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0^+ STATES AND COLLECTIVE BANDS IN DEFORMED ACTINIDE NUCLEI

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By means of the (p, t) reaction we studied the excitation spectra of 0^+ states in the deformed nuclei ^{228}Th, ^{230}Th, and ^{232}U, using the Q3D magnetic spectrograph facility at the Munich tandem accelerator. At small reaction angles the 0^+ transfer angular distributions have steeply rising cross sections which allow identifying these states in otherwise very complicated and dense spectra. For each of these nuclei we resolve typically about ten excited states with safe 0^+ assignments. The studied excitation energies range up to 2.5, 2.7, and 2.3 MeV, respectively. The results are compared with IBA calculations in the *spdf*-boson space. This highly schematic collective model description, including octupole collectivity, but neglecting other relevant degrees of freedom, gives numbers of excited 0^+ states in these actinide nuclei that are rather close to the observed ones. Sequences of states are selected which can be treated as rotational bands. Inertial parameters are obtained at fitting energies of these bands and they are discussed in connection with the IBM calculations.

1. Introduction

In nuclei the appearance of excited 0^+ states is indicative for the presence of specific modes of nuclear excitations, and their study is therefore of considerable interest. For the closed shell nucleus ^{208}Pb only two excited 0^+ states have been observed so far: a neutron pairing vibrational mode [1] and an octupole two-phonon excitation [2 - 4]. Intruder configurations may contribute in addition as known for ^{40}Ca and for less closed nuclei as ^{90}Zr, ^{96}Zr [5], ^{112}Cd [6], and others. In deformed nuclei a large variety of excited 0^+ states is expected as discussed in detail below.

Excited 0^+ states are easily identified via (p, t) and (t, p) reactions in otherwise complicated and dense excitation spectra, because they have steeply rising cross sections at small reaction angles; at the same time only natural parity states are populated stronger. Rich 0^+ spectra had been observed in earlier (p, t) studies of medium weight nuclei as ^{146}Nd [7], ^{146}Sm [8], ^{134}Ba, ^{132}Ba [9], and ^{114}Sn [10], where typically nine excited 0^+ states had been resolved in the excitation energy range up to 2.5 - 4 MeV. To reproduce theoretically the number of 0^+ states in ^{146}Nd and ^{146}Sm and to account for the excitation strengths in (p, t), specific particle-hole and particle-particle correlations had to be considered in addition to particle-core coupling effects [7, 8]. In addition to 0^+ transfer from $J^{\pi} = 0^+$ target nuclei one has to mention 0^+ transfer studies in (p, t) from target nuclei with spin, e.g. the study of such states in ^{229}Pa [11].

In deformed heavy nuclei the lowest 0^+ states had been studied systematically by the Argonne group in the early seventies [12]. For all the actinide nuclei they observed strong excitations of the first excited 0^+ states in the (p, t) reaction. Combined with other available evidences, in particular the strong Coulomb excitation of the associated rotational bands, they conclude that these 0^+ states represent a stable collective excitation different in character from both the β vibration and the most common formulation of the pair vibration. No such excitations were seen in the W-Pt region.

In recent years a few more 0^+ states had been identified [13 - 16], but the understanding of excited 0^+ states is still a challenge for nuclear theory. The work of Soloviev and coworkers [17 - 20] within their QPM model provides some microscopic understanding of the low energy excitation spectra, including the lowest 0^+ states. We have to refer also to the study of Otsuka and Sugita [21] which deals with the *spdf* IBM in application to the actinide nuclei. These calculations were restricted to the first two excited 0^+ excitations (known to this time).

Study of 0^+ states in actinide nuclei was stimulated by observation of at least 8 excitations in ^{229}Pa [11] at the $L = 0$ transfer in the (p, t) reaction. The nucleus ^{229}Pa can be considered as ^{228}Th + p and a question is relevant whether these states are core excitations with a proton as spectator. Additional interest in 0^+ states came from a recent (p, t) study of ^{158}Gd [22], where 13 excited 0^+ states had been identified. In this paper we report on a study of 0^+

states of ^{228}Th, ^{230}Th, and ^{232}U, observed in (p, t) spectra. The choice of the nuclei was restricted by the availability of the respective targets. Our experimental results about states with J^{π} values different from 0^+ or of weakly excited states, where tentative 0^+ assignments may be possible, will be presented in separate publications [23]. We obtained new and detailed experimental information about the 0^+ excitation spectra and their strength distributions in (p, t). A comparison with IBA emphasizes the relevance of octupole collectivity. This, however, cannot replace the need for microscopic calculations. A microscopic understanding of the physics of excited 0^+ states is the only way to ensure that we know all relevant degrees of freedom determining the spectra of heavy deformed nuclei.

2. Excited 0^+ states in deformed nuclei

Up to now, for deformed heavy nuclei we do not know calculations which provide 0^+ excitation energies in a larger range of excitation energies and (p, t) strength function distributions. In comparison to spherical nuclei additional 0^+ states have to be expected in deformed nuclei because of the quantization with respect to the intrinsic axis: an excitation mode with angular momentum J^{π} splits into states distinguished by their K quantum numbers, which range from zero to J, compare Fig. 1.

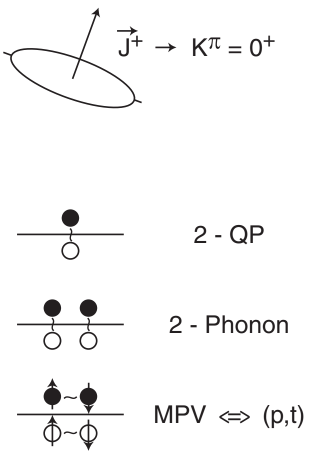

Fig. 1. Schematic presentation of the different kinds of excitations in deformed nuclei, yielding excited $K^{\pi} = 0^+$ states: positive parity two quasiparticle excitations (of collective and of noncollective nature), two phonon excitations and monopole pairing excitations.

For axially symmetric deformed nuclei 0^+ states result from K-splitting of collective and non-collective modes of excitation. For most quantum numbers of natural parity the lowest states are collective; states related to specific two quasiparticle modes are expected to follow at considerably higher excitation energies. The distributions of these two components determine the actual excitation spectra; compare e.g. ^{208}Pb [4]. Thus, low lying 0^+ states are expected to be described within models accounting for the respective collective features; however, additional 0^+ states resulting from specific two quasiparticle modes will contribute in addition.

In addition, there are the two phonon excitations of all the collective modes at low energies. These are the quadrupole, octupole and hexadecupole phonons with $J^{\pi} = 2^+, 3^-$, and 4^+, respectively. Separate from these we have to consider the collective monopole pairing vibration (MPV). In (p, t) the observed excitation strength of excited 0^+ states results from an admixture of the neutron component of the MPV.

