{P}\cdot\hat{P}\right) + a_1\left(\hat{L}\cdot\hat{L}\right) +$$

$$+a_2\left(\hat{Q}\cdot\hat{Q}\right)+a_3\left(T_3\cdot T_3\right)+a_4\left(T_4\cdot T_4\right), \qquad (5)$$

where: n_d - the d-boson number operator (dimensionless); $\hat{P}$ - the monopole pairing operator; $\hat{L}$ - the total angular momentum operator in ($\hbar$); $\hat{Q}$ - the electric quadrupole operator (e·b); T_3, T_4 - the octupole and hexadecapole tensor operators, respectively; ε - d-boson energy parameter, MeV; a_0, a_1, a_2, a_3, a_4 - phenomenological parameters fitted to experimental data, MeV.

The IBM-1 formalism employs boson creation and annihilation operators for monopole (s) and quadrupole (d) bosons:

$s^\dagger$ and $d_m^\dagger$ are **creation operators** for s- and d-bosons, respectively, with magnetic substate $m = -2, -1.0, +1, +2$.

$\tilde{s}$ and $\tilde{d}_m$ are the corresponding **time-reversed annihilation operators**, defined as

$$\tilde{d}_m = (-1)^m d_{-m}, \qquad \tilde{s} = s. \qquad (6)$$

The scalar (dot) product follows angular-momentum coupling rules, e.g.

$$d_m^\dagger\cdot\tilde{d}_m = \sum_m (-1)^m d_m^\dagger d_{-m}. \qquad (7)$$

Using these conventions, the key IBM-1 operators are expressed as [19

$$n_d = d_m^\dagger\cdot\tilde{d}_m{}_2$$

$$\hat{P} = \frac{1}{2}\left(d_m^\dagger\cdot d_m^\dagger - s^\dagger s^\dagger\right)$$

$$\hat{L} = \sqrt{10}\left[d_m^\dagger\cdot\tilde{d}_m\right]^{(1)}$$

$$\hat{Q} = \left[d^\dagger\cdot\tilde{s}+s^\dagger\cdot\hat{d}\right]^{(2)} + \chi\left[d^\dagger\cdot\hat{d}\right]^{(2)}. \qquad (8)$$

Here, χ is the structure parameter controlling the degree of quadrupole deformation. It takes two values $\mp\sqrt{7/2}$: negative sign for the prolate shapes and positive sign for oblate shapes.

The quadrupole operator $\hat{Q}$ shares the same tensor structure as the electric-quadrupole transition operator $\hat{T}(E2)$ introduced in Section 2.1.2, thereby establishing a direct correspondence between the IBM-1 Hamiltonian and *E*2 transition probabilities.

Each term in the Hamiltonian reflects a specific collective property of the nucleus:

– εn_d : governs the energy cost of quadrupole excitations and reflects the deviation from sphericity. Larger ε favors spherical shapes, while smaller values enhance collectivity;

– $a_0\left(\hat{P}\cdot\hat{P}\right)$: represents boson-boson pairing correlations, important for stabilizing vibrational structure;

– $a_1\left(\hat{L}\cdot\hat{L}\right)$: encodes rotational kinetic energy, influencing level spacing in rotational bands, and is dominant in SU(3)-like nuclei;

– $a_2\left(\hat{Q}\cdot\hat{Q}\right)$: accounts for quadrupole-quadrupole interactions, crucial for describing deformation and γ-softness;

– $a_3\left(T_3\cdot T_3\right)$ and $a_4\left(T_4\cdot T_4\right)$: represent octupole and hexadecapole contributions that refine spectra in heavy or subtly deformed nuclei.

The coefficients a_0, a_1, a_2, a_3, and a_4 determine which symmetry limit best represents the nucleus. Adjusting these parameters allows the model to replicate a variety of empirical energy spectra with high accuracy [24].

In essence, IBM-1 provides an algebraic and unified framework for modeling collective excitations in nuclei, enabling both accurate reproduction of experimental spectra and theoretical insight into nuclear deformation and symmetry evolution.

2.1.2. Electric quadrupole transitions and nuclear shape

A fundamental characteristic of atomic nuclei – particularly those exhibiting collective behavior – is their capacity to undergo electromagnetic transitions between energy levels. Within the IBM-1 framework, such transitions are described by the electric quadrupole operator $\hat{T}(E2)$, which governs *E*2-type transitions (i.e., transitions involving a change in angular momentum of $\Delta L = 2$).

The general form of the electric quadrupole transition operator in IBM-1 is defined as [25, 26]:

$$\hat{T}(E2) = \alpha_2\left[\hat{d}^\dagger\cdot\tilde{s}+\hat{s}^\dagger\cdot\tilde{d}\right]^{(2)} + \beta_2\left[\hat{d}^\dagger\cdot\tilde{d}\right]^{(2)}, \qquad (9)$$

where: $\hat{s}^\dagger$, $\hat{d}^\dagger$ - creation operators for s- and d-bosons, respectively; $\tilde{s}$, $\tilde{d}$ - annihilation operators in the time-reversed representation; α_2 and β_2 - effective boson charges that scale the s-d and d-d coupling strengths. The superscript (2) signifies that the operator transforms as a second-rank tensor under rotations.

