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PROTON ELECTROMAGNETIC FORM FACTOR AND RADIUS EXTRACTED FROM ELASTIC PP SCATTERING AT $\sqrt{s} \approx 7, 8$, AND 13 TeV USING THE CHOU-YANG MODEL

Chou-Yang model has been used to obtain the electromagnetic form factor and the root mean square (rms) radius of the proton, using experimental data for proton-proton elastic scattering at $\sqrt{s} \approx 7$, 8, and 13 TeV. The differential cross-section data at low squared four-momentum transfer |t| is fitted to a single exponential function to extract the form factor at the aforementioned center of mass energies. Extracted electromagnetic form factors are used for the prediction of rms radius of the proton. A comparison of electromagnetic form factor and rms charge radius at the different centers of mass energies truly reflects the fact that our results agree well with the experiment and theory. Predicted values of rms radius of the proton confirm its energy-independent nature.

Keywords: Chou-Yang model, elastic scattering of protons, root mean square radius of the proton, the electromagnetic form factor of the proton.

1. Introduction

Hadronic studies play an important role in discovering the most basic constituents of matters and the most fundamental laws obeyed by these constituents. Among hadrons, protons are strongly interacting particles. It is believed that 99.9 % of visible matter in the universe is made up of protons and neutrons. The properties of protons have been under investigation for the last 100 years [1]. The structure of a proton requiring precise knowledge of its the root mean square (rms) radii is an underlying problem in non-perturbative quantum chromo dynamics (QCD) [2]. Two techniques are commonly used to measure the rms radius of the proton: hydrogen spectroscopy, including both ordinary and muonic hydrogen, and electron-proton scattering [3]. The muonic hydrogen spectroscopy [4] and electron-proton (fixed target) scattering [5] were incompatible and gave rise to proton radius controversy [3]. However, the radius of the proton measured through recent ordinary hydrogen spectroscopy [6] is consistent with the electron-proton scattering but is in disagreement with muonic hydrogen spectroscopy [7] which triggers the proton radius puzzle. Several experimental groups at CERN plan to study the internal structure of the proton by analyzing high-energy elastic scattering data, such as that from TOTEM [8 - 10].

Scattering experiments provide insight into the internal structure of hadrons [11]. Besides experimental techniques, theoretical studies exploring the structure of protons are also concerned with the calculation of the form factor and rms radii of protons [12 - 16]. There are two main approaches for studying the scattering of hadrons; an algebraic approach which takes into account the algebraic approach of scattering particles and a geometrical approach in which hadrons are drawn like spherical objects with their internal structure approaching each other [17]. Comparing both these approaches, the Geometric approach is easy to follow as compared to dealing with complex algebra. Chen Yin Yang and his collaborators considered the geometric approach [17] and proposed the Chou-Yang model (CYM) [18] for the interaction of hadrons. In CYM distribution of matter is considered to be identical to the charge distribution inside the hadrons [17, 19]. In addition to this total cross-section is linked to the shape of colliding particles and the square of the radius of colliding particles (Geometric model) [17]. Chou-Yang identified the structure of the proton by using pp elastic scattering data [12]. CYM successfully describes the elastic scattering of hadrons particularly in the diffraction region [15]. The predicted values of form factors and charge radii through CYM agree well with other models such as MIT bag model [13], self-consistent model [14], and experimental data [6, 20].

Moreover, S. Zahra et al. [11] calculated the rms radius of the proton (0.90 and 0.91 fm) for 13 and 4.3 transverse beam distance σ_{Beam} at CM energy = 2.76 TeV employing CYM. H. Zahra et al. [21] in 2011 calculated the radius of hadrons by using the Generalized Chou-Yang Model (GCYM).

In this work form factor and rms radii of proton are calculated by using the simplest version of CYM. Accurate knowledge of the charge radii of proton and its form factor is necessary for understanding the quark-gluon structures for nucleons and lighter nuclei (e.g. Hydrogen nuclei consisting of proton) [2, 20, 22]. The differential cross-section data of the elastic scattering is used to extract the form factors of the proton. The form factors are crucial to probe the electromagnetic structure of protons. These electromagnetic form factors will also provide a perfect basis for testing the QCD [1]. In addition to this, precise measurement of rms radii will also increase the accuracy of transition measurement for hydrogen atoms consequently the precision of fundamental constants will be enhanced.