A. Quadrupole- and octupole collectivity

Accounting for quadrupole- and octupole collectivity only, in [24] the IBA was used within the *spdf* boson space to see how many of the 13 excited 0^+ states of ^{158}Gd identified in [22] can be expected in this way. The calculations of [24] had been restricted to a highly schematic discussion: the simplest form of the Hamiltonian is used and mixing between *d* and *pf* bosons are neglected. The parameters are chosen to reproduce the excitation energies of low lying spectra, especially the bands of negative parity. In the energy range considered, their IBA calculation predicts five excited 0^+ states of pure *sd* (quadrupolar) bosonic structure and three additional excited 0^+ states with two bosons in the *pf* boson space. The latter ones are related to octupole two phonon excitations (OTP). Comparing the 13 observed and the eight predicted excited ^{158}Gd 0^+ states, the model accounts for a considerable fraction of the observed states.

In the present study of actinide nuclei we deal with deformed nuclei of strong and different octupole collectivity. A measure of the octupole collectivity is given by the excitation energy of the first 1^- state. The excitation energies of ^{228}Th, ^{230}Th and ^{232}U are $E_x(1^-) = 328$, 508, and 563 keV, respectively. Thus, we are in a position to compare three neighboring nuclei which differ by one neutron pair and/or by one proton pair only but vary strongly with respect to octupole collectivity. The experimental information on the low-lying states of ^{228,230}Th and ^{232}U is summarized in [25 - 27] and several more recent experimental investigations [13 - 16, 20, 28, 29], including a (p, t) study of the lowest states [15]. Addressed to the theoretical understanding of these nuclei there are a number of recent publications which are related mainly to features of octupole collectivity [30 - 36], triaxiality [37, 38], and to the description within interacting boson approximations [39 - 44].

In axially symmetric statically deformed nuclei any excitation of positive parity causes a $K^\pi = 0^+$ state. Since the nuclei we study are prolate we have to expect, as for Nilsson states, strongest binding for the lowest K state. This enhances the number of 0^+ states at low excitation energies. Large energy shifts result from the K-splitting of the octupole vibrational state, as discussed in [39] for rare earth nuclei: the $K^\pi = 0^-$ and $K^\pi = 1^-$ bandheads are observed at very low excitation energies, whereas the $K^\pi = 3^-$ strength is shifted upwards in energy.

In the IBM study of 0^+ states in ^{158}Gd [24] the octupolar excitations contribute to the number of $K^\pi = 0^+$ states via two-phonon excitations. If the coupling of these two-phonon octupole excitations with quadrupole phonon excitations is neglected, the sdf-IBA calculation gives the $K^\pi = 0^+$ excitations of pure quadrupolar type at exactly the same energies as a pure sd-IBA, and all additionally calculated $K^\pi = 0^+$ states result with pure f^2-boson content, equivalent to octupole-two-phonon excitations.

B. Further degrees of freedom

One has to expect, however, further $K^\pi = 0^+$ states, of both collective and non-collective nature. Collective excitations of multipolarity 0^+ and 4^+, the monopole pairing vibrational excitation (MPV) and some hexadecapole vibrational collectivity, have to be expected and shall lead to a considerable number of additional excited $K^\pi = 0^+$ states.

The monopole pairing vibration is well established for ^{208}Pb. Of the two known 0^+ states of ^{208}Pb, the lower one is the MPV state, and the higher one is the 0^+ octupole-two-phonon excitation [4]. According to the literature one expects at least two kinds of MPV states, one for neutron-pair excitations (n-MPV) and one for proton-pair excitations (p-MPV). The latter one usually is expected at higher excitation energy. Because of its collective nature, in a (p, t) reaction the n-MPV state is expected to be strongly excited. In case of a relatively dense spectrum of 0^+ states the n-MPV state will mix with nearby states. In our case mixing is significant and one may discuss as a center of the transfer strength excitation energy near 1600 keV and equate this with unperturbed excitation energy of the neutron monopole pairing vibrational state (n-MPV). The pairing vibrational excitations result from a particle-particle coupling in the residual interaction, which, however, is not included in the usual RPA- or IBA-like calculations. Within these frames the pairing vibrational excitation has to be considered as a kind of intruder configuration.

Hexadecapole vibrational collectivity is established in spherical nuclei at excitation energies near or slightly above the collective octupole state. In inelastic scattering it is related with large one-step transition strength to excited 4^+ states. The analysis of hexadecapole collectivity is complicated because of the necessity to differentiate against quadrupole-two-phonon 4^+ excitations and related processes. Also the hexadecapole strength may be distributed over a few neighboring states, in contrast to the octupole strength, which is concentrated in the lowest 3^- state. As for the octupole vibration, it is reasonable to assume that the $K^\pi = 0^+$ hexadecapole excitation could be pushed down to rather low energy. This may be treated formally by introducing a g boson and expanding a *sd*-IBA to a *sdg*-IBA, analogous to the case of octupole collectivity. In this way additional $K^\pi = 0^+$ states will derive resulting from quadrupole-hexadecapole coupling.

The very interesting question with respect to the observed (p, t) strength is the contribution of noncollective $K^\pi = 0^+$ states, resulting from noncollective two-quasiparticle (2QP) excitations. For each multipolarity the noncollective 2QP excitations are expected at considerable higher energy than the respective collective vibrational states.

3. Experiments

A. Experimental procedure, spectra

The (p, t) experiments have been performed at the Munich tandem accelerator with a beam of 25 MeV protons. The reaction products have been analyzed with the Q3D magnetic spectrograph [45] and detected in its focal plane. We used two different focal plane detectors which are multiwire proportional chambers with read-out of a cathode foil structure for position determination and $\Delta E/E_{rest}$ particle identification [46 - 48]. The targets ^{230}Th, ^{232}Th, and ^{234}U had a thickness of 100 μg/cm^2 each, evaporated onto 22 μg/cm^2 thick carbon backings. The isotopic purity of the ^{230}Th and ^{234}U was about 99 %. The resulting triton spectra have a resolution of 6 - 7 keV FWHM and are virtually background free. Angular distributions of the cross sections are extracted from spectra at ten different laboratory angles.

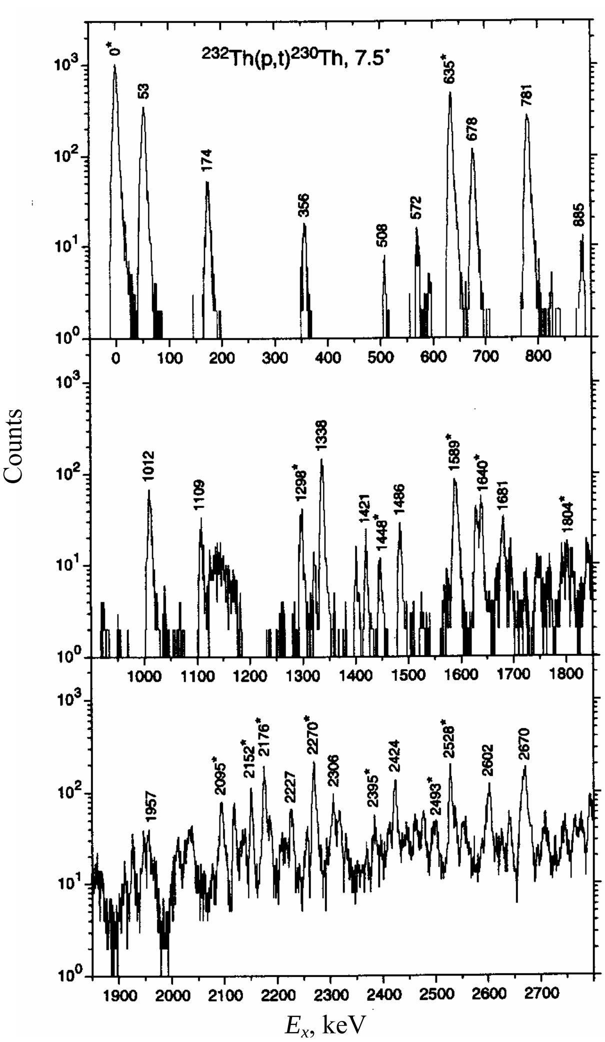

Fig. 2. Complete spectrum for ^{232}Th(p, t) ^{230}Th (E_p = = 25 MeV) in logarithmic scale for a detection angle of 7.5°. Some levels are labeled with their excitation energy in keV. States assigned as 0^+ are marked with an asterisk.