This operator consists of two physically distinct components:

– the first term $\left[\hat{d}^\dagger\cdot\tilde{s}+\hat{s}^\dagger\cdot\tilde{d}\right]^{(2)}$ dominates in vibrational or weakly deformed nuclei, where collec-

tivity arises primarily from interactions involving both monopole (s-boson) and quadrupole (d-boson) excitations;

– the second term $\left[\hat{d}^{\dagger}\cdot\tilde{d}\right]^{(2)}$ becomes increasingly significant in well-deformed, rotational nuclei, where the collectivity is governed by quadrupole dynamics within the d-boson subspace.

The reduced transition probability, B(E2) ($e^2\cdot b^2$), quantifies the probability of an electric quadrupole transition between an initial state L_i and a final state L_f and is given by [27]:

$$B\left(E2;L_i\to L_f\right)=\frac{1}{2L_i+1}\left|\left\langle L_f\left|\hat{T}\left(E2\right)\right|L_i\right\rangle\right|^2. \quad (10)$$

A large B(E2) value signifies strong collectivity and coherent quadrupole motion of valence nucleons, while a small value suggests weak collectivity or a single-particle nature.

Furthermore, the quadrupole moment Q_L, is another essential observable, providing insight into the intrinsic nuclear shape. It is extracted from the diagonal matrix element of the quadrupole operator [28]:

$$Q_L=\left\langle L,M_L=L\left|\sqrt{\frac{16\pi}{5}}\hat{T}\left(E2\right)\right|L,M_L=L\right\rangle. \quad (11)$$

This matrix element evaluates the expectation value of the $E2$ operator in a state with maximal magnetic projection ($M_L = L$). The sign and magnitude of Q_L offer crucial insight into the equilibrium shape of the nucleus:

– if $Q_L > 0$: prolate (rugby-ball shaped nucleus);

– if $Q_L < 0$: oblate (disc-like configuration);

– if $Q_L \approx 0$: spherical or transitional shape, often associated with shape coexistence.

These shape indicators, alongside B(E2) values, are extensively used in IBM-based structure studies to evaluate the validity of the model and deduce the dynamical symmetry (SU(5), SU(3), or O(6)) of a given nucleus.

By fitting α_2 and β_2 to experimental transition data, one can extract quantitative measures of nuclear deformation and transition strengths across isotopic chains. This establishes $\hat{T}\left(E2\right)$ not only as a computational tool but also as a window into the underlying shape evolution and collective dynamics of atomic nuclei.

2.2. VMI models

While the IBM-1 offers a powerful framework for describing collective excitations in even-even nuclei, it assumes a constant moment of inertia. This simplification becomes insufficient for transitional or weakly deformed nuclei, particularly those near closed shells, where the energy spectra often exhibit deviations from rigid rotational behavior at higher angular momenta.

To address this limitation, two phenomenological models – the VMI model and its refined form, the GVMI model – were developed [29]. These models introduce an angular momentum-dependent moment of inertia, offering improved accuracy in reproducing rotational spectra and capturing gradual structural changes associated with increasing spin.

2.2.1. GVMI energy expression and nuclear softness

The VMI model has the assumption of a rotating energy dependence where the moment of inertia slowly changes with the angular momentum. It is described by two parameters: the zero-spin moment of inertia ϑ_0 and the constant of stiffness (compressibility) C, which is the measure of the opposition of the nucleus to shape changes during the rotation.

The GVMI model expands this model by adding another parameter – 2. This parameter adjusts the functional form of the moment of inertia, increasing the degree of flexibility with which the model can explain the non-rigid-rotor behavior of the model, particularly in transitional or γ-soft nuclei.

Concretely, the GVMI formalism may be written as:

$$\vartheta\left(L\right)=\sqrt{\vartheta_0{}^2+Y\cdot L\left(L+1\right)}.$$

So that the total rotational energy in this model is given by [30]:

$$E\left(L\right)=\frac{L\left(L+1\right)}{2\vartheta\left(L\right)}+C\frac{1}{2}\left[\vartheta\left(L\right)-\vartheta_0\right]^2, \quad (12)$$

where L is the nuclear angular momentum ($\hbar$), $\vartheta(L)$ is the moment of inertia at spin L in $\hbar^2$/MeV, ϑ_0 is the ground-state moment of inertia in $\hbar^2$/MeV, and C is the stiffness parameter in $\mathrm{MeV}^{-1}\cdot\hbar^{-4}$, representing the resistance of the nucleus to deviations from its equilibrium moment of inertia. The parameter Y is then chosen so that the moment of inertia can vary with L in a non-linear (square root) manner and hence describe softness or rearrangements of the nuclear structure as it spins.

The first term models the rotational kinetic energy with a variable inertia, while the second introduces a restoring force penalizing large deformations. This formulation allows the GVMI model to describe anharmonic behavior in nuclei undergoing shape transitions or structural softening.

A central quantity derived from this model is the nuclear softness parameter δ, which quantifies the degree of deformation flexibility [30]:

$$\delta = \frac{1}{2C\vartheta_0^3}. \quad (13)$$

– A large δ indicates a soft or transitional nucleus, where the moment of inertia increases noticeably with spin.

– A small δ reflects a rigid rotor, maintaining a nearly constant inertia across angular momenta.

