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A THEORETICAL STUDY OF EVEN-EVEN $^{162-178}_{70}$Yb ISOTOPES USING INTERACTING VECTOR BOSON MODEL

This theoretical study investigates the properties of even-even $^{162-178}_{70}$Yb isotopes using the interacting vector boson model (IVBM). Our study focuses on the ground state band and negative parity band energy-level patterns, which provide insights into the shapes and symmetries of these nuclei. Furthermore, we investigate the collective properties of these isotopes, such as rotational and vibrational motion, as well as their interplay. The results of our theoretical analysis shed light on the structural evolution of ytterbium isotopes with increasing neutron numbers. The comparison of our theoretical predictions with experimental data will provide valuable insights into the nuclear structure of these isotopes and help validate the IVBM model's effectiveness in describing collective phenomena. This theoretical study employs the IVBM to investigate the dynamic symmetry of even-even $^{162-178}_{70}$Yb isotopes. By conducting tests such as the ratio, backbending, and staggering analyses, we aim to determine the underlying symmetries governing the behavior of these isotopes. These results indicate that $^{162}_{70}$Yb possess O(6) symmetry, $^{164-166}_{70}$Yb have transition O(6) – SU(3) symmetry, and $^{168-178}_{70}$Yb possess – SU(3) symmetry. The study's outcomes show that the IVBM is dependable and useful for nuclear physics research because it aligns well with the corresponding experimental data.

Keywords: backbending, dynamic symmetry, midshell, nuclear model, ratio test, staggering.

1. Introduction

Ytterbium has a unique nuclear structure with even-even isotopes. This means that the nucleus consists of an even number of protons and neutrons, which can lead to unique nuclear reactions and properties. This makes ytterbium a good candidate for studying the effects of even-even isotopes on nuclear reactions. Ytterbium is also involved in various nuclear reactions, including the r-process, which leads to the synthesis of heavy elements in the cores of massive stars and in supernova explosions. By studying the energy levels and transition probabilities of ytterbium isotopes using the interacting vector boson model (IVBM), scientists can gain insights into these nuclear reactions and their outcomes [1].

Ytterbium is a chemical element with the atomic number 70. With an atomic mass ranging from 168 to 176, it contains seven naturally occurring isotopes, the most abundant of which is ^{174}Yb (31.83 %). In addition to these stable isotopes, there are ten radioactive isotopes known, with ^{169}Yb being the most stable, having a half-life of 32.026 days. Ytterbium is a rare earth element and has various applications in nuclear physics, as well as other fields such as medicine and technology [2, 3]. In this work, we will study the chain of $^{170-178}_{70}$Yb isotopes with 9, 10, 11, 10, 9 neutron boson numbers and 6 proton boson numbers, so that the chain would have total boson numbers equal to 15, 16, 17, 16 and 15.

The IVBM is a widely used model in nuclear physics; in this model, the nucleus is treated as a system of interacting bosons, which represent the collective motion of nucleons. Half of the valence nucleons counted from the closest closed shells are bosons. As the valence nucleon number N increases and the closed shell (Near Magic) is left behind, the shape shifts from a spherical vibrator (U(5) limit) to a deformed rotor (SU(3) limit), passing through transitional nuclei en route. The characteristic level structure of each shape is a phonon-like level scheme for a vibrator and a clear rotational band for a rotor. These are well represented by the $R_{4/2}$ ratio, which is the ratio of 4_1^+ to 2_1^+ excitation energies.

It is hypothesized within the IVBM in algebra that two kinds of vector "quasiparticles", each of which has a different quantum number that was formerly referred to as a "pseudospin," can be used to explain nuclear dynamics [4]. While the term "pseudospin" is now recognized as having been established for single particle levels in [5], we shall here use the more relevant term "T-spin", which is an analog to the F-spin in IBM-2. These vector bosons differ in their "T-spin" projection, $\alpha = \pm 1/2$ and form a “T-spin” doublet. The SU(3) dynamical symmetry is extended to U(6) with the introduction of this extra degree of freedom. To support this assumption, it is not required to regard the bosons as fermion pairs connected to $L = 1$; instead, they have preferred to treat them as a type of “oscillator quarks” or “Elliott quarks”, similar to Elliott’s well-known model [6 - 8].

For the problem of two interacting vector bosons, the group of dynamical symmetry appears to be the irregular symplectic group Sp(12, *R*). The bigger model spaces that come from altering the number of phonons required to construct collective states are made possible by the symplectic structure and can accommodate the more intricate structural effects. By utilizing three more subgroup chains in the reduction of Sp(12, *R*) to the physical angular momentum subgroup SO(3), the IVBM’s applications are expanded. In the rare earth and actinide regions, complicated nuclear collective spectra of even-even nuclei are described up to states of very high angular momentum by applying the appropriate exactly solvable limiting cases [6, 9].

