УДК 539.142

https://doi.org/10.15407/jnpae2024.01.019

K. A. Gado[1,2,*]

[1] *Department of Physics, Faculty of Science and Arts, Al-Mikhwah, Al-Baha University, Al-Baha, Saudi Arabia*
[2] *Basic Sciences Department, Bilbeis Higher Institute for Engineering, Bilbeis, Sharqia, Egypt*

*Corresponding author: qjado76@gmail.com

NUCLEAR SOFTNESS IN THE VARIABLE MOMENT OF INERTIA MODEL AND ITS APPLICATION TO SUPERDEFORMED BANDS IN THE MASS REGION A ≈ 60 - 90

For superdeformed (SD) bands ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr (b_1), ^{88}Mo (b_1, b_2, b_3) and ^{89}Tc in the A ≈ 60 - 90 mass region, the nuclear softness (NS) parameter, σ, has been calculated using the VMINS3 model. The SD bands ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{62}Zn, ^{65}Zn, and ^{88}Mo (b_2, b_3) have NS parameter values that are greater than those of the normal deformed bands, indicating smaller rigidity. The fluctuation of the NS parameter versus the gamma energy ratio, *R*, of SD bands in the A ≈ 60 - 90 mass region is one of the study's findings. The ratio of transition energies was used to calculate the band head spin, I_0, by the Descartes method (the greatest technique to solve the quartic equation based on an auxiliary cubic equation) which was then confirmed by root mean square deviations. The estimated and observed transition energies are in good agreement.

Keywords: variable moment of inertia model, nuclear softness, spin assignment.

1. Introduction

Due to the small number of particles found in nuclei with A ≈ 60 - 90, and the fact that they have the lightest masses and thus the greatest rotational frequencies, the superdeformed (SD) mass region A ≈ 60 - 90 is of special interest [1]. Furthermore, inadequate theoretical study has been done on these SD nuclei. Most of the SD bands in this mass region behave similarly in terms of their dynamic moment of inertia and rotational frequency i.e., they exhibit a smooth decrease as frequency increases. Sadly, the only publicly available spectroscopic information for the SD bands is gamma energies. The spin value of the rotational bands can only be calculated theoretically because there are insufficient experimental data on them due to non-observation of the discrete linking transitions between the SD states and the low-lying states at normal deformation (ND). Different techniques have been used to assign spins to SD states. The states in SD bands are given a spin using both direct and indirect methods in these schemes [2 - 4]. The nuclear softness (NS) model [5] and the variable moment of inertia (VMI) model [6, 7] both have a particular application and while both are successful in explaining certain phenomena, they fall short in explaining others. Therefore, it made sense to think of each of these models as complementing the others in a single model known as the VMINS3 model, which unifies the two models. According to this model, the nucleus consists of an outer valence nucleon in unfilled shells around a hard core of nucleons. In addition to the successes of each of the two models, this model (VMINS3) has succeeded in formulating an equation to calculate the rotational energy levels to the A ≈ 60 - 90 nuclei, where the energy levels of deformed nuclei are very complicated because there is frequently coupling between the various modes of excitation, but nonetheless some predictions of the VMINS3 model are confirmed experimentally. The VMINS3 model's most significant accomplishments include a description of the NS parameter as a function of super-deformed band angular momentum, transition energy (the only spectroscopic information universally available), and band head spin. Section 2 of this research, which is set up as follows, covers the details of the VMINS3 model. The findings and discussion are presented in Section 3. The final Section of the paper presents the conclusions.

2. Mathematical model

The VMINS3 model gives the following energy expression as the consequence of combining the VMI model with the NS model:

$$E_I = \frac{AI(I+1)}{1+\sigma I} + BI^2. \qquad (1)$$

In this case, $A = \frac{1}{2\vartheta_0}$ and $B = \frac{C\vartheta_0^2\sigma^2}{2}$. Therefore, these parameters included the NS parameter, σ, the stretching constant, *C*, and the ground state moment of inertia, ϑ_0. The NS parameter, σ, is an extra variable $\vartheta_I = \vartheta_0(1+\sigma I)$, as a first-order approximation) whereas the parameters, ϑ_0 and C are for the original VMI model.

