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ALPHA-NUCLEUS INTERACTION POTENTIAL

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The parameters of the interaction potential between alpha-particle and nucleus are evaluated in the framework of unified model for alpha-decay and alpha-capture (UMADAC). The alpha-decay half-lives are evaluated in the framework of the cluster model using the WKB approximation. Both processes, alpha-decay and alpha-capture, are considered as penetration of the alpha-particle through the potential barrier formed by nuclear, Coulomb and centrifugal forces. The spins and the parities of parent and daughter nuclei, the quadrupole and hexadecapole deformations of daughter nuclei are taken into account at evaluation of the alpha-decay half-lives. The alpha-nucleus interaction potential is obtained by fitting experimental data for both the alpha-decay half-lives of 344 nuclei and the alpha-capture cross-sections of ^{40}Ca, ^{44}Ca, ^{59}Co, ^{208}Pb and ^{209}Bi.

1. Introduction

Alpha-decay is very important process in nuclear physics [1 - 10]. The experimental information on alpha-decay half-lives is extensive and is being continually updated [1 - 4, 8]. The theory of alpha-decay was formulated by George Gamow in 1927. Subsequently various microscopic, macroscopic cluster and fission approaches to the description of alpha-decay have been proposed [9] (and Refs cited in this paper). The simple empirical relations, which describe the alpha-decay half-lives, are extensively discussed too [3, 5 - 8, 10].

The alpha-decay process involves sub-barrier penetration of alpha-particle through the barrier, caused by interaction between alpha-particle and nucleus. The alpha-capture reaction proceeds in the opposite direction to the decay. That is why it is natural to use data for both the alpha-decay half-lives and the alpha-capture reactions around barrier for determination of the alpha-nucleus interaction potential [9]. Knowledge of the alpha-nucleus interaction potential is a key for the analysis of various reactions between alpha-particle and nuclei not only in nuclear physics, but in astrophysics also.

We use a combination of updated alpha-decay half-lives data set for the ground-state to ground-state transitions from data compilations *Nubase* [1], [3] and [4] as well as the alpha-capture cross-sections of ^{40}Ca [11, 12], ^{44}Ca [11], ^{59}Co [13], ^{208}Pb [14] and ^{209}Bi [14] around barrier. Thus, we can evaluate the alpha-nucleus potential deeply below and around barrier with high degree of accuracy. The total quantity of the considered alpha-emitters with exactly defined characteristics is 344 nuclei.

Many alpha-emitters are deformed. Therefore corresponding interaction potential should depend on the angle θ between the direction of alpha-emission and the axial-symmetry axis of the deformed nucleus. Both the alpha-decay half-life and the transmission coefficient for tunneling through the barrier are strongly dependent on θ [9], because the transmission coefficient exponentially dependents on the alpha-nucleus potential values. Therefore in the present work we take into account both quadrupole and hexadecapole deformations of daughter nuclei.

The interaction potential between alpha-particle and nuclei consists of nuclear, Coulomb and centrifugal parts. Alpha-transitions between ground states of even-odd, odd-even and odd-odd nuclei can proceed at non-zero values of angular momentum of the alpha-particle, when the spins and/or parities of parent and daughter nuclei are different. As the result, the centrifugal potential distinctly contributes to the total alpha-nucleus potential at small distances between daughter nucleus and alpha-particle. The alpha-decay half-life depends exponentially on the action, which is very sensitive to the alpha-nucleus potential. Therefore accurate consideration of the Alpha-transitions should take into account the spins and parities of parent and daughter nuclei and angular moment of the emitted alpha-particle.

The experimental values and theoretical estimates of the ground-state spins and parities are known for many nuclei [1, 2]. Moreover the number of nuclei with known values of ground-state spin and parity is permanently extended. Therefore we evaluate the alpha-nucleus interaction potential using available updated data for alpha-decay half-lives, the spins and parities of the nuclei's ground-states and alpha-capture reaction cross-sections. Due to this our approach became more accurate.

The unified model for alpha-decay and alpha-capture (UMADAC) is shortly discussed in Section 2. The selection of adjustable parameters and discussion of the results are given in Section 3. The Section 4 is dedicated to conclusions.