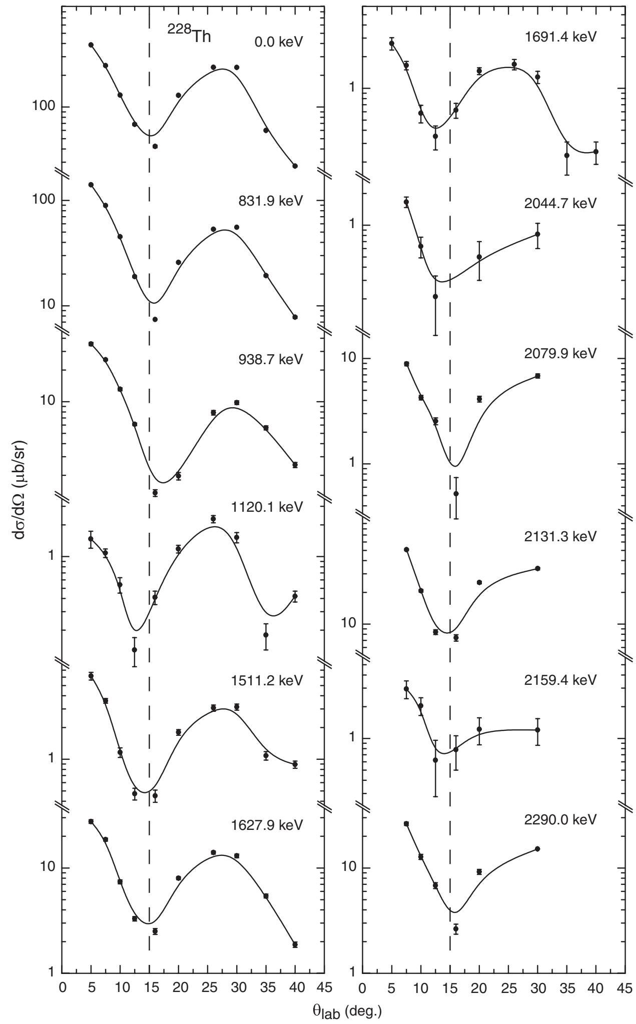

Fig. 3. Angular distributions of assigned 0^+ states in ^{228}Th. The lines are splines drawn to guide the eyes and have no further meaning.

Typical spectra for a detection angle of 7.5° is shown in Fig. 2 for ^{230}Th. At this angle the 0^+ states have comparatively large cross sections. For calibration purposes spectra from different target nuclei had been taken in the same magnetic setting, including the reactions ^{184}W(p, t) and ^{186}W(p, t).

B. Experimentally obtained 0^+ states

The 0^+ transfers are identified from their typical pattern in the differential cross section angular distributions. At very small reaction angles the 0^+ transfer angular distributions have steeply rising cross sections and a sharp minimum at a detection angle of $\approx 15^\circ$. This allows to identify these states in otherwise very complicated and dense spectra.

The differential cross section angular distributions of those observed transfers to ^{228}Th, ^{230}Th, and ^{232}U, which we assign as 0^+ are shown – in logarithmic scale – in Figs. 3, 4, and 5. These are all angular distributions which are very similar in shape to the respective ground state transitions.

One may compare these angular distributions with those for transitions with different quantum numbers: Fig. 6 shows angular distributions of transitions to the first excited 1^-, 2^+, 3^-, 4^+, and 6^+ states of these nuclei. The 1^- transitions are very weak and may result from multistep excitations. All the other rather strong transitions show J^π typical shapes: The angular position of the first minimum in cross section shifts systematically with increasing transferred angular momentum. For a given quantum number the distributions for ^{228}Th, ^{230}Th, and ^{232}U are very similar.

Since we have no knowledge of the microscopic structure of these levels, the assignments of quantum numbers at present is restricted to this kind of pattern recognition. In (p, t) for a given quantum number J^π we can expect – at least in the limit of DWBA – relative cross section angular distributions rather independent of the specific structure of the

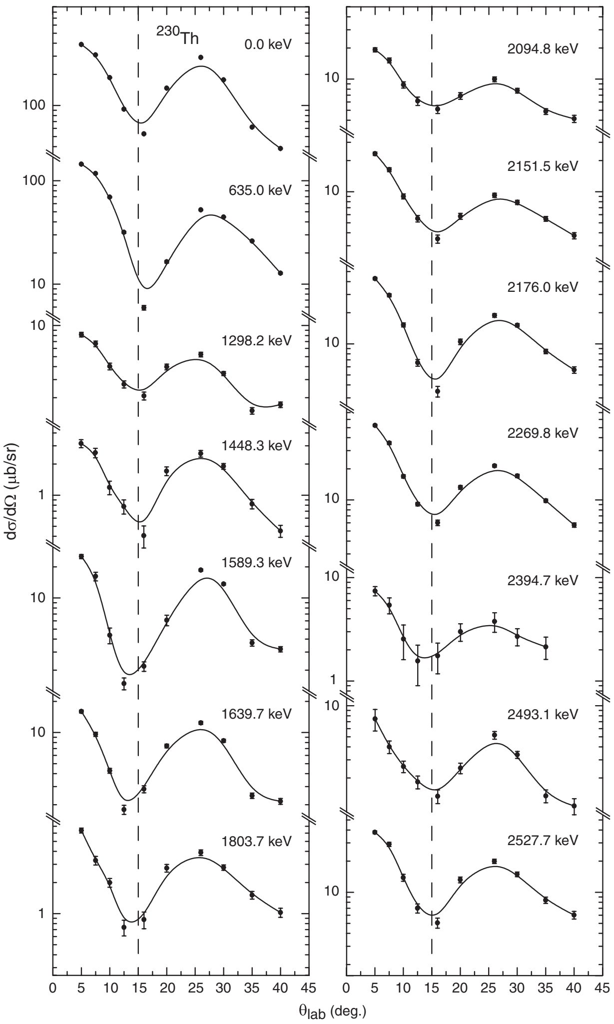

Fig. 4. Angular distributions of assigned 0^+ states in ^{230}Th. The lines are splines drawn to guide the eyes and have no further meaning.