The transition energy for SD bands is written as:

$$E_\gamma(I+2\to I) = E(I+2) - E(I). \quad (2)$$

$$E_\gamma(I+2\to I) = A\left[\frac{2\sigma I^2 + (4+4\sigma)I + 6}{(1+2\sigma+\sigma I)(1+\sigma I)}\right] + B[4I+4]. \quad (3)$$

Similarity,

$$E_\gamma(I\to I-2) = A\left[\frac{2\sigma I^2 + (4-4\sigma)I - 2}{(1-2\sigma+\sigma I)(1+\sigma I)}\right] + B[4I-4]. \quad (4)$$

Finally, we made it

$$E_\gamma(I-2\to I-4) = A\left[\frac{2\sigma I^2 + (4-12\sigma)I + (16\sigma-10)}{(1-2\sigma+\sigma I)(1+\sigma(I-4))}\right] + B[4I-12]. \quad (5)$$

We have carefully worked out the results of the VMINS3 model using Eq. (2), following our procedure of solving the first three transition energy level equations for the three coefficients σ, *A*, and *B*. In such an approach find the value of σ, as an intermediate step (by eliminating *A* and *B* from Eqs. (3), (4), and (5) for $I+2$, I, and $I-2$). One obtains a quartic equation in σ:

$$b_4\sigma^4 + b_3\sigma^3 + b_2\sigma^2 + b_1\sigma + b_0 = 0. \quad (6)$$

These are the formulas for the coefficients b_4, b_3, b_2, b_1 and b_0 in this case, being known in terms of $E_\gamma(I+2\to I)$, $E_\gamma(I\to I-2)$ and $E_\gamma(I-2\to I-4)$:

$$\left.\begin{aligned}
b_4 &= (32r-16)I^5 - (128r-64)I^4 + (448r+64)I^3 + (384r-256)I^2\\
b_3 &= -(8r+8)I^5 + (200r-40)I^4 - (304r-176)I^3 - (64r-384)I^2 - 1088rI - 512\\
b_2 &= -(24r+40)I^4 + (288r-72)I^3 - (200r+48)I^2 - (920r-192)I + 704r\\
b_1 &= -(16r-96)I^3 + (120r-16)I^2 + (8r+176)I - (408r+192)\\
b_0 &= 32r+48
\end{aligned}\right\} \quad (7)$$

To make it simpler to write equations, we defined *r*

$$r = \frac{(4I-4)E_\gamma(I+2\to I) - (4I+4)E_\gamma(I\to I-2)}{(4I-12)E_\gamma(I+2\to I) - (4I+4)E_\gamma(I-2\to I-4)}. \quad (8)$$

Since the coefficients ϑ_0 and *C* are traits of each nucleus and are all positive, the solution of Eq. (6) produces four real roots. Therefore, the smaller value of σ, is favored since, as originally mentioned by Gupta et al. [8], a lower σ, represents a smaller correction to ϑ_0. In order to do this, it is necessary that the discriminant of the cubic equation that results from using the Descartes method [9] to obtain band head spin by solving the quartic equation be always positive.

For a SD band cascade

$$I_0+2n \to I_0+2n-2 \to \ldots I_0+2 \to I_0, \quad (9)$$

The transition energies that were noticed are: $E_\gamma(I_0+2n)$, $E_\gamma(I_0+2n-2)$, $E_\gamma(I_0+2n-4)$, …, $E_\gamma(I_0+4)$, $E_\gamma(I_0+2)$. Eq. (6) fits these transition energies, by using the same equation to fit the observed transition energies, the parameters σ, *A*, and *B* values are determined. The ratio of transition energies may be used to calculate the band head spin as:

$$R(I) = \frac{E_\gamma(I+2\to I)}{E_\gamma(I\to I-2)}. \quad (10)$$

One obtains a quartic equation in I_0

$$h_4 I_0^4 + h_3 I_0^3 + h_2 I_0^2 + h_1 I_0 + h_0 = 0. \quad (11)$$

These are the formulas for the coefficients h_4, h_3, h_2, h_1 and h_0 in this case, being known in terms of σ and *R*:

$$\left.\begin{aligned}
h_4 &= 4B\sigma^3(R-1)\\
h_3 &= 2\sigma^2(AR+6BR-A-2B)-16B\sigma^3\\
h_2 &= 2\sigma(3AR+6BR-A+2B)+4\sigma^2(17BR-5A-11B+3AR)+4B\sigma^3(28R-55)\\
h_1 &= 4\big((A+B)(R+1)\big)+2\sigma(13AR+18BR-9A+10B)+8\sigma^2(13BR+2AR-20B-8A)+16B\sigma^3(32+9R)\\
h_0 &= 2\big(R(3A+2B)+(7A+6B)\big)+2\sigma\big(A(13R-20)+12B(R+1)\big)+16\sigma^2(3BR-12B-4A)
\end{aligned}\right\} \tag{12}$$

whose coefficients are functions of σ, *A*, *B*, and *R*, solved by the Descartes method [9] and having four real roots. Descartes' rule of signs states that roots might be either positive real, negative real, or complex. For this, we must select the greatest $|I_0|$ values. To confirm this choice, the root mean square (rms) deviations of the transition energies computed at various I_0-values were used to confirm the accuracy of the band head spin [10].

$$rms = \left[\frac{1}{N}\sum_{i=1}^{N}\left|\frac{E_\gamma^{cal}(I_i)-E_\gamma^{exp}(I_i)}{E_\gamma^{exp}(I_i)}\right|^2\right]^{\frac{1}{2}}. \tag{13}$$

Here *N* is the total number of fitting transitions.

3. Results and discussion

Since the bandhead energy and spin for the SD bands are often unknown, one might opt to fit the E2 transitions using Eq. (2). The parameters *A* and *B* may now be determined by fitting the E2 transitions for the SD cascades. One may then obtain the NS parameter, σ, by using Eq. (6) and relations in Eq. (7).

By applying the Eq. (6) and relations in Eq. (7) to the first three of the gamma energies of all the SD bands for ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr (b_1), ^{88}Mo (b_1, b_2, b_3) and ^{89}Tc nuclei in A ≈ 60 - 90 mass region, we were able to determine the NS parameter as given in Table 1. If complex roots are found, as in instance ^{61}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr (b_1), ^{88}Mo (b_1), and ^{89}Tc this indicates that VMINS3 cannot be applied to the given nuclei, this is due to the fact that, as in the first and final cases, the cubic equation's root is negative, or as in the other cases, its discriminant is negative. According to Ref. [8], most of the NS parameter, σ, values are observed to lie in the range of $31.4\cdot10^{-2}$ to $263.0\cdot10^{-2}$. These σ, values are at least 10 times larger than those of ND bands. The NS parameter σ for SD bands lies in the range of $10^{-3} \le \sigma \le 10^{-6}$ as compared to ND bands [6, 11] have a range of $10^{-2} \le \sigma \le 10^{-4}$. The NS parameter is related to the extent of rigidity of SD bands. By applying the VMINS3 model, it is determined that the SD bands for ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{62}Zn, ^{65}Zn and ^{88}Mo (b_2, b_3) nuclei are less rigid than the ND bands.

Table 1. **The real root (NS parameter, σ) of Eq. (6) for SD bands for nuclei in A ≈ 60 - 90 mass region together with the experimental transition energy**

SD band	Experimental transition energy, keV			NS parameter, σ			
	$E_\gamma^{exp}(I-2\to I-4)$	$E_\gamma^{exp}(I\to I-2)$	$E_\gamma^{exp}(I+2\to I)$	Real roots of Eq. (6)			
				First	Second	Third	Fourth
^{58}Ni(b_1)	1663	1989	2350	0. 853	4.720	2.450	3.130
^{58}Cu	830	1197	1576	1.610	7.670	3.380	5.900
^{59}Cu(b_1)	1599	1900	2242	1.380	4.090	3.180	2.290
^{61}Zn	1432	1626	1845	–	–	–	–
^{62}Zn	1993	2215	2440	0.680	4.740	2.710	2.700
^{65}Zn	1341	1491	1668	–	3.790	2.630	–
^{68}Zn	1506	1717	1918	–	–	–	–
^{84}Zr	1526	1663	1808	–	–	–	–
^{86}Zr(b_1)	1518	1646	1785	–	–	–	–
^{88}Mo(b_1)	1238	1343	1481	–	–	–	–
^{88}Mo(b_2)	1458	1596	1743	0.358	4.450	3.400	1.410
^{88}Mo(b_3)	1260	1384	1521	0.314	3.740	2.820	1.230
^{89}Tc	1147	1259	1384	–	–	–	–