By fitting the parameters C and ϑ_0 to experimental spectra, one can extract δ and characterize the shape evolution and collective response of the nucleus under rotation [29].

2.3. Statistical validation of model accuracy

To quantitatively compare theoretical predictions with experimental spectra, the chi-square deviation (χ^2) (MeV^2) is used [31]:

$$\chi^2 = \frac{\left(E_{\text{cal}}(i) - E_{\text{exp}}(i)\right)^2}{\left(E_{\text{cal}}(i)\right)^2}, \quad (14)$$

where $E_{\text{exp}}(i)$ is the experimental energy of the i^{th} level, $E_{\text{cal}}(i)$ is the corresponding calculated energy, and N is the number of energy levels considered.

A lower χ^2 value reflects a better agreement between theory and experiment, and thus a more reliable parameter set.

3. Results and discussion

In this section, the theoretical energy levels and structural characteristics of the even-even polonium isotopes $^{190}_{84}\text{Po}_{106}$ and $^{192}_{84}\text{Po}_{108}$ are examined using three nuclear models: the Interacting IBM-1, the VMI model, and the GVMI model. These models were employed to reproduce experimental energy spectra, extract structural indicators, and explore deformation effects associated with collective behavior and angular momentum evolution.

3.1. Energy level computations via IBM-1

The IBM-1 was employed to calculate the low-lying collective excitation levels for the even-even polonium isotopes $^{190}_{84}\text{Po}_{106}$ and $^{192}_{84}\text{Po}_{108}$. Calculations were performed using the FN95-Plato computational suite, which allows for the numerical diagonalization of the IBM-1 Hamiltonian with user-defined parameters through an input file (pos1.inp) [32]. The theoretical energy spectra were subsequently compared with available experimental data to validate model accuracy and to assess the underlying nuclear structure.

In IBM-1, the nucleus is treated as a system of interacting s-bosons ($L = 0$) and d-bosons ($L = 2$) representing correlated valence nucleon pairs outside closed shells. The total boson number $N = n_s + n_d$ was derived from the number of valence protons and neutrons, calculated relative to the doubly magic ^{208}Pb core. For ^{190}Po and ^{192}Po, the valence proton number $N_\pi = 1$ remains constant, while the valence neutron numbers $N_\nu = 10$ and $N_\nu = 9$, respectively, determine the bosonic configuration space.

To achieve optimal agreement with empirical energy levels, a systematic parameter fitting procedure was conducted. The Hamiltonian parameters were fine-tuned to minimize the deviation between calculated and experimental levels, particularly in the ground-state band. The selected values for the strength constants governing the various boson-boson interactions (monopole, quadrupole, pairing, octupole, and hexadecapole) are presented in Table 1.

Table 1. **Optimal IBM-1 parameters for ^{190}Po and ^{192}Po (all energies in MeV)**

Isotope	N_π	N_ν	N	ε	a_0	a_1	a_2	a_3	a	χ	SO(6)
^{190}Po	1	10	11	0.0470	0.0012	0.0132	0.0290	0.0011	0.0030	0.0201	0.0151
^{192}Po	1	9	10	0.0360	0.0028	0.0180	0.0240	0.0011	0.0030	0.0201	0.0151

The calculated energy levels showed good agreement with experimental values up to spin $L = 10\ \hbar$, with deviations typically less than a few hundred keV. This level of agreement validates the IBM-1 assumption that the nuclear structure in these isotopes is predominantly collective and can be effectively modeled using bosonic degrees of freedom (see Figs. 1 and 2).

Furthermore, the IBM-1 model predicts additional excited states beyond the experimentally confirmed levels. These predictions can guide future experimental nuclear spectroscopy studies aimed at discovering new transitions and refining the structural systematics of neutron-deficient Po isotopes.

Notably, the results suggest that ^{190}Po exhibits more pronounced quadrupole collectivity than ^{192}Po, as indicated by its lower fitted ε and higher quadrupole interaction coefficient a_2. This finding aligns with expectations, given the slightly larger number of valence neutrons in ^{190}Po, which enhances collective behavior.

3.2. Energy predictions using VMI and GVMI models

To complement the bosonic approach of IBM-1 and to gain further insight into the rotational structure and deformation properties of the studied nuclei, the VMI and GVMI models were employed. These models are particularly effective in capturing the non-rigid rotational behavior and angular momentum-dependent structural changes often observed in transitional and soft nuclei.

The VMI model assumes a rotational energy dependence that accounts for a gradually changing moment of inertia as the angular momentum increases. This model incorporates two main parameters: the zero-spin moment of inertia ϑ_0 and a compressibility constant C, which quantifies how the nuclear shape responds to increasing rotation. GVMI extends this framework by adding parameter Y, which enhances the ability of the model to describe deviations from ideal rigid rotor behavior, especially in nuclei near closed shells or those exhibiting γ-softness.

Parameter optimization was performed using dedicated computational routines – VMI.FOR and GVMI.FOR – which employed a least-squares fitting approach to minimize the chi-square deviation between calculated and experimental energy levels. The optimized parameters for each isotope and associated statistical indicators are summarized in Table 2.