The elastic scattering data is taken from the TOTEM experiment [23 - 25]. In elastic scattering processes, measurable are total cross-sections (σ) and differential cross-section $\frac{d\sigma}{dt}$. The total cross-section of two colliding particles depends on a single variable in CYM it is the square of total energy called the center of mass (CM) energy [17]. Differential cross-section approximation can be carried out by fitting differential cross-section data to a single Gaussian [17]. For this particular case, the differential cross-section is:

$$\frac{d\sigma}{dt} = a\,e^{bt}. \tag{1}$$

Energy dependence is considered to be lying in parameter a, which is obtained from the fitting of differential cross-section data against a four-momentum transferred square [26]. The product of form factors for elastically scattered hadrons A and B is given as:

$$F_A(t)F_B(t) = Constant\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{a}{\pi}\right)^{\frac{n}{2}}\left(\frac{1}{b}\right)^{n}\frac{b}{n}e^{\frac{bt}{3n}}. \tag{2}$$

The radius of a hadron is linked to the form factor of a hadron by the formula:

$$F(t) = 1 - \frac{1}{6\left(\hbar c^2\right)}t\left\langle r^2\right\rangle + \ldots \tag{3}$$

2. Calculations

In this work the latest data of proton-proton elastic scattering from the TOTEM experiment at $\sqrt{s}$ ≈ 7, 8 and 13 TeV is used. Complete details of these experiments are given in [23 - 25]. In the TOTEM experiment [23], the differential cross section is measured in the range 0.0025 GeV^2 < −t < < 0.38 GeV^2. The dip around −t = 0.5 GeV^2 [26] and deviations from the exponential form are expected. Since the deviations from the exponential form are expected to increase at larger t, the upper limit of the fit range is increased to −t = 0.3 GeV^2 when fitting the alternative forms. All alternative forms have at least one more free parameter which improves the quality of the fits at larger t, where best sensitivity for additional parameters is required.

Experimental data of p-p elastic scattering in the diffraction peak region is fitted to a single exponential function and parameters, a and b are extracted and their values for each data set are listed in Table 1.

Table 1. **Parameters of fitting for $\sqrt{s}$ = 7, 8, and 13 TeV**

$\sqrt{s}$, TeV	j	a_j, mb^{-1}/GeV	b_j, GeV^{-2}
7	1	512.09 ± 0.004	20.15 ± 2·10^{-5}
8	2	700.13 ± 0.372	20.11 ± 0.002·10^{-5}
13	3	623.41 ± 0.001	20.49 ± 3.7·10^{-5}

Slope parameter b depends on the fit range of four-momentum transferred squared. As one moves forward from |t| = 0, $\frac{d\sigma}{dt}$ decreases exponentially for a certain range of |t|. A straight line will be obtained for the $\ln\left(\frac{d\sigma}{dt}\right)$ vs |t| plot in this range. When one moves further away from this value exponential behavior of the slope vanishes. This is known as a slope break. The value of b depends on energy, reaction, and fit range of |t| [27]. This analysis of slope break is based on lower values of |t|. In diffraction peak region shrinkage of $d\sigma/dt$ is universal for all colliding hadrons and is independent of the nature of hadrons making interaction [28].

The protons form factor can be expressed in the following form:

$$F_j(t) = \sqrt{C_j\sum_{i=1}^{3}c_{ji}e^{d_{ji}t}}. \tag{4}$$

Here j = 1, 2, and 3 for $\sqrt{s}$ ≈ 7, 8, and 13 TeV energies respectively and C_j is a normalizing constant. The computed values of c_{ji} and d_{ji} are given in Table 2, whereas the computed values of C_j are given in Table 3. The slope of the form factor when the four-momentum approaches zero $(t=0)$ yields the charge radius of the proton [11]. The obtained values of constants are tabulated in (3). We can rewrite Eq. (3) for radius calculation as:

$$\left\langle r^2 \right\rangle = -6\hbar^2 \left(\frac{dF(t)}{dt} \right)\Bigg|_{t=0}. \qquad (5)$$

Electromagnetic form factor values for $\sqrt{s}$ ≈ 7, 8, and 13 TeV are inserted into Eq. (5) to predict rms radius of the proton at the different CM energies, given in Table 4.