In Table 2, we have summarized the band head spin in the mass range A ≈ 60 - 90 in the present study and prior studies [12, 13]. The VMINS3 model is not able to assign the I_0 for some nuclei as ^{61}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr (b_1), ^{88}Mo (b_1), and ^{89}Tc due to complex roots found in Eq. (11). It is helpful to try to connect the data in terms of some theory of nuclear structure search like the VMINS3 model in order to systematize what would otherwise be a perplexing mass of data. It is clear from this comparison that our present results for the band head spin appear to relatively satisfy the experimental data available when compared to findings from earlier investigations. The VMINS3 model formula has been used to fit the E2 gamma energies of all the SD bands for ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr (b_1), ^{88}Mo (b_1, b_2, b_3) and ^{89}Tc nuclei in A ≈ 60 - 90 mass region.

Table 2. **The band head spin I_0 for SD bands as well as the values from the available theoretical models, together with the computed transition energy and the parameters A and B utilized in the fitting**

SD band	$E_\gamma^{exp}(I_0 \to I_0-2)$, keV	$E_\gamma^{VMINS3}(I_0 \to I_0-2)$, keV	$A\cdot10^2$, keV	$B\cdot10^1$, keV	I_0			
					Present assigned	Ref. [12]	Ref. [13]	Exp. [14]
^{58}Ni(b_1)	1663	1718	–3.470	3.95	10	13	13	15
^{58}Cu	830	887	–6.700	4.30	8	8	4	9
^{59}Cu(b_1)	1599	1615	–5.590	3.92	8.5	13.5	12.5	–
^{61}Zn	1432	–	–	–	–	17.5	13.5	12.5
^{62}Zn	1993	2001	–0.297	2.75	14	22	18	–
^{65}Zn	1341	1310	1.320	2.24	17.5	12.5	20.5	–
^{68}Zn	1506	–	–	–	–	18	14	–
^{84}Zr	1526	–	–	–	–	25	21	–
^{86}Zr(b_1)	1518	–	–	–	–	27	28	–
^{88}Mo(b_1)	1238	–	–	–	–	27	–	–
^{88}Mo(b_2)	1458	1503	–0.644	1.50	21	24	32	–
^{88}Mo(b_3)	1260	1313	–0.641	1.30	21	23	–	–
^{89}Tc	1147	–	–	–	–	23.5	20.5	–