2. UMADAC

The alpha-decay half-life $T_{1/2}$ is calculated as [9]

$$T_{1/2} = \hbar \ln 2 / \Gamma,$$

where

$$\Gamma = (1/4\pi)\int \gamma(\theta,\phi)\,d\Omega$$

is the total width of decay, $\gamma(\theta, \varphi)$ is the partial width of alpha-emission in direction θ and φ, Ω is the space angle. The width of alpha-emission in direction θ for axial-symmetric nuclei is given by the following:

$$\gamma(\theta) = \hbar 10^{\nu}\, t(Q_\alpha, \theta, \ell),$$

where 10^{ν} is the alpha-particle frequency assaults the barrier, which takes into account the alpha-particle preformation, $t(Q_\alpha, \theta, \ell)$ is the transmission coefficient, which shows the probability of penetration through the barrier and Q_α is the energy of emitted alpha-particle.

The transmission coefficient can be obtained in the semiclassical WKB approximation:

$$t(Q_\alpha, \theta, \ell) = 1/\left\{1 + \exp\left[\frac{2}{\hbar}\int_{a(\theta)}^{b(\theta)} dr \sqrt{2\mu\left(v(r,\theta,\ell,Q_\alpha) - Q_\alpha\right)}\right]\right\},$$

where $a(\theta)$ and $b(\theta)$ are the inner and outer turning points determined from the equations $v(r,\theta,\ell,Q_\alpha)\big|_{r=a(\theta),\, b(\theta)} = Q_\alpha$, μ is the reduced mass. The alpha-nucleus potential $v(r, \theta, \ell, Q_\alpha)$ consists of Coulomb $v_C(r, \theta)$, nuclear $v_N(r, \theta, Q_\alpha)$ and centrifugal $v_\ell(r)$ parts, i.e.

$$v(r, \theta, \ell, Q_\alpha) = v_C(r, \theta) + v_N(r, \theta, Q_\alpha) + v_\ell(r),$$

where

$$v_C(r,\theta) = \frac{2Ze^2}{r}\left[1 + \frac{2R^2}{5r^2}\beta_2 Y_{20}(\theta) + \frac{3R^4}{9r^4}\beta_4 Y_{40}(\theta)\right], \qquad \text{for } r \geq r_C,$$

$$v_C(r,\theta) \approx \frac{2Ze^2}{r_C}\left[\frac{3}{2} - \frac{r^2}{2r_C^2} + \frac{3R^2}{5r_C^2}\beta_2 Y_{20}(\theta)\left(2 - \frac{r^3}{r_C^3}\right) + \frac{3R^4}{9r_C^4}\beta_4 Y_{40}(\theta)\left(\frac{7}{2} - \frac{5r^4}{2r_C^2}\right)\right], \quad \text{for } r \leq r_C,$$

$$v_N(r,\theta,Q_\alpha) = \frac{V(Q_\alpha)}{1 + \exp\left[\left(r - r_m(\theta)\right)/d\right]}, \qquad v_\ell(r) = \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}.$$

Here Z, R, β_2, β_4 are, respectively, the number of protons, the radius, the quadrupole and hexadecapole deformation parameters of the nucleus, interacting with the alpha-particle; e is the charge of proton, $Y_{20}(\theta)$ and $Y_{40}(\theta)$ are harmonic functions; $V(Q_\alpha)$ and $r_m(\theta)$ are, correspondingly, the strength and effective radius of the nuclear part of alpha-nucleus potential. The inner turning point $a(\theta)$ is close to both $r_m(\theta)$ and r_C.

The alpha-particle emission from nuclei obeys the spin-parity selection rule. Let mark j_p, π_p and j_d, π_d as spin and parity values of the parent and daughter nuclei respectively. The alpha-particle has zero value of spin and positive parity, therefore the minimal value of angular momentum $\ell_{\min}$ at the Alpha-transition between states with j_p, π_p and j_d, π_d is

$$l_{\min} = \begin{cases} \Delta_j & \text{for even } \Delta_j \text{ and } \quad \pi_p = \pi_d, \text{ and for odd } \Delta_j \text{ and } \quad \pi_p \neq \pi_d, \\ \Delta_j + 1 & \text{for odd } \Delta_j \text{ and } \quad \pi_p = \pi_d, \text{ and for even } \Delta_j \text{ and } \quad \pi_p \neq \pi_d, \end{cases} \quad \text{where } \Delta_{\mathrm{j}} = \left|j_p - j_d\right|.$$