In this study, the IVBM was used to calculate the energy levels, for the ground state band (GSB) of the chain ${}^{162\text{-}178}_{70}$Yb as well as it was used to calculate the negative parity band (NPB), and then we have applied some tests on it [10, 11]. The results of this study provide valuable insights into the properties of these isotopes and their potential applications in nuclear research. The calculated results are also compared with the previous experimental results, providing a validation of the accuracy of the IVBM model. Both positive and NPB $\left(I^{\pi} = 0^{+}, 2^{+}, 4^{+}, \ldots\right.$ and $\left.I^{\pi} = 1^{-}, 3^{-}, 5^{-}, \ldots\right)$ for the ground state, respectively, overlap to form a single band called the octupole band $I^{\pi} = 0^{+}, 1^{-}, 2^{+}, 3^{-}, 4^{+}, 5^{-}, \ldots$; this overlap is a good example of the staggering of energies.

In addition, the study discusses the backbending phenomena for these isotopes. Backbending is a phenomenon that occurs in nuclear reactions where the shape of the nucleus is altered. Also, the overlap between positive and NPB so-called staggering has been discussed as well. This can affect the stability of the nucleus and the transition probabilities of the isotopes [10].

2. The theory

The initial formulas of the interacting boson model were developed to study the spectrum of even-even nuclei with positive and negative symmetry. The IVBM was used to calculate energy levels and quantum transfers between them with high angular momentum. It proved successful in describing the phenomenon of backbending as well as the phenomenon of oscillation in the energy differences between positive and negative parity bands of the ground state (staggering of energies).

2.1. The Hamiltonian

The Hamiltonian effect in this model is given according to the descriptive analysis of multipolarity by the following equation [12]:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2, \qquad (1)$$

where the GSB is described by *a*, *b*, β_3, and the octupole band is described by α_1 and α_3. The total number of bosons is represented by the Hermitian operator *N*, while the quantum numbers T^2 and T_0 characterize the pseudospin, which was introduced to differentiate between two types of vector bosons that form the fundamental building blocks of the model's algebraic structure. In the IVBM, the allowed values for the two bands (GSB and NPB) energy states are given by [13, 14]:

$$E(I)_{GSB} = \beta I (I+1) + \gamma I \ . \qquad (2)$$

Additionally,

$$E(I)_{NPB} = \beta I (I+1) + (\gamma + \eta) I + \zeta. \qquad (3)$$

The intensity of the rotational properties' influence on the nuclei is indicated by the β parameter, while the intensity of the vibrational properties' influence is indicated by the γ parameter. The parameters η and ζ are crucial additions to compute the values of the energy levels in the NPB beam [15].

2.2. The backbending

To ascertain whether an isotope possesses backbending property and, if it does, where it is located, we should apply the relationship between the gamma energy E_{γ} and the moment of inertia ($2J/\hbar 2$), which can be expressed as follows [16, 17]:

$$\frac{2J}{\hbar^2} = \frac{4I - 2}{E_{\gamma}}. \qquad (4)$$

Conversely, the relationship between $\hbar\omega$ and E_{γ} is provided by [18, 19]:

$$\hbar\omega = \frac{E_{\gamma}}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}}. \qquad (5)$$

2.3. The staggering

The staggering pattern provides insights into the nuclear structure and the transitions between different nuclear shapes. It helps in identifying the presence of phase/shape transitions and understanding the evolution of nuclear structure under different conditions. The oscillation between the energy levels of the NPB and GSB called staggering is evidence of the overlap of the energy levels between them. The relationship of the staggering patterns between the NPB and GSB is given by Bonatsos et al. [20]:

$$\Delta E_{1,\gamma}(I)=\frac{1}{16}\Big(6E_{1,\gamma}(I)-4E_{1,\gamma}(I-1)-$$

$$-4E_{1,\gamma}(I+1)+E_{1,\gamma}(I-2)+E_{1,\gamma}(I+2)\Big) \qquad (6)$$

and

$$E_{1,\gamma}(I)=E\left(I+1\right)-E\left(I\right). \qquad (7)$$

All states in the NPB are higher or lower in energy than their state in the GSB. In other words, the staggering $\Delta I = 1$ takes an oscillation pattern between positive and negative values, and this oscillation may reach zero, followed by another one [21, 22].

3. Results and discussion

The study of ytterbium isotopes, particularly that of even-even ytterbium isotopes, has provided valuable insights into nuclear structure. For that, the ytterbium isotopes chain ${}^{162-178}_{70}\mathrm{Yb}$ have been examined. Table 1 displays the number of (neutron and proton) bosons for the isotope series under investigation. It also displays the parameters used to determine the energy levels of the two bands (positive and negative), which were determined by fitting them to get the best results using Eqs. (2) and (3). In order to understand the behavior of the nuclei of this series of isotopes, we have calculated the energy levels given these parameters.