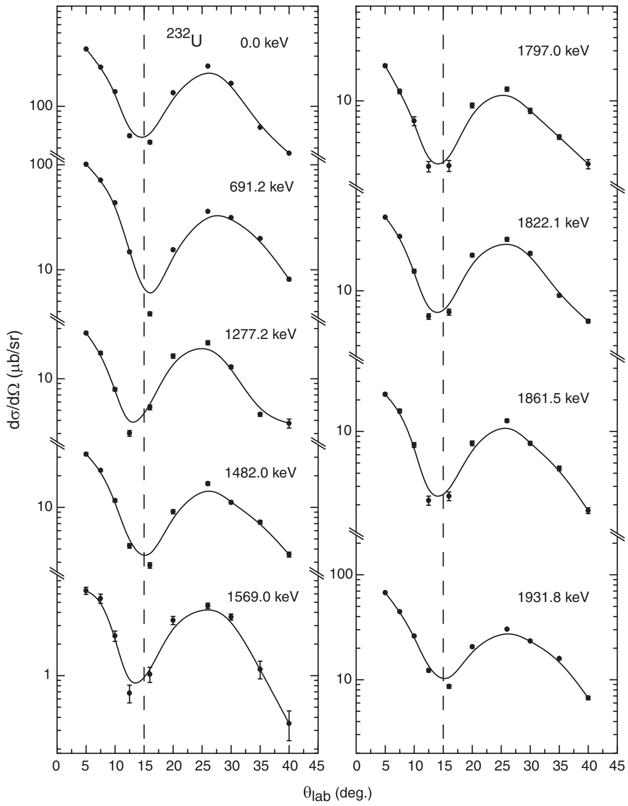

Fig. 5. Angular distribution of assigned 0^+ states in ^{232}U. The lines are splines drawn to guide the eyes and have no further meaning.

individual state, since the wave function of the outgoing triton is restricted to the nuclear exterior, and thus to the tails of the transition form factors. In the shaded area of Fig. 6 we compare the angular distributions of the 927 keV excited state in ^{232}U with the 0^+ ground state. The angular distribution of the 927 keV state gives no firm evidence for a 0^+ assignment proposed by Ardisson et al. [49].

C. DWBA analysis and transfer strengths

To determine spectroscopic factors we need a form factor as reference. Since in the present state of analysis we do not know the relative contributions of the specific j^2 transfer configurations to each of the observed excited 0^+ states – according to their microscopic structure – we arbitrarily chose one configuration which provides the best reproduction of the ground state transition, which is the $(2g_{9/2})^2$ transfer, as a reference. For each state the binding energies of the two neutrons are calculated to match the outgoing proton energy observed in (p, t). We used the code CHUCK3 of Kunz [50].

In Fig. 7 DWBA cross section angular distributions are compared with data for the observed transfers to the ground states of ^{228}Th, ^{230}Th, and ^{232}U. Their obtained spectroscopic factors, i.e. $(d\sigma/d\Omega)^{\text{exp.}}/(d\sigma/d\Omega)^{\text{CHUCK3}}$, of 7.7, 8.7, and 8.0, respectively, are nearly the same, in agreement with earlier observations. Anyhow, the values depend strongly on the chosen potential parameters. We used the optical potential parameters listed in Table 1.

To determine the spectroscopic factors of the excited states, we compare the experimental values of the differential cross section at the detection angle $\theta = 7.5^\circ$ with the DWBA-calculated value at the respective kinematic condition. The energy dependences of the ratios of the DWBA-calculated cross sections of excited states to the ground state cross sections are shown in Fig. 8.

The results are summarized in Table 2 where the spectroscopic factors are normalized to the respective observed ground state transfer strength, and given in percent of the latter.

There are a few transitions with very low cross sections in the 1 μbarn/sr range which show for very

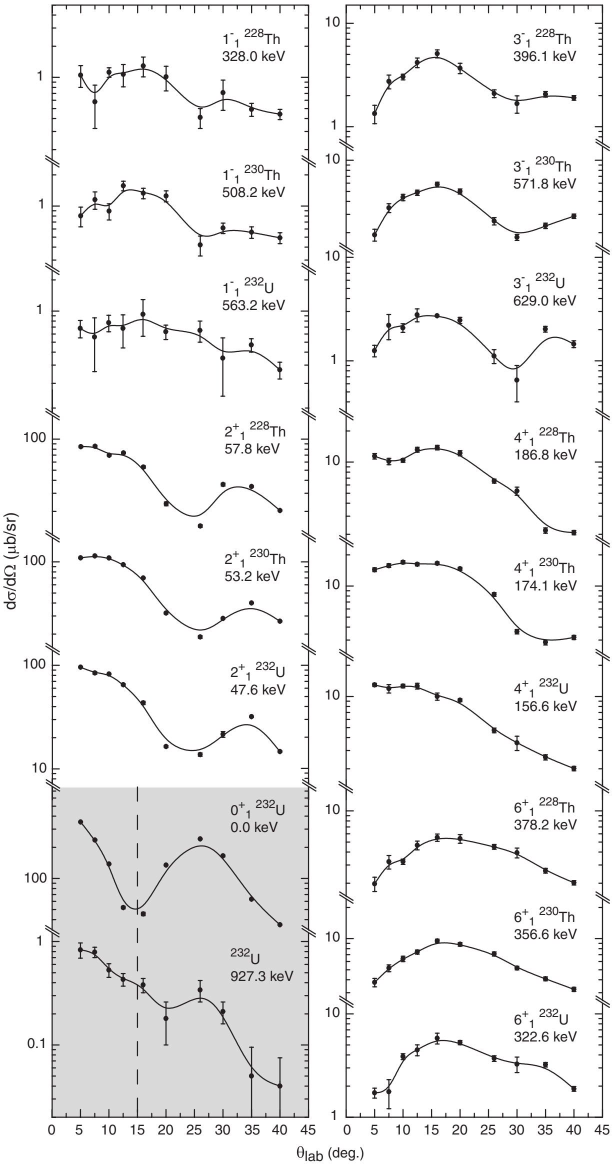

Fig. 6. Angular distributions of the first excited 1^-, 2^+, 3^-, 4^+, and 6^+ states. The lines are splines drawn to guide the eyes and have no further meaning.

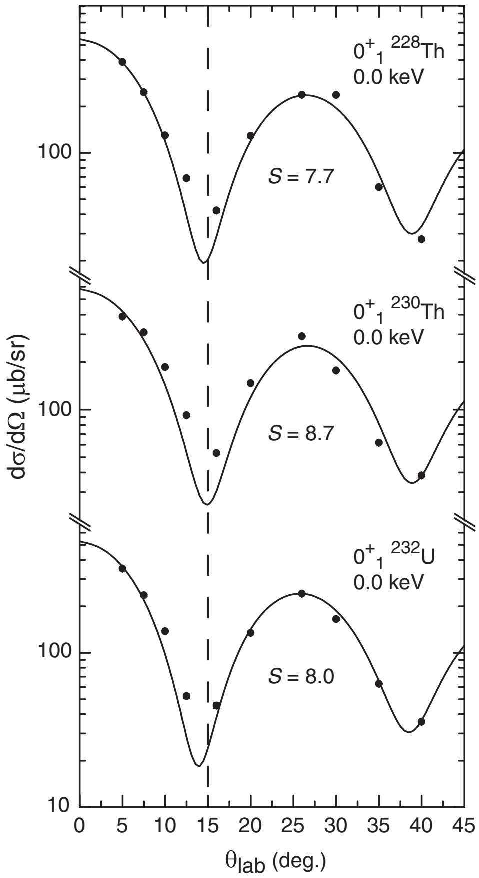

Fig. 7. Ground state transition data with DWBA fits. The spectroscopic factors $S = (d\sigma/d\Omega)^{\text{exp.}}/(d\sigma/d\Omega)^{\text{CHUCK3}}$ are given.