Table 2. **Calculated values of c_{ji} and d_{ji}**

Indices	$\sqrt{s}$ ≈ 7 TeV, j = 1		$\sqrt{s}$ ≈ 8 TeV, j = 2		For $\sqrt{s}$ ≈ 13 TeV, j = 3	
i	c_{ji}	d_{ji}	c_{ji}	d_{ji}	c_{ji}	d_{ji}
1	−2.022 ± 0.00006	−3.358 ± 0.0003	−2.770 ± 0.0015	3.351 ± 0.0003	−2.420 ± 0.00004	−3.41 ± 0.00006
2	0.569 ± 0.00007	−2.239 ± 0.00022	0.9141 ± 0.0008	−2.2344 ± 0.0002	0.740 ± 0.000027	−2.276 ± 0.00004
3	12.767 ± 0.00005	−6.716 ± 0.00007	14.928 ± 0.004	−6.7033 ± 0.0007	14.08 ± 0.00001	−6.83 ± 0.00012

Table 3. **Calculated values of C_j**

$\sqrt{s}$, TeV	C_j
7	0.088 ± 0.00004
8	0.076 ± 0.000023
13	0.080 ± 0.0005

3. Results and discussion

CYM can be applied for higher as well as lower values of CM energy and it produces excellent results. Both the electromagnetic form factor and rms radii of the proton can be predicted accurately at lower values of four-momentum transferred squared (t) and at high $\sqrt{s}$. Fig. 1 shows experimental data obtained from the TOTEM [23 - 25] and the fitted curve is a solid black line and the same method is applied for $\sqrt{s}$ ≈ 8 and 13 TeV.

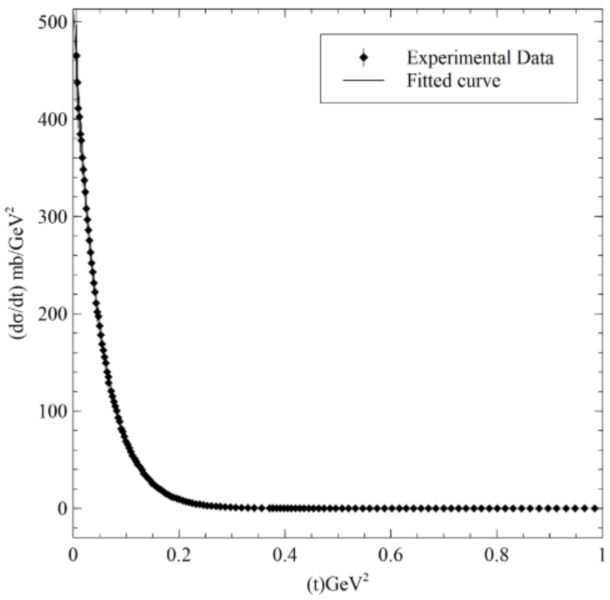

Fig. 1. Experimental differential cross-section data of proton-proton scattering fitted to a single exponential function at $\sqrt{s} = 7$ TeV.

Table 4. **Calculated rms radii of proton**

$\sqrt{s}$, TeV	rms radius of proton, fm
7	0.91 ± 0.004
8	0.91 ± 0.0002
13	0.918 ± 0.0032

A comparison of the electromagnetic form factor's behavior at $|t|$ for all three energies is exhibited in Fig. 2. Black, red, and blue lines represent form factors in data sets $\sqrt{s}$ ≈ 7, 8, and 13 TeV. At very high energy behavior of all the form factors is the same. Y. H. Lin et al. [22] in their scholarly work presented a combined analysis of the electromagnetic form factors of the nucleon in the space- and time-like regions using dispersion theory. They also extracted proton charge radius the extracted proton charge radius r p = 0.840 fm which is small. Their method is slightly complicated. In this work, we adopted the simplest approach to predict the form factor as well as the charge radius of the proton by using the simplest version of CYM. Implications of a simple method for calculation of proton's rms radius that agree well with experimental and theoretical results make this method more favorable.