Table 1. **Number of bosons and IVBM parameters of GSB and NPB, keV**

Isotope	N_ν	Total number of bosons	GSB		NPB	
			β	γ	η	ζ
${}^{162}_{70}\mathrm{Yb}$	5	11	4.8	157.1	–32.2	624.8
${}^{164}_{70}\mathrm{Yb}$	6	12	8.3	86.3	–71.7	1119.9
${}^{166}_{70}\mathrm{Yb}$	7	13	7.0	89.9	–94.2	1349.2
${}^{168}_{70}\mathrm{Yb}$	8	14	6.4	83.9	17	1046
${}^{170}_{70}\mathrm{Yb}$	9	15	7.1	73.4	–71.4	1348.3
${}^{172}_{70}\mathrm{Yb}$	10	16	10.2	25.9	55.5	1164.1
${}^{174}_{70}\mathrm{Yb}$	11	17	9.4	32.5	–22	1237.8
${}^{176}_{70}\mathrm{Yb}$	10	16	8.9	49.9	–31.9	1052.4
${}^{178}_{70}\mathrm{Yb}$	9	15	13.0	3.0	–	–

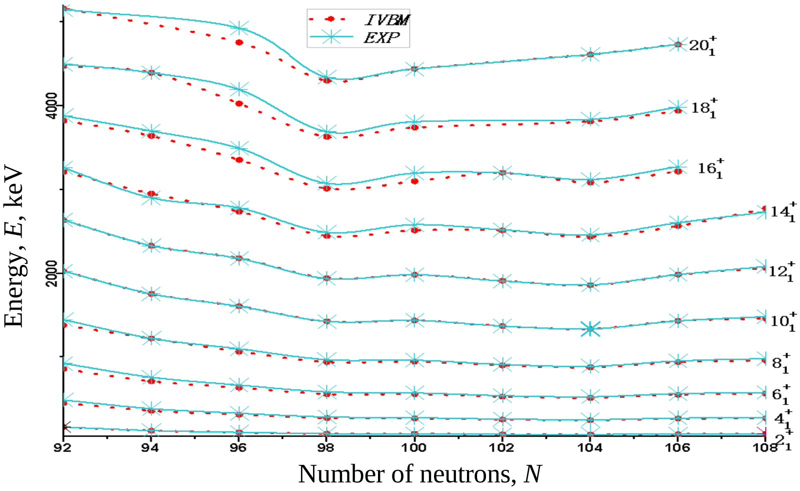
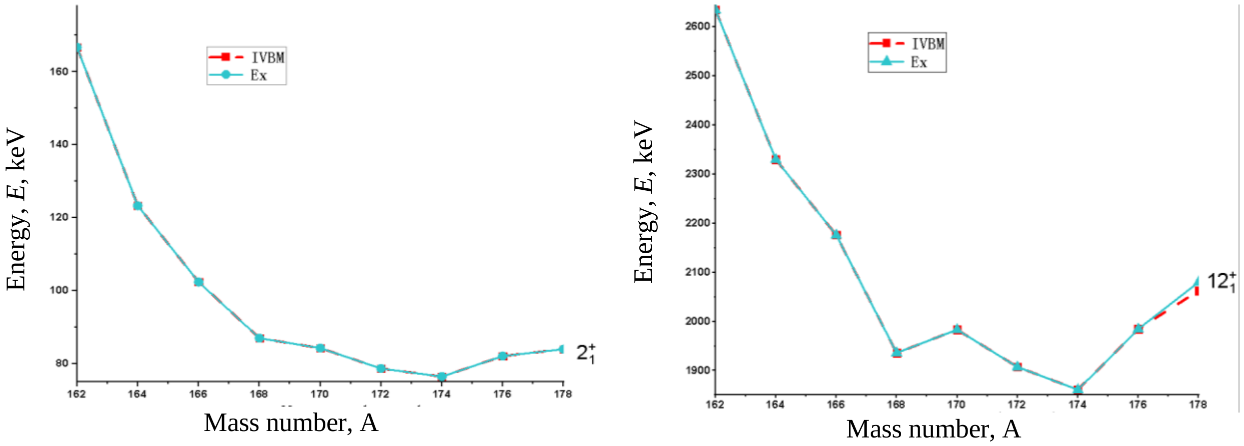


Fig. 1. Comparison between the number of neutrons and experimental and calculated energy levels for the Yrast band. (See color Figure on the journal website.)