Table 1. **Optical potential parameters used in the DWBA calculations. The values were chosen according to [51]**

E_p	MeV		(p, t)Th, U			(p, t)^{158}Gd	
			25			27	
		p	t	n	p	t	n
V_r	(MeV)	57.10	166.70		58.88	160.03	a)
$4W_d$	(MeV)	32.46			29.80		
W_0	(MeV)	2.80	10.28		3.24	17.83	
$4V_{so}$	(MeV)	24.80		$\lambda = 25$	24.80		$\lambda = 25$
r_r	(fm)	1.17	1.16	1.17	1.23	1.20	1.17
r_D	(fm)	1.32			1.32		
r_0	(fm)	1.32	1.50		1.32	1.40	
r_{so}	(fm)	1.01			1.01		
R_c	(fm)	1.30	1.30		1.25	1.30	
a_r	(fm)	0.75	0.75		0.75	0.72	0.75
a_D	(fm)	0.51			0.65		
a_0	(fm)	0.51	0.82		0.65	0.84	
a_{so}	(fm)	0.75			0.75		
nlc		0.85	0.25		0.85	0.25	

Table 2. **Assigned 0^+ states in ^{228}Th, ^{230}Th, and ^{232}U. The transfer strength of the excited states adds up to 64, 64, and 72 % of the observed ground state strength, respectively**

E_x, keV this work	E_x, keV NDS [25, 26, 27]	$\sigma(\theta = 7.5^\circ)$, μbarn/sr	Spectroscopic factor, %
	^{228}Th		
0.0	0.0	246.6	100.0[a]
831.9(3)	831.823(10)	89.4	28.9
938.7(3)	938.58(7)	25.0	7.8
1120.1(3)	1120.09(10)[b]	1.1	.3
1511.2(3)		3.6	1.0
1627.9(3)		18.6	4.9
1691.4(4)		1.7	.4
2044.7(5)		1.7	.4
2079.9(5)		8.9	2.1
2131.3(6)		50.7	11.8
2159.4(5)		3.0	.7
2290.0(7)		26.3	5.9
Σ			164.2
	^{230}Th		
0.0	0.0	308.5	100.0[a]
635.0(3)	634.9(1)	117.3	31.9
1298.2(10)		6.7	1.5
1448.3(10)		2.6	.6
1589.3(5)	1589.8(3)	16.2	3.4
1639.7(10)	1638.5(2)[c]	9.6	2.0
1803.7(10)		3.3	.7
2094.8(5)		15.2	2.8
2151.5(5)		16.3	3.0
2176.0(10)		29.5	5.4
2269.8(5)		35.5	6.4
2394.7(20)		5.4	.9
2493.1(10)		4.0	.7
2527.7(5)		29.1	4.9
Σ			164.2
	^{232}U		
0.0	0.0	235.5	100.0[a]
691.4(3)	691.21(24)	71.4	26.0
1277.2(4)		17.5	5.7
1482.0(4)		22.5	7.1
1569.0(4)		5.4	1.7
1797.0(4)		12.3	3.7
1822.1(4)		32.9	9.8
1861.5(4)		15.7	4.6
1931.8(4)		44.6	13.0
Σ			171.6

[a] By definition, see text.
[b] From [20].
[c] Assigned as $(2, 0^+)$ in [26].

small scattering angles an increasing cross section with decreasing angle, but otherwise not the typical pattern for a 0^+ excitation. We leave the discussion of these states to a forthcoming paper [23] and discuss here only states where the data provide firm evidence for 0^+ excitations. In the range of low cross sections, higher order and coupled channel effects may produce angular distributions which

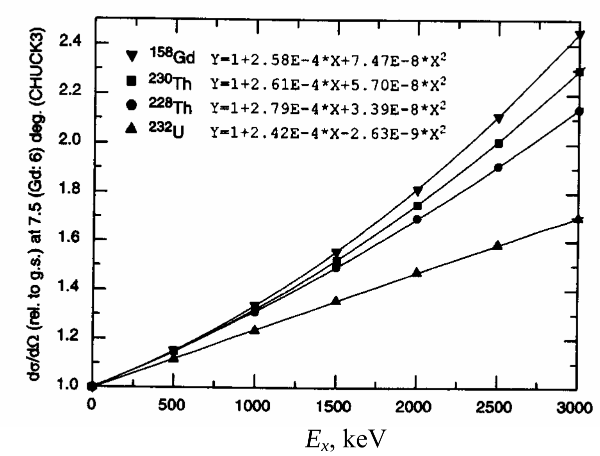

Fig. 8. DWBA calculations of cross sections of excited 0^+ states at 7.5° normalized to the ground state cross section (calculated with CHUCK3 using optical potential

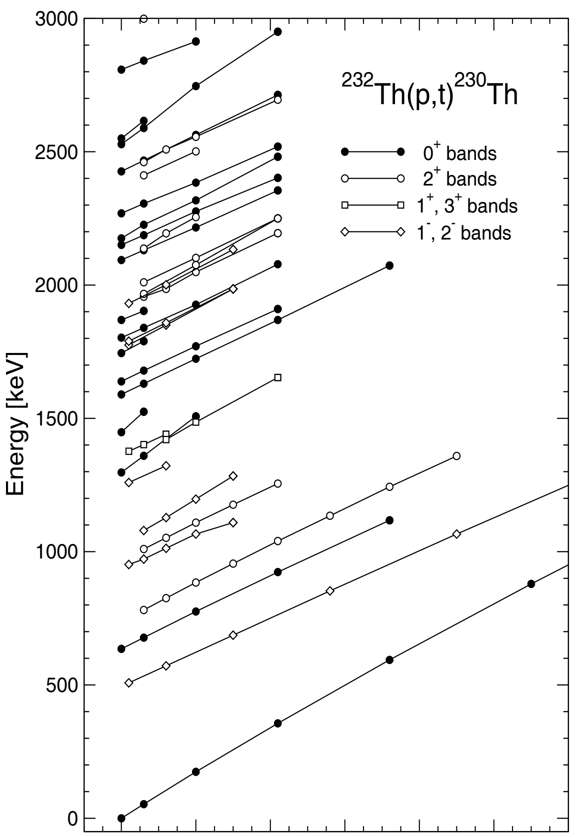

Fig. 9. Collective bands in ^{230}Th based on the 0^+, 2^+, 1^-, 2^- and 3^- excited states assigned from the DWBA fit of the angular distribution from the (p, t) reaction.

deviate strongly from those for one-step direct excitations. Thus we cannot claim to observe all excited 0^+ states. Our analysis is restricted to those showing up with a spectroscopic strength of about 0.5% of the ground state excitation strength or more, defining in this way a class of states.