Note that the value of alpha-particle angular momentum ℓ can be large $\ell_{\min}$. This is related to the intrinsic structure of the single-particle levels around Fermi levels in parent and daughter nuclei and the way of alpha-particle formation in parent nuclei. There are many cases of Alpha-transition between ground states with non-zero value of angular momentum. For the sake of simplicity we suppose that the angular momentum of Alpha-transition between ground states ℓ equals to $\ell_{\min}$. So, the centrifugal part of the alpha-nucleus potential is determined according to the spin-parity selection rule for Alpha-transition. The centrifugal contribution to the potential is very important for alpha-emission from even-odd, odd-even and odd-odd nuclei.

The alpha-capture cross section of axial-symmetric nucleus at around-barrier collision energy Q_α is equal to [9]:

$$\sigma(Q_\alpha) = \frac{\pi \hbar^2}{2\mu Q_\alpha}\int_0^{\pi/2} \sum_\ell (2\ell + 1)\, t(Q_\alpha, \theta, \ell) \sin(\theta)\, d\theta.$$

The transmission coefficient $t(Q_\alpha, \theta, \ell)$ is evaluated using the semiclassical WKB approximation in the case of collision between alpha-particle and stiff magic or near-magic spherical nuclei at collision energies Q_α below barrier. The transmission coefficient is approximated by an expression for a parabolic barrier at collision energies higher than the barrier energy.

3. Discussion and results

3.1. Input data

We chose data for $T_{1/2}$ of 344 alpha-decay transitions between the ground states of parent and daughter nuclei with exact values of both the half-live and the alpha-decay branch ratio. As the result, 136 even-even, 84 even-odd, 76 odd-even and 48 odd-odd alpha-emitters in large mass 106 < A < 261 and charge 52 < Z < 107 ranges were fitting to the experimental data.

The energy of alpha-particle, emitted from nucleus at alpha-decay, is calculated using recent evaluation of atomic mass data [1]. Naturally, atoms participate in experiments. Therefore, the effect of atomic electrons on the alpha-particle's energy should be taken into account:

$$Q_\alpha = \Delta M_p - \left(\Delta M_d + \Delta M_\alpha\right) + 10^{-6} k \left(Z_p^\varepsilon - Z_d^\varepsilon\right),$$

where ΔM_p, ΔM_d, ΔM_α are, correspondingly, the mass-excess of parent, daughter nuclei and alpha particle. The last term in equation describes the effect of atomic electrons, kZ^ε represents the total binding energy of Z electrons in the atom, k = 8.7 eV and ε = 2.517 for nuclei with $Z \geq 60$ and k = 13.6 eV and ε = 2.408 for nuclei with $Z < 60$ [10].

The experimental data on deformation parameters β_2 and β_4 are taken mostly from the RIPL-2 database [15]. When no experimental data exist for a nuclide in the RIPL-2 compilation, values of the deformation parameters are picked up from the macroscopic-microscopic model [5]. The experimental values of spin and parity for nuclei are taken from [1] and, when there are no data exist, from [2]. Unfortunately, for some nuclei the values of spin and parity are absent in both Refs. [1] and [2]. In these cases we take value 0^+ and accept corresponding value of the alpha-particle's angular moment equals zero. The data for alpha-capture cross sections of ^{40}Ca, ^{44}Ca, ^{59}Co, ^{208}Pb and ^{209}Bi were taken from Refs. [11 - 14]. Alpha-capture cross sections considered using the same approach as in Ref. [9].