Fig. 1 shows good agreement between experimental and calculated results for all the chains of isotopes. To further clarify the behavior of these isotopes, we have plotted the relationship between the atomic weight and energy for spins 2 and 12 separately, as shown in Fig. 2, where it turns out that $^{174}_{70}\mathrm{Yb}$ isotope has the lowest energy, and this is due to the presence of the effective neutron bosons for this isotope in the midshell exactly. We notice the presence of neutron bosons of the isotope $^{174}_{70}\mathrm{Yb}$ in the midshell between 82 and 126.

For more information about the nuclei, we have tested which dynamic symmetry every isotope might follow. In order to do that we performed a ratio test ($E(I+2)/E2$), and as Fig. 3 illustrates that $^{162}_{70}\mathrm{Yb}$ has critical X(5) symmetry at low spin (less than 10), and it has gamma unstable O(6) symmetry at high spin (greater than 10), while $^{164}_{70}\mathrm{Yb}$ has X(5) limit. For $^{166}_{70}\mathrm{Yb}$, it seems to be in the transition region O(6) - SU(3). Regarding the remaining isotopes $^{168-178}_{70}\mathrm{Yb}$, we could observe that they follow rotational symmetry SU(3).



a *b*

Fig. 2. The relation between the atomic weight and the energy levels for the spin 2^+ (*a*) and 12^+ (*b*). (See color Figure on the journal website.)

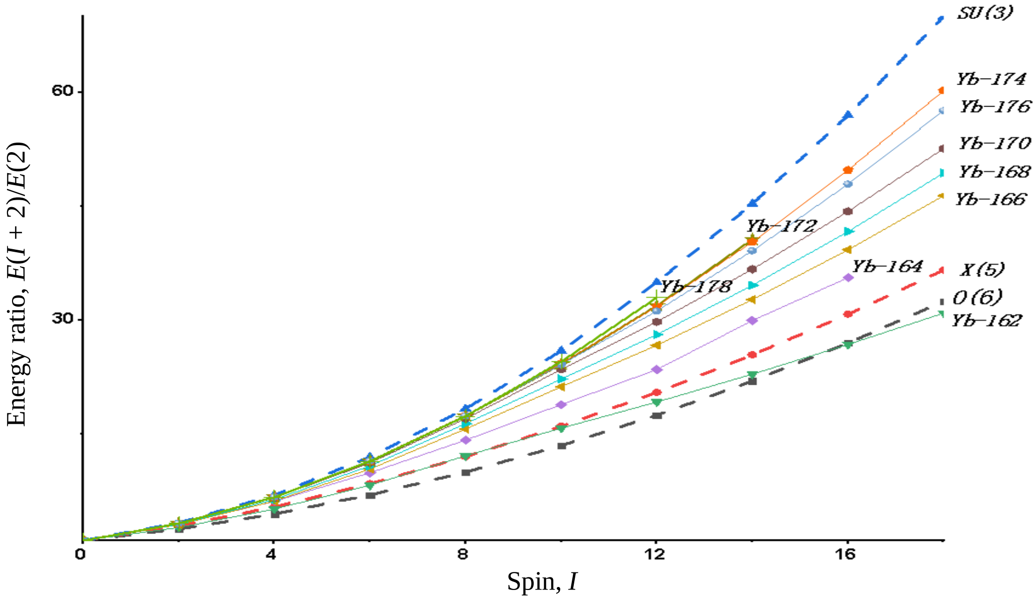
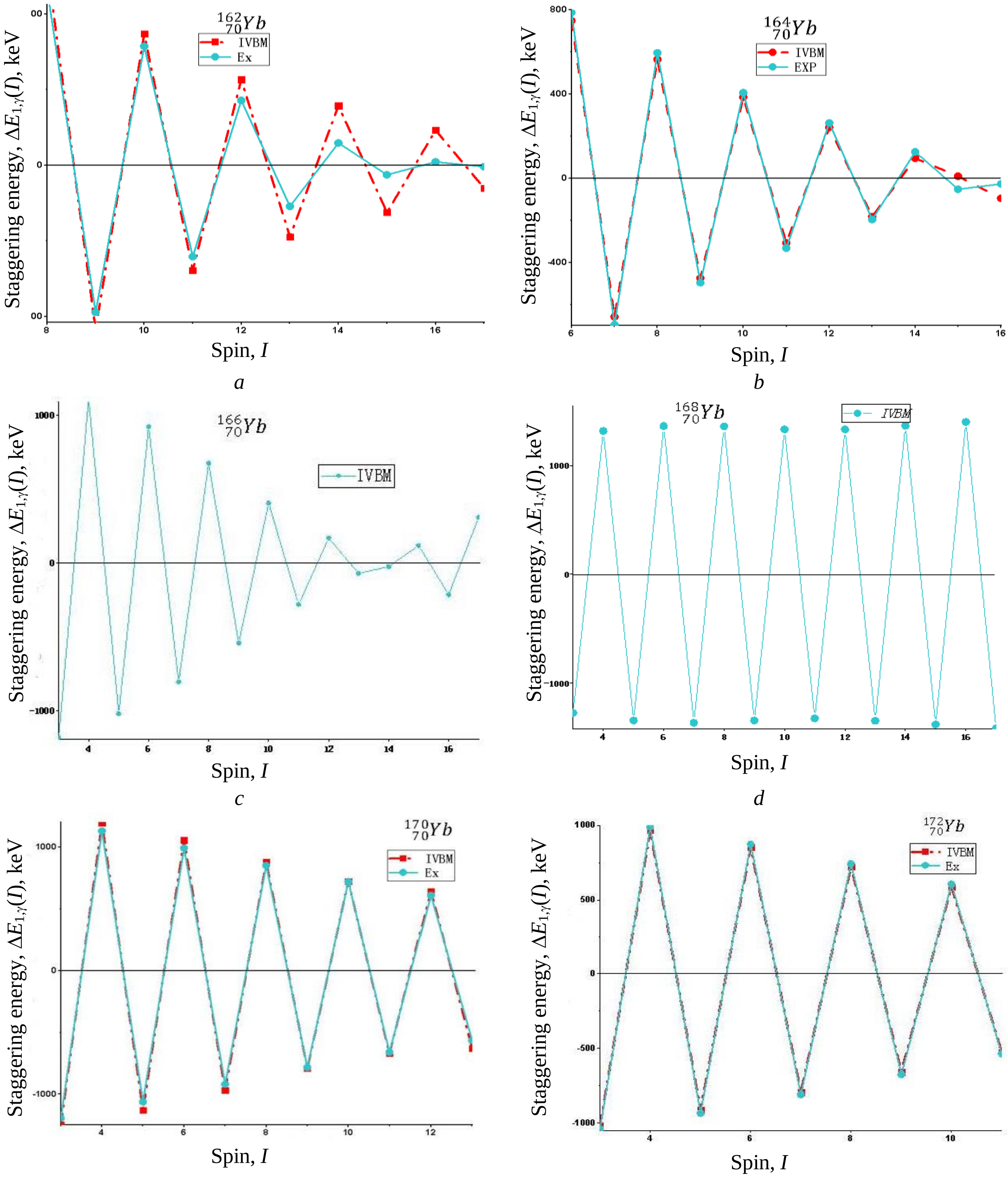