For ^{228}Th, ^{230}Th, and ^{232}U, which differ by one neutron or one proton pair only, the summed 0^+ transfer strengths to the excited states add up to 64 %, 64 %, and 72%, respectively. These are much higher values than observed for ^{158}Gd [22], where the excited states carry only 26 % of the ground state transfer strength. Note the identical values for the two Th isotopes and the larger value for ^{232}U with one additional proton pair.

D. Collective bands based on the 0^+ and 2^+, 1^-, 2^-, 3^- excited states in ^{230}Th

Identification of the states belonging to the collective bands was made on the following bases:

a) the angular distribution can be fitted by the DWBA calculations for corresponding spin;

b) the cross section of the transfer to the states in the bands decreases with spin;

c) the energies of the states in the band follow approximately to the dependence of spin as $E \approx I(I+1)$.

Identified in such a way collective bands are shown in Fig. 9.

4. Comparison of 0^+ states with IBA calculations

The role of the octupole degree of freedom in heavy deformed nuclei and the related description with *f* bosons, added to the established IBA in the *sd*-boson space (*sd*-IBA), has been systematically studied for deformed rare earth nuclei [39] and for deformed actinides [40]. For the rare earth nuclei the IBA in the *sdf*-boson space (*sdf*-IBA) reproduces reasonably well the main features of the observed low lying negative parity states, for the actinide nuclei a better reproduction of the respective data is obtained if one allows in addition to the *f* boson for a *p* boson (*spdf*-IBA). The physical nature of the *p* boson is not clear. It may result as an artifact, or an anharmonicity, of an octupole excitation in a quadrupolar deformed potential. In the present context we treat the *f* or the combination of a *p* and an *f* boson (*pf* boson) as a technical way to describe octupole collectivity.

A simple IBA Hamiltonian in the *spdf* space including vibrational contributions and a quadrupole interaction in the simple form, is

$$H = \varepsilon_d \hat{n}_d + \varepsilon_p \hat{n}_p + \varepsilon_f \hat{n}_f - \kappa \hat{Q}_{spdf} \cdot \hat{Q}_{spdf}, \qquad (1)$$

where ε_d, ε_p, and ε_f are the boson energies and $\hat{n}_d$, $\hat{n}_p$, and $\hat{n}_f$ are the boson number operators. The same strength κ of the quadrupole interaction describes the *sd* bosons and the *pf* bosons. The $\hat{Q}_{spdf}$ quadrupole operator

$$\hat{Q}_{spdf} = \hat{Q}_{sd} + \hat{Q}_{pf} = [s^\dagger \tilde{d} + d^\dagger s]^{(2)} -$$

$$-(1/2)\sqrt{7}[d^\dagger \tilde{d}]^{(2)} + (3/5)\sqrt{7}[p^\dagger \tilde{f} + f^\dagger \tilde{p}]^{(2)} -$$

$$-(9/10)\sqrt{3}[p^\dagger \tilde{p}]^{(2)} - (3/10)\sqrt{42}[f^\dagger \tilde{f}]^{(2)} \quad (2)$$

is used as in [43]; the $-\sqrt{7}/2$ factor in front of the $[d^\dagger \tilde{d}]^{(2)}$ may be adjusted introducing an additional parameter χ_{sd}. This Hamiltonian was used in Refs. [24, 43].

In Figs. 10 and 11 we display excitation energies of negative and positive parity states in ^{228}Th, ^{230}Th, and ^{232}U, comparing an *spdf*-IBA calculation with experimental data from the ND compilations [25 - 27]. The *pf* boson parameters are chosen to reproduce the $K^\pi = 0^-$ and $K^\pi = 1^-$ bandheads; they are determined by the experimental energies of the $J^\pi = 1_1^-, K^\pi = 0^-$ and $J^\pi = 1_2^-, K^\pi = 1^-$ states. For ^{228}Th the $J^\pi = 1_1^-, K^\pi = 0^-$ excitation energy is 328 keV and thus significantly lower than 508 keV and 563 keV for ^{230}Th and ^{232}U, respectively. This is in contrast to the $J^\pi = 1_2^-, K^\pi = 1^-$ excitation energies, which are about the same in the three nuclei, comparing Fig. 10.

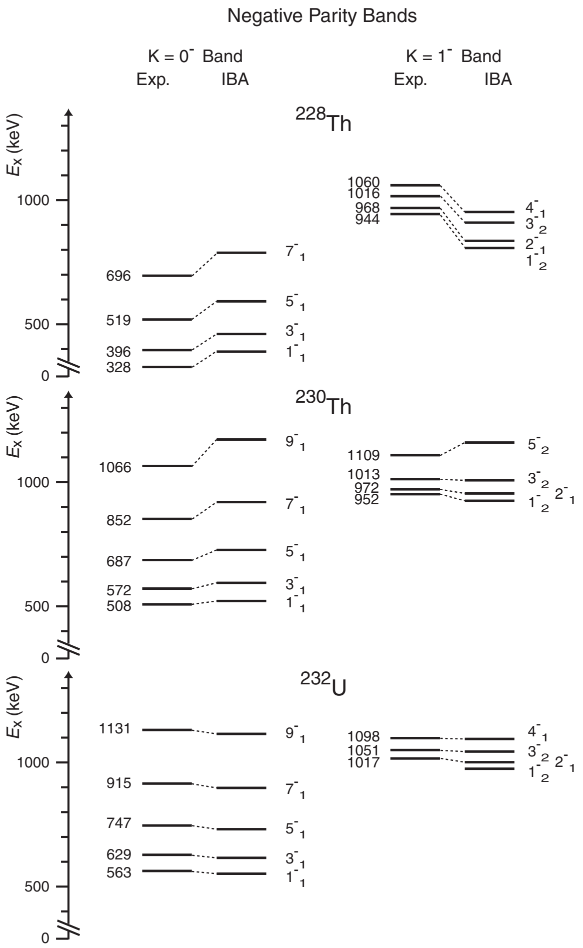

Fig. 10. Lowest negative parity bands in ^{228}Th, ^{230}Th, and ^{232}U, comparing the *spdf*-IBA calculation with known excitation energies from the NDS [25 - 27] and Ref. [20]. Left side experimental, right side calculated levels.