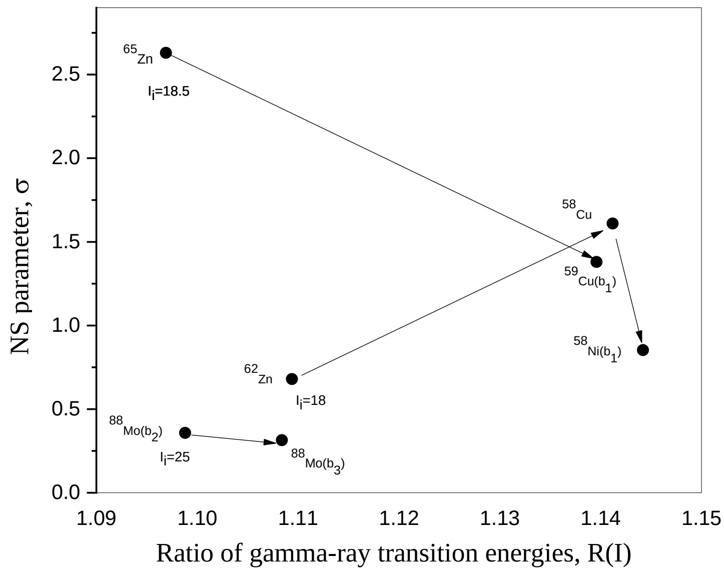
For these nuclei, we took into account only the SD bands for which conditions were satisfied. Firstly: the roots of Eq. (6) are real either positive or negative (the NS parameter, σ). Secondly: they are subject to the constraint Gupta et al. [8], (the smallest value among the roots). Finally: our constraint is to select the largest I_0 – value regardless of its sign (resulting from solving quartic Eq. (11)). Due to the fact that the rms deviation of the larger I_0 is less than the mean square deviation of the smaller I_0, Table 3 demonstrates that the SD bands for the nuclei ^{58}Ni (b_1), ^{59}Cu (b_1), ^{62}Zn, and ^{88}Mo (b_2) fulfill all requirements. ^{88}Mo (b_3) fulfills all requirements due to no other value for I_0 appears, where the number of negative real roots and complex roots is two, as stated by Descartes' rule for signs. Since the negative second root's value is so tiny ~0.5, it cannot adequately reflect the value of I_0. SD bands for the nuclei ^{58}Cu, and ^{65}Zn satisfy all conditions except that rms deviation of the smaller I_0 is bigger than the rms deviation of the larger I_0, may be due to that SD bands for the nuclei ^{58}Cu, and ^{65}Zn configurations involve the single-particle states $\pi 3^m$ where m is the number of protons in the $\mathcal{N} = 3$ intruder levels, where $\mathcal{N}$ represents the principal oscillator quantum number. The $R_4 = \frac{E_4}{E_2}$, energy ratio in the deformed even-even nuclei is one of the most prevalent indicators of rigidity. Since the SD bands are high spin bands with unknown band head energies, we are unable to determine such an energy ratio R_4. We instead make use of the gamma-ray transition energy ratios. Therefore, using experimental gamma-ray transition energies [14], we compute the ratios as shown in Eq. (10) and plot the NS parameter vs these ratios in the SD bands where the ratio could be computed (this is only possible in those SD bands where the same set of spins are known, i.e., it shares the same angular momenta).

Table 3. **Comparison of the calculated and experimental E_γ, keV. The calculated transition energies for different band head spin (I_0) with the rms deviation value of SD bands for nuclei under study**

$E_\gamma^{exp}(I \to I-2)$	I	$E_\gamma^{VMINS3}(I \to I-2)$	I	$E_\gamma^{VMINS3}(I \to I-2)$	I	$E_\gamma^{VMINS3}(I \to I-2)$
Nucleus	$^{58}Ni(b_1)$					
Band head spin			8		10	
1663	15	1718	10	927	12	1243
1989	17	2034	12	1243	14	1560
2350	19	2350	14	1560	16	1876
2750	21	2666	16	1876	18	2192
3157	23	2982	18	2192	20	2508
rms	0.033367732		0.358854505		0.216548835	
Nucleus	^{58}Cu					
Band head spin			4		8	
8300	9	8870	6	7140	10	1059
1197	11	1231	8	1059	12	1404
1576	13	1576	10	1404	14	1748
1955	15	1920	12	1748	16	2093
2342	17	2265	14	2093	18	2437
2748	19	2609	16	2437	20	2781
rms	0.039670261		0.115484718		0.144171304	
Nucleus	$^{59}Cu(b_1)$					
Band head spin			6.5		8.5	
1599	14.5	1615	8.5	675	10.5	988
1900	16.5	1929	10.5	988	12.5	1302
2242	18.5	2242	12.5	1302	14.5	1615
2611	20.5	2555	14.5	1615	16.5	1929
3004	22.5	2869	16.5	1929	18.5	2242
3424	24.5	3182	18.5	2242	20.5	2555
3827	26.5	3495	20.5	2555	22.5	2869
rms	0.046812415		0.42145265		0.288538876	
Nucleus	^{62}Zn					
Band head spin			11		14	
1993	18	2001	13	1451	16	1781
2215	20	2220	15	1671	18	2001
2440	22	2440	17	1891	20	2220
2690	24	2660	19	2110	22	2440
2939	26	2880	21	2330	24	2660
3236	28	3099	23	2550	26	2880
rms	0.01975576		0.230661883		0.098849569	
Nucleus	^{65}Zn					
Band head spin			12.5		17.5	
1341	12.5	1310	14.5	1489	19.5	1937
1491	14.5	1489	16.5	1668	21.5	2116
1668	16.5	1668	18.5	1847	23.5	2295
1887	18.5	1847	20.5	2026	25.5	2474
2121	20.5	2026	22.5	2205	27.5	2653
2362	22.5	2205	24.5	2384	29.5	2832
2963	24.5	2384	26.5	2563	31.5	3011
3005	26.5	2563	28.5	2743	33.5	3190
3349	28.5	2743	30.5	2922	35.5	3369
rms	0.105359684		0.098311827		0.282340067	
Nucleus	$^{88}Mo(b_2)$					
Band head spin			9		21	
1458	30	1503	11	369	23	1084
1596	32	1623	13	487	25	1204
1743	34	1743	15	606	27	1323
1895	36	1863	17	725	29	1443
2051	38	1983	19	845	31	1563
2229	40	2103	21	964	33	1683
rms	0.031181183		0.647413223		0.244171972	