Table 2. **Optimized model parameters and statistical fitting results for VMI and GVMI**

Isotope	Band	Model	ϑ_0, MeV^{-1}	C, MeV^3	E_k, MeV	Y, MeV	δ	Δ	χ^2
^{190}Po	g	VMI	8.490	0.000999	0.000	–	0.8179	0.0358	0.0071
		GVMI	8.830	0.099992	0.000	0.0994	0.0116	0.0096	0.0008
^{192}Po	g	VMI	3.0003	0.000989	0.000	–	18.7182	0.0404	0.0061
		GVMI	7.372	0.019977	0.000	0.0999	0.0200	0.0112	0.0006

From Table 2, it is evident that the GVMI model provides a markedly better agreement with experimental data than the classical VMI model, as reflected in both the lower standard deviation and significantly reduced chi-square values. This is particularly noticeable in the case of ^{192}Po, a nucleus close to the neutron shell closure ($N = 108$), where the deformation is expected to be soft and the rotational alignment effects more subtle.

The nuclear softness parameter δ, calculated using Eq. (10), offers physical insight into the degree of rigidity in the rotational response of the nucleus. Physically, the δ values within the range of about 10^{-3} to 10^{-1} are realistic degrees of softness as usually seen in transitional or moderately deformed nuclei. Those values that are many times greater than unity indicate an unrealistically great sensitivity of the moment of inertia to angular momentum, which is essentially an implication that the nucleus should act as a fluid with zero resistance to deformation, non-empirical behaviour in even-even nuclei [30, 33].

For ^{192}Po, VMI predicts a very large $\delta = 18.7182$, which is therefore considered physically unreasonable. A very large value would mean that the nucleus is unrealistically soft, and the moment of inertia would grow too rapidly with the spin. The GVMI model, by contrast, makes this description more refined: $\delta = 0.0200$, a physically acceptable value, corresponding to a nucleus that is soft and yet has a finite rigidity.

Similarly, for ^{190}Po, GVMI provides a more moderate and realistic value of $\delta = 0.0116$, aligning with expectations for a heavier, more collective isotope with enhanced quadrupole correlations due to a larger number of valence neutrons.

Physically, the improvement offered by GVMI can be attributed to its inclusion of the Y parameter, which introduces an additional term to account for deviations in the moment of inertia due to microscopic structural changes, such as the onset of nucleon alignment or pairing breaking at moderate angular momenta. This added flexibility enables the model to more accurately reproduce not just the ground-state band energies but also the curvature and inflection patterns observed in the experimental spectra.

3.3. Energy level schemes for ^{190}Po and ^{192}Po

The energy level diagram for the even-even nucleus ^{190}Po was constructed up to spin-parity 20^+ using the IBM-1, VMI, and GVMI models, with reference to available experimental data [34], as shown in Fig. 1. This expanded representation enables a deeper assessment of how each model captures the collective structure across a wide angular momentum range.

Up to the 14^+ state, where experimental data are accessible, the IBM-1 model exhibits strong agreement with measured levels – most notably at the 6^+) state, where the IBM-1 prediction (0.876 MeV) nearly coincides with the experimental value (0.901 MeV). The VMI model slightly overestimates this level at 0.930 MeV, while the GVMI model demonstrates excellent conformity with an identical prediction of 0.902 MeV, affirming its effectiveness in the mid-spin region.

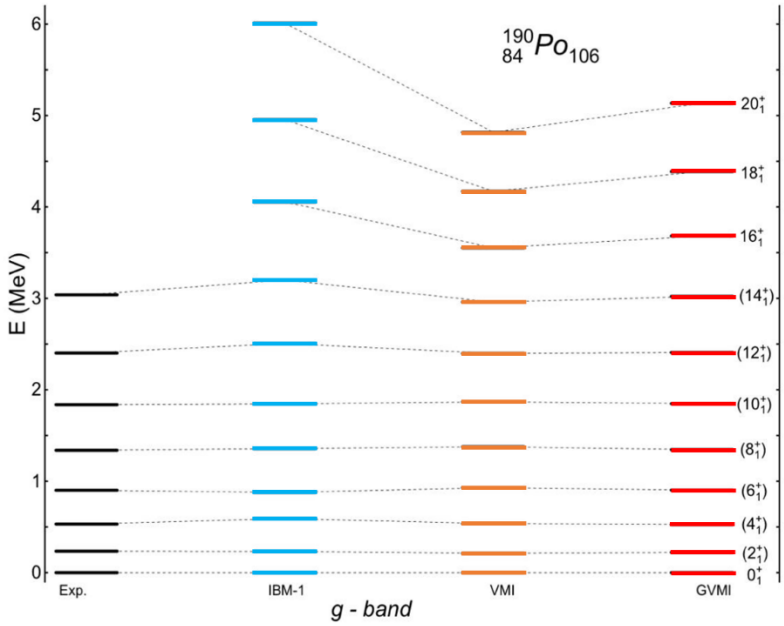


Fig. 1. Energy level diagram for ^{190}Po. Experimental data (*black*) [34] and predictions from IBM-1 (*blue*), VMI (*orange*), and GVMI (*red*) models. The levels correspond to the ground-state band transitions from 0^+ to 20^+, providing a direct visual comparison of model fidelity to empirical structure. (See color Figure on the journal website.)