Fig. 3. Ratio values $E(I+2)/E2$ calculated for the chain of isotopes $^{162-178}_{70}\mathrm{Yb}$ and typical values for each limit near to the chain O(6), X(5), and SU(3). (See color Figure on the journal website.)

The alternating sign values are displayed by the staggered $E_{1,\gamma}(I)$ throughout the extended region of the angular momentum. Generally speaking, as I rises, the staggering first begins at somewhat high levels and then progressively drops. The staggering then begins to rise before falling once more. When the staggering reaches a vanishing value, the phase change manifests itself [23]. Fig. 4, *a*, *b*, and *c* displays the odd-even staggered results for the $^{162-166}_{70}\mathrm{Yb}$ nuclei. The phase change O(6) - SU(3) for those isotopes is confirmed by the IVBM results, which drop with increasing I; whereas the staggered curves almost reach zero, but we could note that it reaches zero at I = 16 or more, for $^{162}_{70}$Yb and $^{164}_{70}$Yb. Accordingly we could say that those isotopes possess O(6) limit; and for $^{166}_{70}$Yb the staggering curve reach zero at I = 14, which tells that this isotope has transitional phase O(6) - SU(3). In contrast, the IVBM results in Fig. 4, *d*, *e*, *f*, *g*, and *h* show a slight decrease with increasing I, but the staggering curves do not reach zero which confirms that $^{168-178}_{70}$Yb and $^{172}_{70}$Yb possess SU(3) properties. Fig. 4 shows good agreement between calculated results using experimental and calculated data [22], we could note the presence of some isotopes without laboratory results due to poor available measured data of NPB for some isotopes.