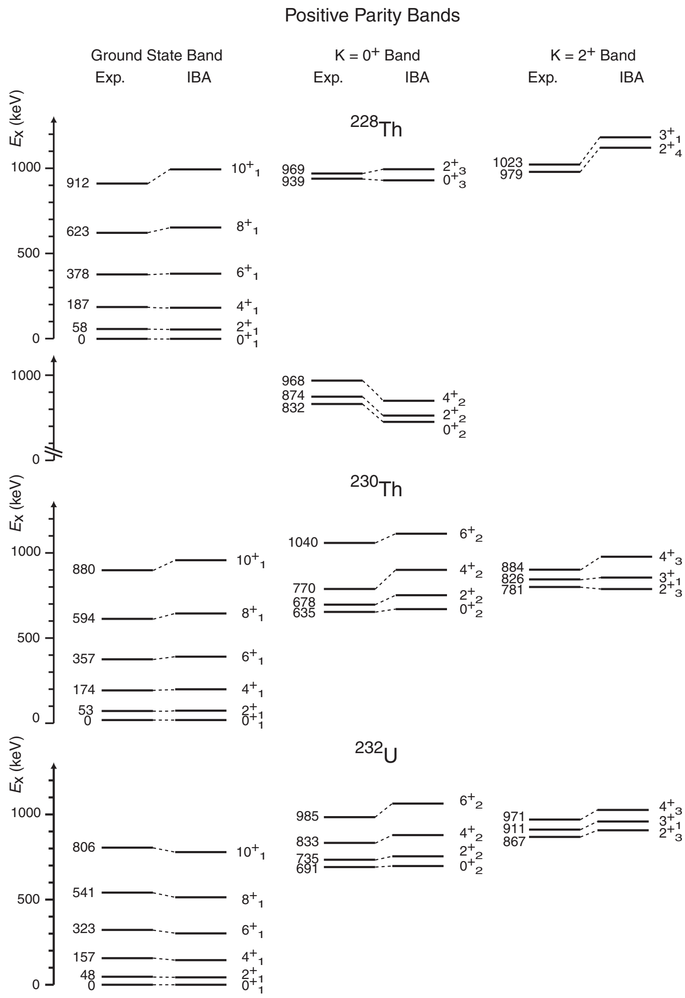

Fig. 11. Lowest positive parity bands in ^{228}Th, ^{230}Th, and ^{232}U, comparing the *spdf*-IBA calculation with known excitation energies from the NDS [25 - 27] and Ref. [20]. For ^{228}Th we show two 0^+ bands. According to the schematic IBA calculation the octupole two phonon band is the one with the lower excitation energies. In reality the two bands are expected to mix.

The IBA parameters in the *sd* boson space are determined by the low energy spacing of the ground state band and the $J^{\pi}=2_1^+, K^{\pi}=0^+$ and $J^{\pi}=2_1^+, K^{\pi}=2^+$ bandheads, respectively. The values of the parameters are listed in Table 3.

Table 3. **Multipole parameters of the *spdf*-boson IBA calculation. The number of negative parity bosons is allowed to range from 0 to 3**

Nucleus Total Number of Bosons	^{232}U	^{230}Th	^{228}Th
	12	11	10
ε_s	0.0000	0.0000	0.0000
ε_p	0.9900	1.0000	1.0500
ε_d	0.2500	0.2500	0.2100
ε_f	0.9400	0.9000	0.6500
κ	0.0120	0.0140	0.0180
χ_{sd}	1.3228	1.0000	1.3228

Table 4. **Moments of inertia for the bands in ^{230}Th as assigned from the angular distributions from the ^{232}Th(p, t)^{230}Th reaction**

Energy	J(0$^+$)	Energy	J(2$^+$)	Energy	J(1$^-$,2$^-$)
0.0	56.8	781 ^{b}	67.6	508 ^{c}	78.1
635 ^{b}	70.4	971 ^{b}	70.4	951 ^{c}	99.5
1297 ^{a}	48.6	1956 ^{b}	75.7	1079	61.7
1589 ^{b}	74.6	1967 ^{a}	65.0	1259 ^{c}	79.4
1639 ^{b}	75.7	2138 ^{a}	60.0		
1745 ^{b}	67.6	2461 ^{b}	73.5		
1802 ^{c}	80.6	2666 ^{a}	56.2		
2093 ^{c}	82.0	2999 ^{a}	56.2		
2050 ^{c}	82.0				
2175 ^{a}	65.0				
2269 ^{c}	82.0				
2528 ^{a}	49.0				
2808 ^{a}	62.5				

The experimental spectra of the 0^+ states obtained for ^{228}Th, ^{230}Th, and ^{232}U, and the results of *spdf*-IBA calculations are compared in Fig. 12; as for ^{158}Gd [24], in the *spdf*-IBA calculations mixing between *d* and *pf* bosons is neglected and the *f* (and *p*) bosons account for octupole collectivity. The key quantities for octupole collectivity, the 1^- excitation energies, are also indicated.

For ^{228}Th, ^{230}Th, and ^{232}U in the energy ranges covered experimentally (2.5, 2.7, and 2.3 MeV, respectively) the IBA predicts four, six, and six excited 0^+ states, respectively, of pure *sd* (quadrupolar) bosonic structure, and additionally six, seven, and four excited 0^+ states, respectively, which have two bosons in the *pf* boson space. They are related to – or represent – octupole two phonon excitations (OTP).

Inspecting the lowest excited 0^+ states of ^{228}Th, ^{230}Th, and ^{232}U we have a reasonable correlation in excitation energy between experiment and calculation.

For all of these three nuclei this schematic calculation predicts one of the two lowest excited 0^+ states as an octupole two phonon excitation and the other one as an *sd* space excitation; the sequence, however, changes. Because of the larger octupole collectivity of ^{228}Th, expressed by the low value of the excitation energy of the lowest 1^- state at 328 keV, the predicted octupole two phonon 0^+ excitation is lower in excitation energy than the predicted lowest excited 0^+ states in the *sd* space. The situation is reversed for ^{230}Th and ^{232}U, where the lowest 1^- states have higher excitation energies, compare Fig. 12.

Taking the IBA calculation and the parameterization used literally, the IBA predicts in the energy ranges considered 10, 13, and 10 excited 0^+ states. Accounting in addition for the presence of a monopole pairing vibrational state, and perhaps one state from hexadecupole collectivity, both not included in the calculation, we have nearly perfect agreement with the numbers of 11, 13, and 8 observed and safely assigned states for ^{228}Th, ^{230}Th, and ^{232}U, respectively.

The IBA, however, fails completely to reproduce the (p, t) spectroscopic factors. The calculated first excited 0^+ state comes with about one percent of the transfer strength of the ground state, and the higher states are even weaker, whereas experimentally the excited states show up with about 60 % of the ground state transfer strength. The IBA as well as most RPA calculations do not include the monopole pairing vibrational configurations. The spreading of this strength, however, is the mechanism which provides the (p, t) transfer.