At higher angular momentum, where experimental data are lacking, the divergence among theoretical models becomes more pronounced. For instance, at the (16^+) state, IBM-1 predicts an energy of 4.05 MeV – substantially higher than the VMI result (3.55 MeV) – while the GVMI prediction (3.68 MeV) remains closely aligned with IBM-1, suggesting that GVMI more accurately captures the nonlinear increase in level spacing associated with evolving nuclear deformation.

Overall, the IBM-1 and GVMI models provide consistently reliable descriptions of the ^{190}Po level scheme in the low- and mid-spin regimes. At higher spins, GVMI's incorporation of nuclear softness and deformation compressibility enables a more realistic representation of collective structure, reinforcing its suitability for transitional nuclei. These findings support the classification of ^{190}Po within the O(6) dynamical symmetry limit.

The energy level diagram for the even-even isotope ^{192}Po was constructed up to spin-parity (20^+) using predictions from the IBM-1, VMI, and GVMI models, as shown in Fig. 2. These theoretical results were compared with available experimental data [35] up to 14^+, providing a comprehensive evaluation of collective excitations and symmetry behavior across increasing angular momentum.

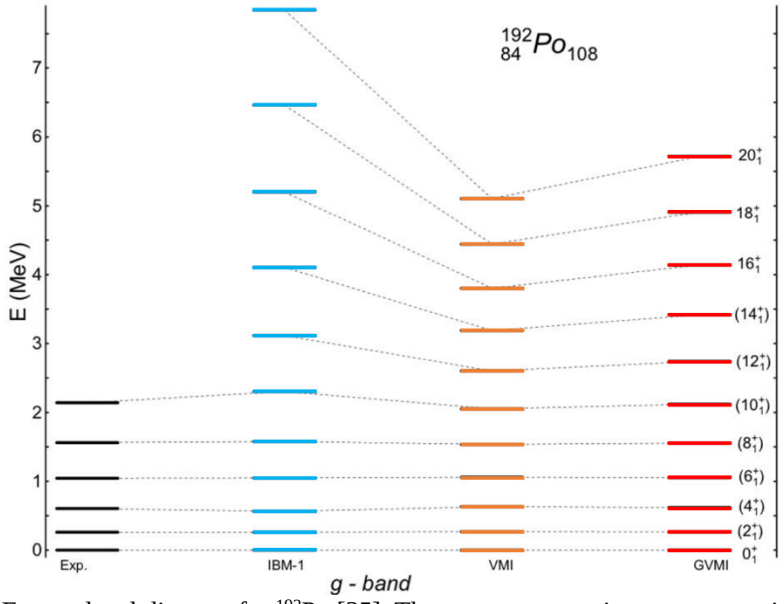


Fig. 2. Energy level diagram for ^{192}Po [35]. The same comparative structure as in Fig. 1. (See color Figure on the journal website.)

For the lower-lying states, all three models demonstrate acceptable consistency with experimental values. At the (4^+) level, for example, the IBM-1 prediction (0.564 MeV) lies within ~0.04 MeV of the experimental value (0.605 MeV), while the VMI and GVMI predictions (0.632 MeV and 0.618 MeV, respectively) show similar proximity, affirming their reliability in the low-spin regime. This agreement continues at the (6^+) state, where IBM-1 (1.047 MeV), VMI (1.058 MeV), and GVMI (1.051 MeV) closely replicate the experimental energy of 1.043 MeV, differing by less than 0.02 MeV in all cases.

The overall pattern indicates that ^{192}Po exhibits stronger collectivity than ^{190}Po, consistent with its placement further from the closed neutron shell (N = 126). The GVMI model, in particular, proves effective in capturing this enhanced deformation, especially in the upper-spin region. These findings reinforce the transitional nature of ^{192}Po and underscore the importance of employing angular momentum-dependent moment of inertia models for such nuclei.

3.4. Energy level ratio analysis results

The comparative analysis of the energy ratios $E(4^+)/E(2^+)$, $E(6^+)/E(2^+)$, and $E(8^+)/E(2^+)$ for the isotopes ^{190}Po and ^{192}Po, as shown in Fig. 3, offers critical insight into their underlying collective structures and symmetry characteristics. As illustrated in the bar chart, the experimental values for ^{190}Po exhibit a ratio progression of approximately 2.27, 3.85, and 5.72, respectively. These values align most closely with the theoretical expectations of the O(6) dynamical symmetry limit within the IBM-1, indicative of a γ-soft nuclear shape. The O(6) symmetry, corresponding to a soft potential in the γ degree of freedom, typically manifests in nuclei that are transitional between vibrational and rotational regimes.