a

b

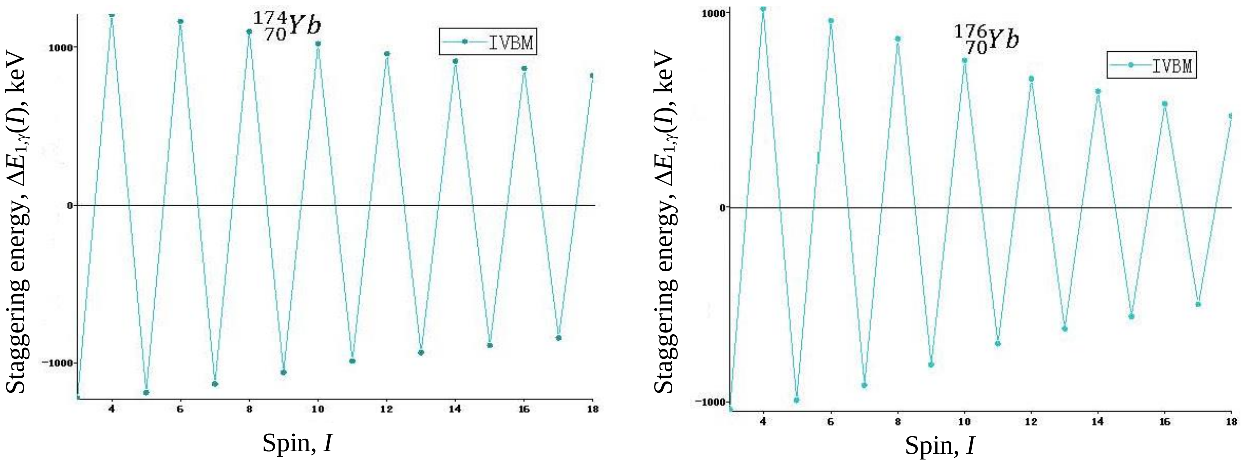
c

d

e

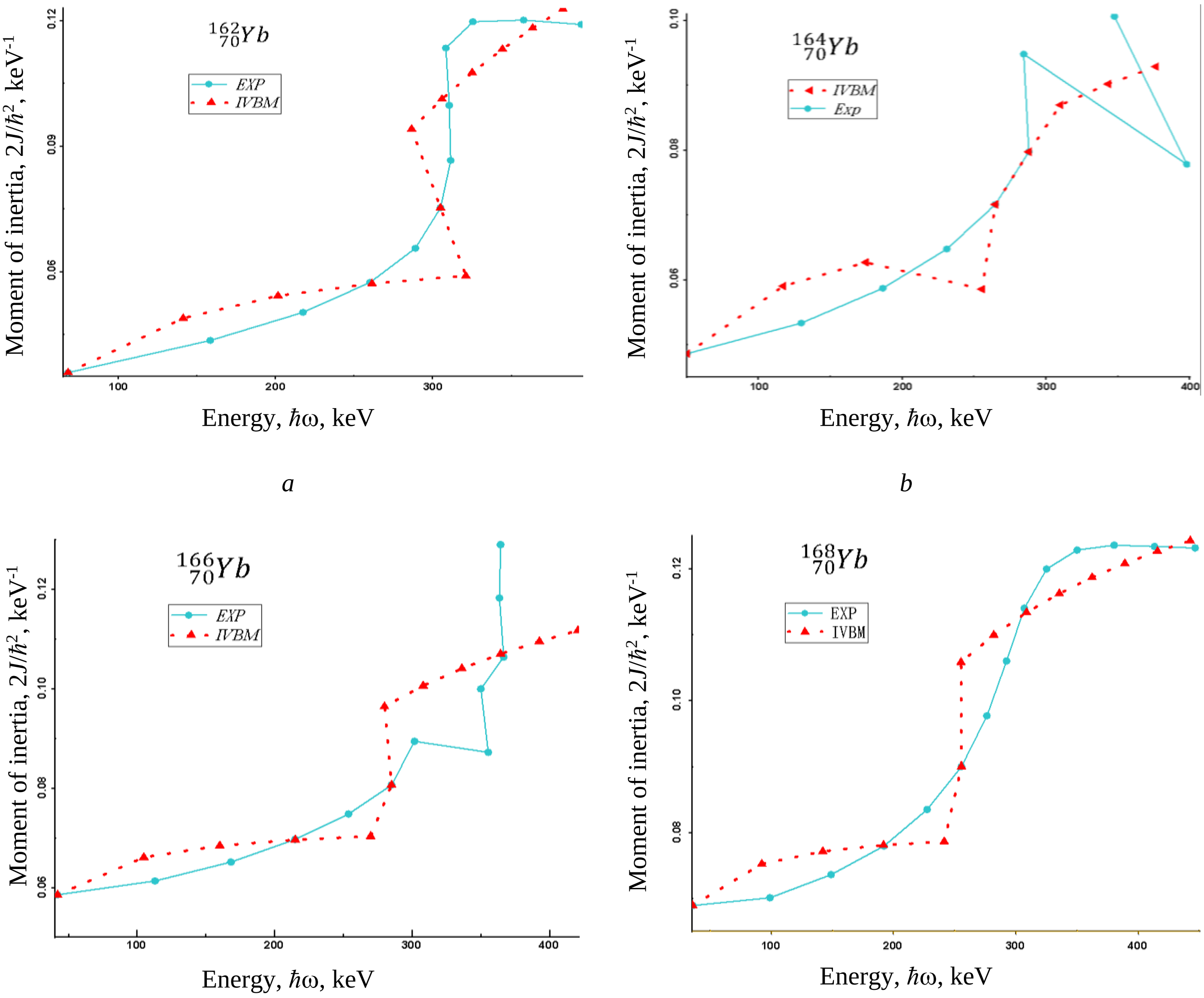
f

Continuation of Fig. 4



g *h*

Fig. 4. *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* – staggering calculated from Eq. (6) for $^{162-176}_{70}$Yb isotopes.