3.2. Parameters

Describing both the half-lives for ground-state to ground-state Alpha-transitions in 344 nuclei and alpha-capture cross-sections for nuclei mentioned above, we parameterized $V(Q_\alpha)$, $r_m(\theta)$, d, r_C and determined these parameters by searching the minimum of the function

$$F = \left(5D_{e-e} + D_{e-o} + D_{o-e} + D_{o-o}\right) +$$

$$+20\left[3D_\sigma^{^{208}Pb} + D_\sigma^{^{40}Ca,1} + D_\sigma^{^{40}Ca,2} + D_\sigma^{^{44}Ca} + 3D_\sigma^{^{209}Bi} + D_\sigma^{^{59}Co}\right].$$

Here $D_{e-e} = \sum\limits_{e-e} \left[\log_{10}\left(T_{1/2}^{theor}\right) - \log_{10}\left(T_{1/2}^{\exp}\right)\right]^2$ is the difference between decimal logarithm of theoretical $T_{1/2}^{theor}$ and experimental $T_{1/2}^{\exp}$ values of alpha-decay half-lives for the set of even-even nuclei. $D_{e-o}, D_{o-e}, D_{o-o}$ are differences similar to D_{e-e} for even-odd, odd-even and odd-odd data sets respectively. $D_\sigma = \sum\limits_k \log_{10}\left[\sigma^{theor}(E_k) - \sigma^{\exp}(E_k)\right]$, where $\sigma^{theor}(E_k)$ and $\sigma^{\exp}(E_k)$ are theoretical and experimental values of alpha-capture cross section of corresponding nucleus at energy E_k. Different weights in the last equation of function F are related to different accuracy of various experimental data. As the result of minimization for various forms of parameters $V(Q_\alpha)$, $r_m(\theta)$, d and r_c we found the F function's minimum at:

$$V(Q_\alpha) = \left[-40.103 - \frac{0.102}{A^{1/3}} - 9.193 I + 0.0001\frac{Q_\alpha}{A^{1/3}} + \frac{0.061 Y_{20}(\theta)\beta_2}{A^{1/6}}\right], \quad d = 0.687 - 0.366/A^{1/3},$$

$$r_m(\theta) = 1.168 + R\left(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta)\right), \quad R = 1.291 A^{1/3}\left(1 + \frac{1.4088}{A} - 0.0994 I\right),$$

$$\nu = 19 + S - 0.135\sqrt{Z}\,A^{1/6} + 0.9132\left(\left(-1\right)^{\ell} - 1\right) - 0.041\frac{Z}{\sqrt{Q_\alpha}} + 0.656\,I - 1.644\beta_2 - 1.211\beta_4 + 0.069\frac{\ell\left(\ell+1\right)}{A^{1/6}},$$

where A, Z are the number of nucleons and protons in nucleus interacting with alpha-particle, $I = \left(A-2Z\right)/A = \left(N-Z\right)/A$, $S = 4.1382$, $S = 3.5701$, $S = 3.8246$ and $S = 3.6625$ for even-even, even-odd, odd-even and odd-odd nuclei correspondingly. Note that our calculations contain 22 parameters. The results of our UMADAC are presented in Figs. 1 and 2.

3.3. Alpha-captures cross sections

The alpha-capture cross-sections are compared with experimental data in Fig. 1. The data for alpha-capture of ^{208}Pb and ^{209}Bi are precisely described in the framework of the UMADAC. The cross section for alpha-capture of ^{40}Ca, ^{44}Ca, and ^{59}Co are reproduced well the experimental data in the confined range of energy. In the framework of UMADAC a one-dimensional model for evaluation of the fusion cross-section between an alpha-particle and a spherical nucleus is used. We also made result of the coupled-channel calculation of the fusion cross-section for reaction $\alpha + {}^{208}$Pb by using CCFULL code, and presented the result in Fig. 1.