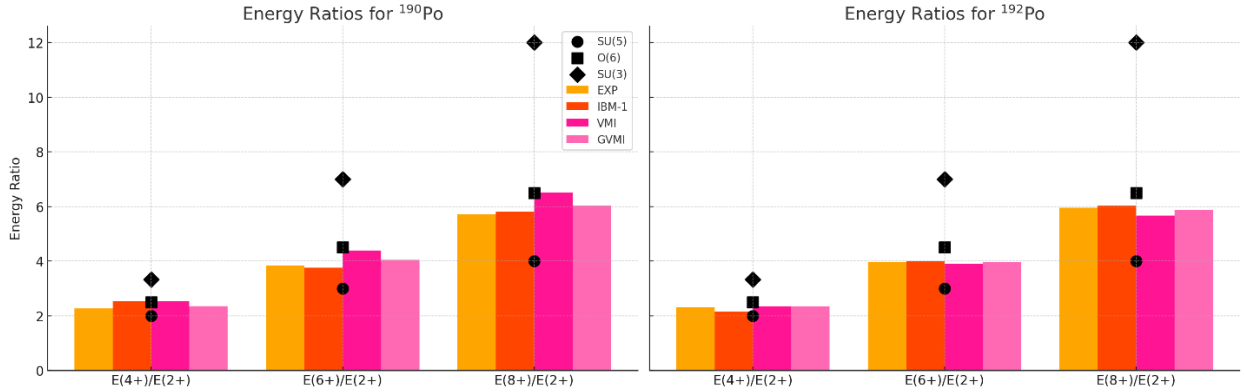


Fig. 3. Comparison of experimental [34, 35] and calculated $E(4^+)/E(2^+)$, $E(6^+)/E(2^+)$, and $E(8^+)/E(2^+)$ energy ratios with the ideal U(5), O(6), and SU(3) dynamical symmetry limits [36] for the even-even isotopes ^{190}Po and ^{192}Po. (See color Figure on the journal website.)

Model-based predictions further corroborate this classification. The IBM-1 model achieves close agreement with experimental data, particularly at lower spin states, while modest deviations emerge at higher spins. The GVMI model, incorporating deformation-dependent softness parameters, improves the description of higher-spin energy levels, reflecting enhanced centrifugal stretching effects not captured in simpler models. The VMI model, although less flexible, still maintains reasonable agreement with experimental observations, but tends to overestimate energy ratios at higher angular momentum states. Collectively, these model results reinforce the suitability of the O(6) limit in characterizing the low-lying spectrum of ^{190}Po.

For ^{192}Po, the observed energy ratios – 2.31, 3.98, and 5.96 – suggest a slight enhancement in collectivity relative to ^{190}Po. This evolution points to a progressive structural transition within the even-even polonium isotopic chain. Despite the increase in collectivity, the ratios remain well below the rotational SU(3) limit and deviate significantly from the vibrational SU(5) expectations, thereby reinforcing the dominance of O(6)-like behavior in both nuclei. Once again, the IBM-1 and GVMI models successfully reproduce the experimental data, with GVMI offering marginally improved performance at higher spins. This suggests that while the collective behavior intensifies in ^{192}Po, the γ-soft structural character is retained.

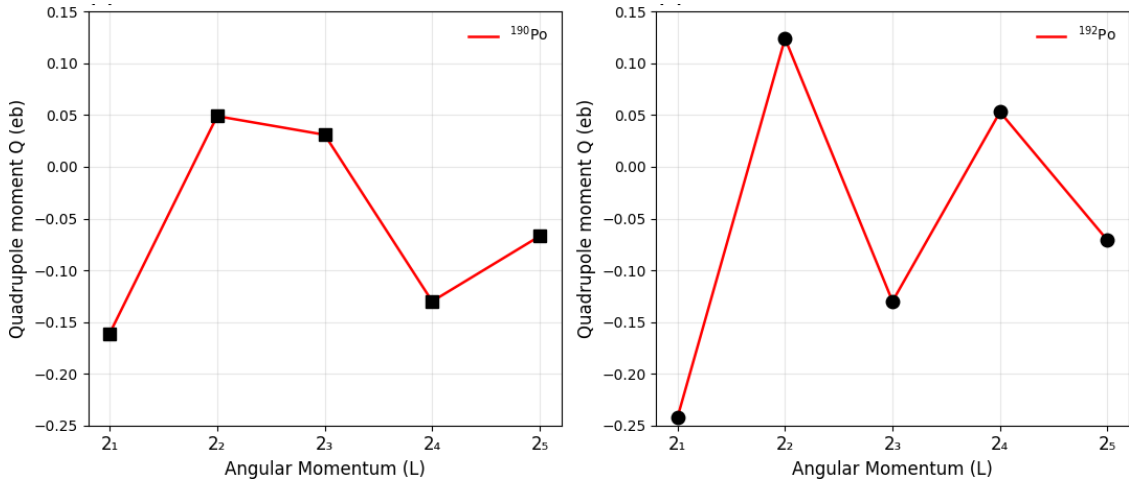
Notably, the symmetry limit markers – SU(5), O(6), and SU(3) – plotted using distinctive symbols (circle, square, and diamond, respectively), serve as visual benchmarks against which the model predictions and experimental values can be directly compared. Both isotopes fall in close proximity to the O(6) symmetry line, supporting previous interpretations based on collective energy systematics and IBM-1 predictions. The results suggest that neither ^{190}Po nor ^{192}Po exhibits purely vibrational SU(5) or

rotational SU(3) character; rather, they occupy a transitional region. In particular, ^{192}Po demonstrates a slightly enhanced degree of collectivity and deformation relative to ^{190}Po, consistent with its gradual evolution toward more pronounced γ-soft behavior.

In summary, the graphical comparison of energy ratios affirms the relevance of the O(6) dynamical symmetry in describing the collective structure of both ^{190}Po and ^{192}Po. The success of the IBM-1 and GVMI models in capturing the experimental trends underscores their effectiveness in modeling transitional nuclei. This analysis supports the broader hypothesis of structural evolution along the polonium isotopic chain and provides a cohesive framework for interpreting related observables, such as energy level spacings and electric quadrupole moments.