(See color Figure on the journal website.)



a *b*

c *d*

Continuation of Fig. 5

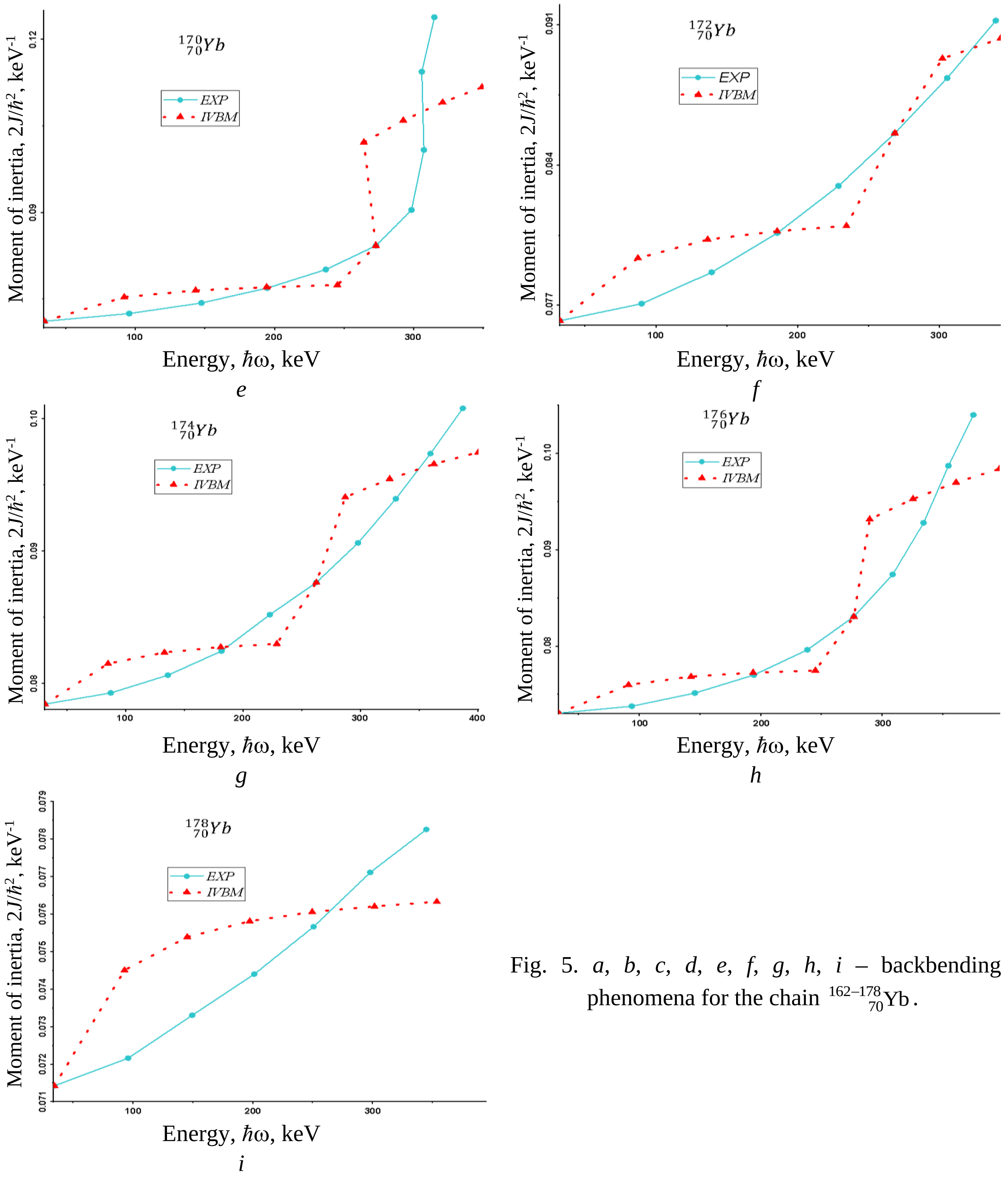


Fig. 5. *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i* – backbending phenomena for the chain $^{162-178}_{70}\text{Yb}$.