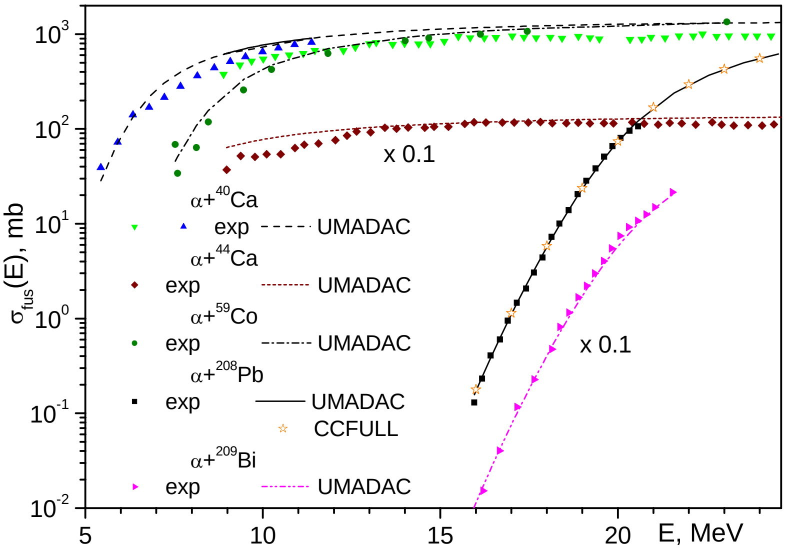

Fig. 1. Experimental and theoretical alpha-capture cross sections of ^{208}Pb, ^{209}Bi, ^{40}Ca, ^{44}Ca, and ^{59}Co.

3.4. Alpha-decay half-lives

The evaluated alpha-decay half-lives agree well with 344 experimental data points, see Fig. 2. The experimental values of half-lives are scattered over an extremely wide range from 10^{-8} to 10^{27} s. The alpha-decay half-lives are very nicely described in the case of even-even parent nuclei.

Using the alpha-nucleus potential obtained in the framework of the UMADAC, we evaluate alpha-decay half-lives for all possible alpha-emitters within the range $10^{-9} \le T_{1/2} \le 10^{38}$ s. As the result, there are 1246 possible alpha-emitters. The upper limit we obtained for the value of $T_{1/2} \le 10^{38}$ s gives adequate margin for planning experiments in foreseeable future.

4. Conclusions

We determined the alpha-nucleus potential by using the data for alpha-decay half-lives of 344 alpha-emitters and around-barrier alpha-capture cross sections of ^{40}Ca, ^{44}Ca, ^{59}Co, ^{208}Pb, ^{209}Bi, taking into account deformation and spin-parity effects. We predicted alpha-decay half-lives for the ground-state to ground-state transitions in 902 nuclei. By taking into account the spins and parities of parent and daughter nuclei we obtain spectacular improvement of description of the alpha-decay half-lives in even-odd, odd-even and odd-odd nuclei.