3.5. Quadrupole moment results

The presented plot (Fig. 4, *a*) depicts the variation of the electric quadrupole moment Q as a function of angular momentum L for the isotope ^{190}Po, as predicted by the IBM-1. The five plotted points correspond to the quadrupole moments Q_{2i} of the low-lying 2^+ states, denoted from 2_1^+ to 2_5^+. To obtain a clear picture of the development of Q as a function of L, the calculated values were plotted using straight lines. Such a representation is a fair representation of the observed non-monotonic fluctuations in Q as were observed without the artificial extrema it introduces.



a *b*

Fig. 4. Electric quadrupole moments Q as a function of angular momentum $L(\hbar)$ for the isotope ^{190}Po (*a*) and ^{192}Po (*b*). The calculated IBM-1 values are connected in lines to emphasize the overall trend in nuclear shape evolution. (See color Figure on the journal website.)

The trend demonstrates an initial rise in the quadrupole moment from a negative value at 2_1^+, reaching a maximum positive value at 2_2^+, before declining through 2_3^+ and 2_4^+, followed by a slight recovery at 2_5^+. This alternating pattern in Q-values suggests a dynamic shape evolution in which the nucleus exhibits oblate deformation at low spins, transitions to a more prolate-like configuration near 2_2^+, and then undergoes a gradual reversal. The observed oscillatory nature in quadrupole moment behavior points to potential shape coexistence or gamma-soft characteristics, which are consistent with the O(6) dynamical symmetry limit of the IBM framework.

Overall, the analysis of the quadrupole moment distribution serves as a sensitive probe for identifying the underlying symmetry and deformation features in transitional nuclear systems.

The electric quadrupole moment Q provides critical insights into the deformation and collective behavior of atomic nuclei. For the ^{192}Po isotope, the present calculated work of quadrupole moments exhibits a pronounced oscillatory trend as the spin increases from 2_1^+ to 2_5^+. Initially, the moment is strongly negative at 2_1^+, indicating a predominant prolate deformation. This is followed by a positive peak at 2_2^+, suggesting a shift toward oblate-like or γ-soft deformation. The moment sharply reverses and reaches a minimum at 2_3^+, reaffirming dynamic shape changes that could be attributed to vibrational mixing or transitional behavior in the collective motion of the nucleus. The return to less negative and then slightly positive values at 2_4^+ and 2_5^+, respectively, hints at partial restoration of a more symmetric charge distribution.

Such a sinusoidal pattern in $Q(J)$ reflects a nuclear structure that does not conform rigidly to a simple axial rotor or vibrator but rather demonstrates γ-softness, where the energy surface is flat with respect to triaxial deformation. This interpretation is

consistent with the O(6)-like symmetry observed in the energy ratio systematics (e.g., $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$, etc.), reinforcing the notion of a γ-soft collective regime in ^{192}Po.

When juxtaposed with ^{190}Po, key differences emerge in the evolution of quadrupole moments. In ^{190}Po, the present calculated work of IBM-1 predicted quadrupole moments showed a more moderate oscillation with smaller amplitude fluctuations between the states 2_2^+ to 2_5^+. While both isotopes start with negative quadrupole moments at 2_1^+, indicative of prolate deformation, the amplitude of deformation in ^{192}Po is significantly larger, suggesting a greater degree of collectivity or softness in the nuclear potential energy surface. Moreover, the sharper reversals in the ^{192}Po sequence, particularly the extremum at 2_3^+, point to enhanced dynamic effects – possibly due to a higher number of valence nucleons or enhanced proton-neutron interactions as the isotope becomes more neutron-rich. These characteristics imply that ^{192}Po occupies a more transitional zone in the IBM-1 phase space between vibrational and γ-soft regimes, whereas ^{190}Po leans closer to a more vibrationally dominated structure.

3.6. Structural analysis with model evaluation

The combined interpretation of energy level patterns and collective energy ratios strengthens the comparative evaluation of the IBM-1, VMI, and GVMI models presented in Section 3.6. The IBM-1 model demonstrates a strong capacity to reproduce the level spacings and energy ratio trends observed in ^{190}Po, which exhibit signatures consistent with a γ-soft or O(6)-like symmetry. This reinforces IBM-1's effectiveness in describing nuclei with modest deformation and well-developed quadrupole collectivity. In contrast, the GVMI model offers greater adaptability in accounting for the structural evolution seen in ^{192}Po, particularly the gradual increase in energy ratios indicative of enhanced deformation. The model's incorporation of a softness parameter (δ) and deformation compressibility term (*Y*) enables it to capture subtle changes in nuclear moment of inertia and shape dynamics more effectively than the classical VMI framework.

Although the VMI model provides a useful reference for spherical or weakly deformed systems, its simplified formalism underestimates the complexity of structural transitions in transitional nuclei such as ^{192}Po. These distinctions are quantitatively reflected in the statistical comparisons provided in Table 2, where the GVMI model yields the lowest chi-square and standard deviation values, affirming its superior predictive accuracy. Overall, the collective insights from energy level reproduction and quadrupole moment behavior establish a well-founded model hierarchy, with GVMI emerging as the most versatile for describing even-even polonium isotopes near the mid-shell region. This integrated assessment offers a solid basis for guiding future theoretical modeling and spectroscopic studies in heavy nuclei.