The backbending phenomenon has been observed in some nuclei, which is a deviation from the expected linear increase in moment of inertia with increasing rotational frequency. This phenomenon has been studied using IVBM, and it has been suggested that it may be due to rapid quasiparticle alignment in the considered nuclei. In this work, we have used Eqs. (4) and (5) to examine whether backbending existed in the studied chain of isotopes to learn more about the ytterbium nuclei. As shown in Fig. 5, *a*, *b*, *c*, and *d*, the $^{162}_{70}\text{Yb}$ isotope has good backbending, whereas for those isotopes $^{164}_{70}\text{Yb}$ and $^{166}_{70}\text{Yb}$, a backbending was hardly discernible [10], but for the rest of the isotopes $^{168-178}_{70}\text{Yb}$, an unbinding is noted.

Table 2 shows the structural evolution of $^{162-178}_{70}\text{Yb}$ isotopes using IVBM model.

Table 2. **Isotopes $^{162-178}_{70}$Yb classification**

O(6)	O(6) - SU(3)	SU(3)
$^{162}_{70}$Yb	$^{164}_{70}$Yb	$^{168}_{70}$Yb
	$^{166}_{70}$Yb	$^{170}_{70}$Yb
		$^{172}_{70}$Yb
		$^{174}_{70}$Yb
		$^{176}_{70}$Yb
		$^{178}_{70}$Yb

4. Conclusion

In conclusion, the IVBM provides a powerful tool for calculating the energy levels and transition probabilities for specific isotopes. These calculations provide valuable insights into the properties of the isotopes and their potential applications in nuclear research. The results of these calculations have contributed to our understanding of nuclear deformation and the role of quantum effects in nuclear physics. We were able to conclude that $^{162}_{70}$Yb isotope has O(6) symmetry, $^{164}_{70}$Yb and $^{166}_{70}$Yb isotopes have transitional symmetry O(6) - SU(3), and the rest of the chain of isotopes $^{168-178}_{70}$Yb has rotational symmetry SU(3). The parameters we have mentioned reflect the unique interactions and dynamics within the nuclear structure that give rise to the observed staggering patterns. Furthermore, the agreement between our theoretical predictions and experimental data not only supports the validity of the IVBM model but also creates new avenues for investigating isotope nuclear structure. Thus, this work demonstrates the value of theoretical modeling in expanding our knowledge of nuclear physics and opens the door for more investigations and discoveries in this exciting area. When we come to an end, we are reminded of the path we took, and the lessons learned in the process of shedding light on the hidden symmetries and group behaviors of ytterbium isotopes. This project not only advances our knowledge of nuclear physics but also serves as an example of the mutually beneficial relationship between theory and experiment, whereby each advances the other and advances the field.

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ТЕОРЕТИЧНЕ ДОСЛІДЖЕННЯ ПАРНО-ПАРНИХ ІЗОТОПІВ $^{162-178}_{70}$Yb З ВИКОРИСТАННЯМ МОДЕЛІ ВЗАЄМОДІЮЧИХ ВЕКТОРНИХ БОЗОНІВ

Це теоретичне дослідження присвячено вивченню властивостей парно-парних ізотопів $^{162-178}_{70}$Yb з використанням моделі взаємодіючих векторних бозонів (IVBM). Наша робота зосереджена на будові енергетичної смуги, пов’язаної з основним станом, та смуги з негативною парністю, що забезпечує розуміння форм і симетрії цих ядер. Крім того, ми досліджуємо колективні властивості цих ізотопів, такі як обертальний і коливальний рух, а також їх поєднання. Результати нашого теоретичного аналізу висвітлюють еволюцію структури ізотопів ітербію зі збільшенням числа нейтронів. Порівняння наших теоретичних результатів з експериментальними даними дає цінну інформацію про ядерну структуру цих ізотопів і підтверджує ефективність IVBM в описі колективних явищ. Ми використовуємо IVBM для дослідження динамічної симетрії парно-парних ізотопів $^{162-178}_{70}$Yb. Аналізуючи параметри зворотного згинання, розхитування, ми визначаємо основні симетрії, що керують поведінкою цих ізотопів. Результати показують, що $^{162}_{70}$Yb має симетрію O(6), $^{164-166}_{70}$Yb мають перехідну O(6) – SU(3) симетрію, а $^{168-178}_{70}$Yb − SU(3) симетрію. У підсумку, IVBM є надійним і корисним методом для досліджень в ядерній фізиці, оскільки ця модель забезпечує результати, що добре узгоджуються з відповідними експериментальними даними.

Ключові слова: зворотне згинання, динамічна симетрія, серединна оболонка, ядерна модель, тест на співвідношення, розхитування.

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