Continuation of Table 3

$E_\gamma^{exp}(I \to I-2)$	I	$E_\gamma^{VMINS3}(I \to I-2)$	I	$E_\gamma^{VMINS3}(I \to I-2)$	I	$E_\gamma^{VMINS3}(I \to I-2)$
Nucleus	$^{88}Mo(b_3)$					
Band head spin			21			
1260	32	1313	23	846		
1384	34	1417	25	950		
1521	36	1521	27	1053		
1671	38	1625	29	1157		
1816	40	1729	31	1261		
1971	42	1833	33	1365		
2135	44	1937	35	1469		
2298	46	2041	37	1573		
rms	0.062598033		0.31232299			



The variation of NS parameter versus the ratio of gamma-ray transition energies.

In Figure the NS parameter, σ is plotted against the calculated energy ratio $E_\gamma(20 \to 18)/E_\gamma(18 \to 16)$ for the SD bands for the nuclei ^{58}Ni (b_1), ^{58}Cu and ^{62}Zn, $E_\gamma(20.5 \to 18.5)/E_\gamma(18.5 \to 16.5)$ for the SD bands for the nuclei ^{65}Zn and ^{59}Cu (b_1), and $E_\gamma(27 \to 25)/E_\gamma(25 \to 23)$ for the SD bands for the nuclei ^{88}Mo (b_2) and ^{88}Mo (b_3). The NS parameter decreases as the energy ratio increases except in the case of the SD band of an odd-odd nucleus ^{58}Cu for the first group. As for the other two groups the NS parameter decreases as the energy ratio increases. It suggests that with the increasing value of the energy ratio, the rigidity of SD bands increases.

4. Conclusion

In this paper, we converted the original VMI Hamiltonian into an equivalent VMINS3 Hamiltonian wherein the moment of inertia does not appear in the rotational kinetic energy term with explicit expression given by Eq. (1), showing its dependence on the Hamiltonian parameters and the NS parameter, σ. We have investigated the band head spin for SD bands in the mass region A ≅ 60 - 90. Our main motivation was to prove that the VMINS3 model is an improvement of the VMI model. To that goal, we have employed the Descartes method to solve the quartic equation of band head spin. We provided new analytical formulae for the NS parameter, σ. By applying the VMINS3 model, it is determined that the SD bands for ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{62}Zn, ^{65}Zn and ^{88}Mo (b_2, b_3) nuclei are less rigid than the ND bands. The value of the transition energy ratio increases with increasing the rigidity of SD bands. The estimated and observed transition energies are in fairly satisfactory agreement. This method for spin assignment of SD rotational bands may help to design future experiments for SD bands.