The IBM-1 model is capable of calculating many nuclear properties for stable even–even nuclei, particularly those of medium and light mass. Its success for heavy nuclei, however, is rather limited. All polonium isotopes are heavy nuclei, and most are unstable and lack a number of experimentally determined nuclear properties, most importantly, measured energy levels. This scarcity explains the limited theoretical studies of these isotopes. In the present study, we were able to compute several nuclear properties that have not been previously reported. The VMI and GVMI models, by contrast, succeeded in reproducing all energy levels theoretically and in predicting many nuclear observables that IBM-1 could not, because the IBM-1 formalism does not distinguish between proton bosons and neutron bosons. Among the phenomenological models, GVMI provided the best overall theoretical agreement with experiment.

All three nuclear models give their closest agreement with experimental data at low-lying levels; however, a clear divergence appears beginning at angular momentum $J = 12\ \hbar$.

4. Conclusion

This study demonstrates that the three nuclear models – IBM-1, VMI, and GVMI – achieved a satisfactory level of accuracy in describing the collective behavior of the even-even polonium isotopes ^{190}Po and ^{192}Po. A good agreement was observed between the calculated and experimental energy levels and transition energies as functions of angular momentum, confirming the reliability of the theoretical approaches employed. Dynamical symmetry analysis, based on characteristic energy ratios $E(4^+)/E(2^+)$, $E(6^+)/E(2^+)$, and $E(8^+)/E(2^+)$, revealed that the isotope ^{190}Po is best described by the O(6) γ-soft limit, while ^{192}Po exhibits transitional features between SU(5) and O(6) symmetries. This finding provides a clear indication of symmetry evolution within the polonium isotopic chain. Moreover, theoretical calculations of the electric quadrupole moment Q_l for the 2^+ states offer insight into the intrinsic shapes of these nuclei. For ^{190}Po, oblate deformation (flattened spheroid) was identified at the 2_1^+, 2_4^+, and 2_5^+ states, while prolate deforma-

tion (elongated spheroid) appeared at the 2_2^+ and 2_3^+ states. Similarly, ^{192}Po displayed oblate shapes at the 2_1^+, 2_3^+, and 2_5^+ states, and prolate deformation at the 2_2^+ and 2_4^+ states. Notably, these quadrupole moments were derived exclusively from theoretical modeling, as experimental values for these observables remain unavailable. Overall, the results contribute to a deeper understanding of nuclear collectivity and shape coexistence in medium-heavy nuclei and validate the effectiveness of algebraic and phenomenological models in describing the structural evolution of polonium isotopes.

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ЯДЕРНА СТРУКТУРА ТА ЕВОЛЮЦІЯ СИМЕТРІЇ В $^{190,192}_{84}$Po : ВИВЧЕННЯ З МОДЕЛЯМИ IBM-1, VMI ТА GVMI

У цій роботі три ядерні моделі – модель взаємодіючих бозонів-1 (IBM-1), модель змінного моменту інерції (VMI) та її узагальнена форма (узагальнений змінний момент інерції, GVMI) – було використано для дослідження низькорозташованих колективних станів парно-парних ізотопів полонію ^{190}Po та ^{192}Po. Моделі було використано для обчислення рівнів енергії з позитивною парністю $E(L)$, енергій переходів $E\gamma$, приведених ймовірностей електричних квадрупольних переходів $B(E2)$ та електричних квадрупольних моментів Q_L на основі доступних експериментальних даних та схем ідеальних рівнів. Характерні енергетичні співвідношення $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$ та $E(8_1^+)/E(2_1^+)$ було використано для оцінки динамічної симетрії ядер та порівняно з теоретичними межами в групах симетрії SU(5), SU(3) та O(6). Результати показують, що ^{190}Po проявляє домінантну γ-м'яку симетрію O(6), тоді як ^{192}Po демонструє перехідну поведінку між SU(5) та O(6). Порівняльний аналіз показав гарну відповідність між експериментальними даними та теоретичними прогнозами, зокрема з IBM-1 та GVMI, що підтверджує їхню надійність у моделюванні структурної еволюції в ядрах середньої важкості. Крім того, теоретичні рівні енергії були екстрапольовані для високоспінових станів, які ще не спостерігалися експериментально. Для ^{190}Po, IBM-1 прогнозує стан 16_1^+ при 4,06 МеВ, стан 18_1^+ при 4,96 МеВ та стан 20_1^+ при 6,01 МеВ. Для ^{192}Po передбачені рівні включають 12_1^+ при 3,12 МеВ, 14_1^+ при 4,11 МеВ та 16_1^+ при 5,20 МеВ. Ці передбачення розширюють відомі спектри до спіну 20^+ та пропонують теоретичну основу для майбутньої експериментальної перевірки. Ці результати сприяють глибшому розумінню колективних збуджень та динаміки симетрії в ізотопному ланцюзі полонію.

Ключові слова: IBM-1, VMI, GVMI, ^{190}Po, ^{192}Po, енергетичні рівні, динамічна симетрія.

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