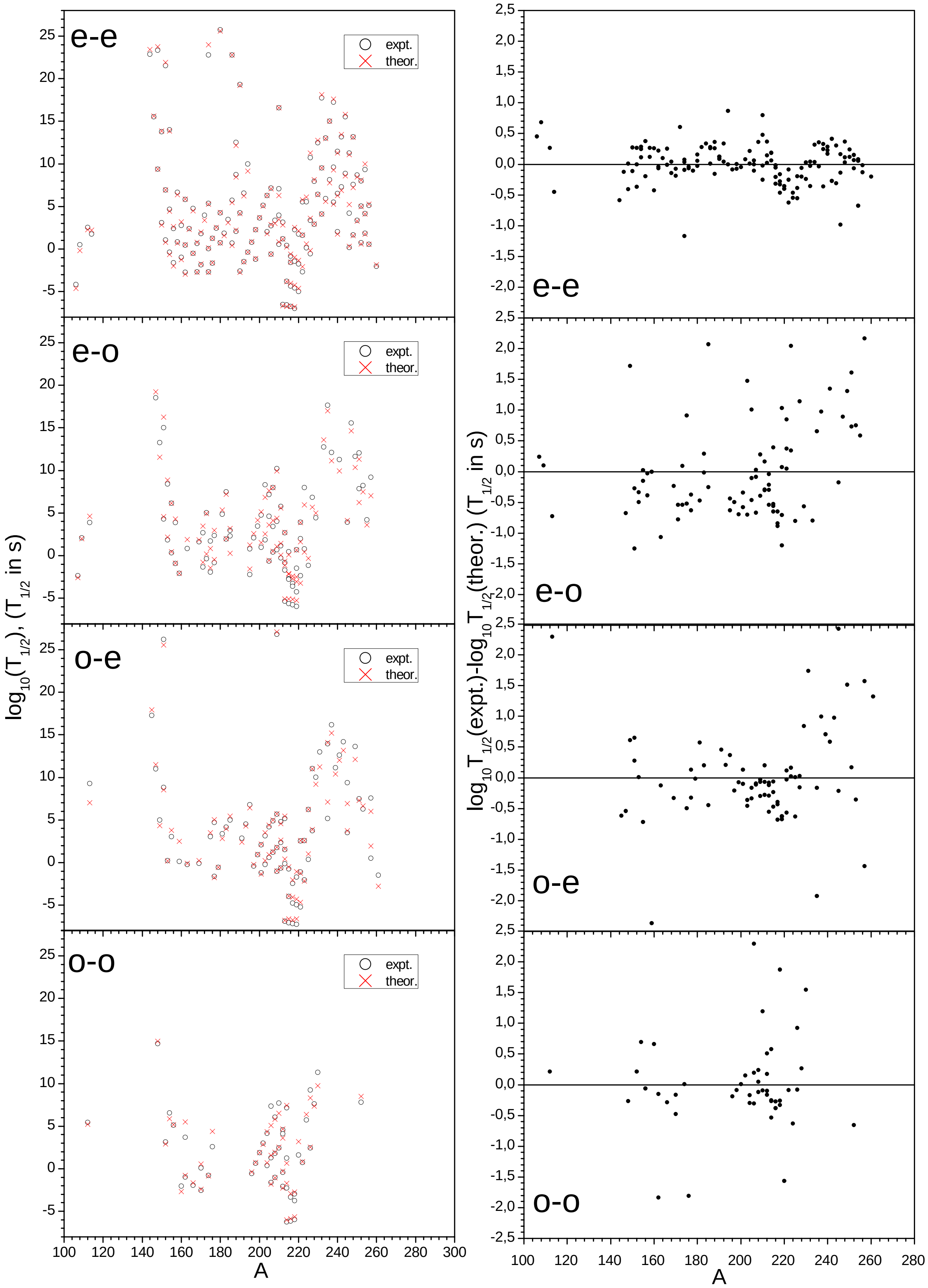

Fig. 2. Differences between experimental and theoretical values of $\log\left(T_{1/2}\right)$ for alpha-decays in even-even (e-e), even-odd (e-o), odd-even (o-e) and odd-odd (o-o) parent nuclei.

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АЛЬФА-ЯДЕРНИЙ ПОТЕНЦІАЛ ВЗАЄМОДІЇ

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У рамках моделі (UMADAC), що одночасно описує альфа-розпад та захват альфа-частинки ядром, визначено параметри альфа-ядерного потенціалу взаємодії. Обидва процеси, альфа-розпад та захват альфа-частинки ядром, розглядаються як проникнення альфа-частинки крізь потенціальний бар’єр, утворений кулонівськими, ядерними та відцентровими силами. Використовуючи кластерну модель та наближення WKB, визначено періоди напіврозпаду альфа-активних ядер. У розрахунках враховано спіни, парності материнського та дочірнього ядра, квадрупольні та гексадекапольні деформації дочірнього ядра. Альфа-ядерний потенціал взаємодії визначено в процесі підгонки експериментальних даних про періоди напіврозпаду 344 альфа-активних ядер та перерізів захвату альфа-частинок ядрами ^{40}Ca, ^{44}Ca, ^{59}Co, ^{208}Pb, ^{209}Bi.

АЛЬФА-ЯДЕРНЫЙ ПОТЕНЦИАЛ ВЗАИМОДЕЙСТВИЯ

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В рамках модели (UMADAC), которая одновременно описывает альфа-распад и захват альфа-частицы ядром, определены параметры альфа-ядерного потенциала взаимодействия. Оба процесса, альфа-распад и захват альфа-частицы ядром, рассматриваются как проникновение альфа-частицы сквозь потенциальный барьер, образованный кулоновскими, ядерными и центробежными силами. Используя кластерную модель и WKB приближение, определены периоды полураспада альфа-активных ядер. В расчетах учитываются спины, четности материнского и дочернего ядер, квадрупольная и гексадекапольная деформации дочернего ядра. Альфа-ядерный потенциал взаимодействия определен в процессе подгонки экспериментальных данных о периодах полураспада 344 альфа-активных ядер и сечений захвата альфа-частиц ядрами ^{40}Ca, ^{44}Ca, ^{59}Co, ^{208}Pb, ^{209}Bi.

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