REFERENCES

1. T. Bäck et al. Observation of superdeformed states in ^{88}Mo. Eur. Phys. J. A 6 (1999) 391.
2. F.S. Stephens. Spin alignment in superdeformed rotational bands. Nucl. Phys. A 520 (1990) c91.
3. J.A. Becker et al. Level spin and moments of inertia in superdeformed nuclei near A = 194. Nucl. Phys. A 520 (1990) c187.
4. C.S. Wu et al. Spin determination and calculation of nuclear superdeformed bands in A ~ 190 region. Phys. Rev. C 45 (1992) 261.
5. K.A. Gado. Macroscopic investigation of rotations for some deformed even-even nuclei. J. Radiat. Res. Appl. Sci. 13(1) (2020) 37.
6. M.A.J. Mariscotti, G. Scharff-Goldhaber, B. Buck. Phenomenological analysis of ground-state bands in even-even nuclei. Phys. Rev. 178 (1969) 1864.
7. A. Goel, U. Nair, A. Yadav. Band head spin assignment of Tl isotopes of superdeformed rotational bands. Cent. Eur. J. Phys. 12(9) (2014) 693.
8. J.B. Gupta, A.K. Kavathekar, Y.P. Sabharwal. Reexamination of the variable moment of inertia nuclear softness model. Phys. Rev. C 56 (1997) 3417.
9. A. Fathi, P. Mobadersany, R. Fathi. A simple method to solve quartic equations. Australian Journal of Basic and Applied Sciences 6(6) (2012) 331.
10. A.S. Shalaby. Theoretical spin assignment and study of the A ∼ 100 - 140 superdeformed mass region by using ab formula. Int. J. Phys. Sci. 9(7) (2014) 154.
11. G. Scharff-Goldhaber, C.B. Dover, A.L. Goodman. The Variable Moment of Inertia (VMI) Model and Theories of Nuclear Collective Motion. Ann. Rev. Nucl. Sci. 26 (1976) 239.
12. A.S. Shalaby. Simple model calculations of spin and quantized alignment for the A ∼ 60 - 90 superdeformed mass region. Acta Phys. Hung. A 25 (2006) 117.
13. K.A. Gado. Importation of band head spin for superdeformed bands in mass region A ∼ 60 - 90 using the variable moment of inertia model. Nucl. Phys. At. Energy 24 (2023) 336.
14. B. Singh, R. Zywina, R.B. Firestone. Table of superdeformed nuclear bands and fission isomers: Third Edition. Nucl. Data Sheets 97(2) (2002) 241.

К. А. Гадо[1,2,*]

[1] Кафедра фізики, факультет природничих наук і мистецтв, Аль-Міхва, Університет Аль-Баха, Аль-Баха, Саудівська Аравія

[2] Кафедра фундаментальних наук, Більбейський Вищий інженерний інститут, Більбейс, Шаркія, Єгипет

*Відповідальний автор: qjado76@gmail.com

ЯДЕРНА М'ЯКІСТЬ У МОДЕЛІ ЗІ ЗМІННИМ МОМЕНТОМ ІНЕРЦІЇ ТА ЇЇ ЗАСТОСУВАННЯ ДО НАДДЕФОРМОВАНИХ СМУГ В ОБЛАСТІ МАС А ≈ 60 – 90

Для наддеформованих (SD) смуг ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr (b_1), ^{88}Mo (b_1, b_2, b_3) і ^{89}Tc в області мас A ≈ 60 - 90, параметр ядерної м'якості (NS), σ, був розрахований за допомогою моделі VMINS3. Смуги SD ^{58}Ni (b_1), ^{58}Cu, ^{59}Cu (b_1), ^{62}Zn, ^{65}Zn і ^{88}Mo (b_2, b_3) мають значення параметра NS, які перевищують значення нормально деформованих смуг, що вказує на меншу жорсткість. Залежність флуктуації параметра NS від відношення енергій гамма-випромінювання, *R*, в області мас A ≈ 60 - 90 є одним із результатів дослідження. Відношення енергій переходів було використано для розрахунку головного спіну, I_0, за методом Декарта (найкращий метод розв'язання рівняння четвертої степені за допомогою кубічного рівняння), з послідуючим використанням середньоквадратичного відхилення. Оцінені та спостережені енергії переходів добре узгоджуються.

Ключові слова: модель зі змінним моментом інерції, ядерна м'якість, визначення спіну.

Надійшла/Received 10.09.2023