4. Conclusion

The values calculated for the rms radius of the proton are 0.91, 0.9, and 0.92 fm at $\sqrt{s}$ ≈ 7, 8, and 13 TeV. The accuracy of the obtained results depends on available data points at lower values of |t| and the systematic and statistical errors. Fig. 2 shows that the electromagnetic form factors of colliding protons have similar behavior in elastic collision at the higher CM energies. From here we can conclude that there is no change in the underlying structure of the proton during p-p collisions, even at high CM energies. A comparison of the rms radii of the proton is shown in Fig. 3. It is clear that our results are in good agreement with

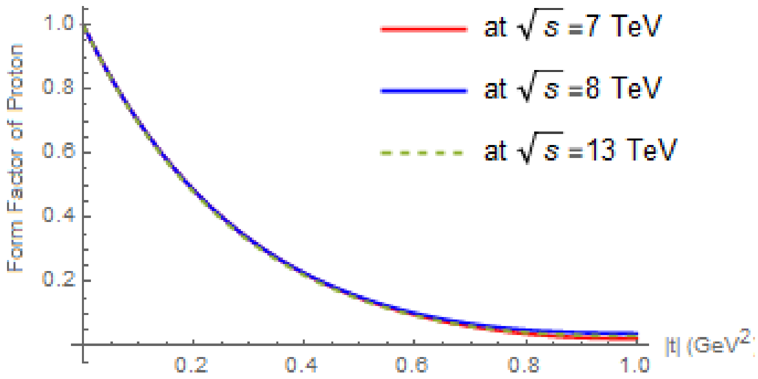

Fig. 2. Dependence of form factors on $|t|$ at $\sqrt{s} \approx 7$, 8, and 13 TeV.
(See color Figure on the journal website.)

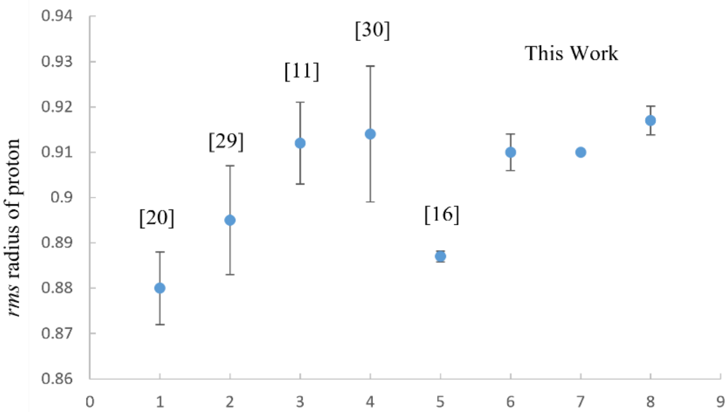

Fig. 3. Comparison of our computed proton's rms radii with the available theoretical and experimental values.
(See color Figure on the journal website.)

theory and experimentally available data. Again, it shows that the rms radius is independent of the scattering energy in the hadron-hadron collision. At very high energies interaction strength increases resulting increase in the total cross-section but not in rms radius of the proton.

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ЕЛЕКТРОМАГНІТНИЙ ФОРМ-ФАКТОР ТА РАДІУС ПРОТОНА, ВИЗНАЧЕНИЙ З ПРУЖНОГО *pp*-РОЗСІЯННЯ ПРИ $\sqrt{s} \approx 7, 8$ ТА 13 ТеВ З ВИКОРИСТАННЯМ МОДЕЛІ ЧОУ-ЯНГА

Модель Чоу-Янга була використана для отримання електромагнітного форм-фактора та середньоквадратичного радіуса протона, використовуючи експериментальні дані протон-протонного пружного розсіяння при $\sqrt{s} \approx 7, 8$ і 13 ТеВ. Диференціальні перерізи при низьких чотири-імпульсах |t| наближувались однією експоненціальною функцією для визначення форм-фактора у системі центра мас енергій. Отримані значення використовуються для передбачення середньоквадратичного радіуса протона. Порівняння електромагнітних форм-факторів та середньоквадратичного зарядового радіуса при різних енергіях центра мас показує, що наші результати добре узгоджуються з експериментом і теорією. Передбачені значення середньоквадратичного радіуса протона підтверджують його незалежність від енергії.

Ключові слова: модель Чоу-Янга, пружне розсіяння протонів, середньоквадратичний радіус протона, електромагнітний формфактор протона.

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