In Table 4 we present moments of inertia obtained at fitting the level energies of the bands displayed in Fig. 9 by the expression $E = E_0 + AI(I+1)$. The moments of inertia vary in wide ranges from 49 to 100. If the negative parity bands could be assumed to be the octupole phonon bands then the largest moments of inertia in the range from 78 to 100 can be related to the octupole phonon structure. Then if the 0^+ state at 1297 keV is β-vibration state [13, 15], then the smallest moments of inertia in the range from 48 to 65 can be related to the one-phonon quadrupole bands. At last on can assume that the intermediate values of

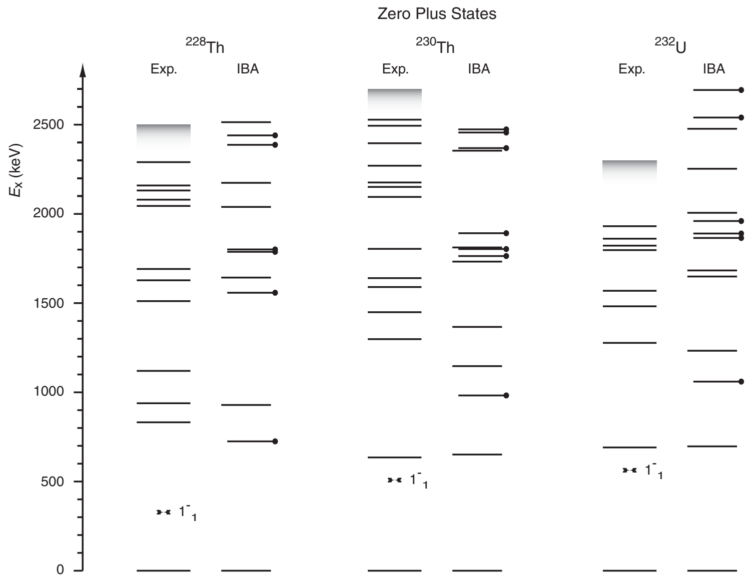

Fig. 12. Excitation energies of all safely assigned 0^+ states in ^{228}Th, ^{230}Th and ^{234}U, compared with *spdf*-IBA calculations. OTP states are marked by a dot. The shadowed areas indicate the upper range of the experimental evaluation. The positions of the 1_1^- states are indicated, too.

the moments of inertia in the range 65 to 78 can be related to the two-phonon quadrupole excitations. Though overlapping is possible as well as the bands with large moments of inertia can have two quasiparticle nature. If it is true then this empirical observation is in contradiction with the prediction of the IBM calculation: numbers of the 0^+ excitations with the structure assumed in such a way differs from that predicted by the IBM. A recent calculation of the 0^+ excitations in actinide nuclei in the frame of the quasiparticle-phonon model [52] was more successful in reproducing the absolute cross section of the (p, t) reaction. But the nature of these excitation predicted in this calculation is in contradiction as with the IBM calculation and with the above mentioned empirical observation.

5. Conclusion

We have performed (p, t) transfer reactions to study excited 0^+ states in ^{228}Th, ^{230}Th, and ^{232}U. In each of these three nuclei we found several excited 0^+ states that have not been experimentally observed before. This allowed the identification of considerable sets of excited 0^+ states which were compared to the predictions of the *spdf* IBA. We hope data of this kind will stimulate further and microscopically motivated studies, as those in the QPM model [17 - 20] but in a large configuration space. At present we have the interesting result that collective model descriptions of these actinide nuclei predict nearly quantitatively the number of the observed excited 0^+ states.

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REFERENCES

1. *Julin R., Kantele J., Kumpulainen J. et al.* // Phys. Rev. - 1987. - Vol. C36. - P. 1129.
2. *Yeh M., Garrett P.E., McGrath C.A. et al.* // Phys. Rev. Lett. - 1996. - Vol. 76. - P. 1208.
3. *Yates S.W., Yeh M., Kadi M. et al.* // J. Phys. - 1999. - Vol. G25. - P. 691.
4. *Valnion B.D., Ponomarev V.Yu., Eisermann Y. et al.* // Phys. Rev. - 2001. - Vol. C63. - P. 024318.
5. *Fayans S.A., Platonov A.P., Graw G., Hofer D.* // Nucl. Phys. - 1994. - Vol. A577. - P. 557.
6. *Hertenberger R., Eckle G., Eckle F.J. et al.* // Nucl. Phys. - 1994. - Vol. A574. - P. 414.
7. *Ponomarev V.Yu., Pignanelli M., Blasi N. et al.* // Nucl. Phys. - 1996. - Vol. A601. - P. 1.
8. *Oros A.M., Von Brentano P., Jolos R.V. et al.* // Nucl. Phys. - 1997. - Vol. A613. - P. 209.
9. *Cata-Danil Gh., Bucurescu D., Trache L. et al.* // Phys. Rev. - 1996. - Vol. C54. - P. 2059.
10. *Jaskola M., Guazzoni P., Zetta L. et al.* // Acta Phys. Pol. - 2002. - Vol. B33. - P. 363.
11. *Levon A.I., De Boer J., Graw G. et al.* // Nucl. Phys. - 1994. - Vol. A576. -P. 267.

12. *Maher J.V., Erskine J.R., Friedman A.M. et al.* // Phys. Rev. - 1972. - Vol. C5. - P. 051305.
13. *Ackermann B., Baltzer H., Freitag K. et al.* // Z. Phys. - 1994. - Vol. A350. - P. 13.
14. *Baltzer H., Freitag K., Günther C. et al.* // Z. Phys. - 1995. - Vol. A352. - P. 47.
15. *Baltzer H., De Boer J., Gollwitzer A. et al.* // Z. Phys. - 1996. - Vol. A356. - P. 13.
16. *Weber T., De Boer J., Freitag K. et al.* // Z.Phys. - 1997. - Vol. A358. - P. 281.
17. *Soloviev V.G.* // Z. Phys. - 1989. - Vol. A334. - P. 143.
18. *Shirikova N.Yu.* Private communication.
19. *Soloviev V.G., Sushkov A.V., Shirikova N.Yu.* // Nucl. Phys. - 1994. - Vol. A568. - P. 244; Phys. Part. Nucl. - 1996. - Vol. 27(6). - P. 667; Prog. Part. Nucl. Phys. - 1997. - Vol. 38. - P. 53.
20. *Weber T., De Boer J., Freitag K. et al.* // Eur. Phys. J. - 1998. - Vol. A3. P. 25.
21. *Otsuka T., Sugita M.* // J. Phys. Soc. J. - 1989. - Vol. 58 (Suppl.). - P. 530 - 537.
22. *Lesher S.R., Aprahamian A., Trache L. et al.* // Phys. Rev. - 2002. - Vol. C66. - P. 051305.
23. *A.I. Levon et al.*, to be published.
24. *Zamfir N.V., Zhang J.-Y., Casten R.F.* // Phys. Rev. - 2002. - Vol. C66. - P. 057303.
25. *Artna-Cohen A.* // Nucl. Data Sheets. - 1997. - Vol. 80. - P. 723.
26. *Akovali Y.A.* // Nucl. Data Sheets. - 1993. - Vol. 69. - P. 155.
27. *Schmorak M.R.* // Nucl. Data Sheets. - 1991. - Vol. 63. - P. 139.
28. *Amzal N., Cocks J.F.C., Butler P.A. et al.* // J. Phys. - 1999. - Vol. G25. - P. 831.
29. *Cocks J.F.C., Hawcroft D., Amzal N. et al.* // Nucl.Phys. - 1999. - Vol. A645. - P. 61.
30. *Butler P.A., Nazarewicz W.* // Rev. Mod. Phys. - 1996. - Vol. 68. - P. 349.
31. *Raduta A.A., Lo Iudice N., Ursu I.I.* // Nucl. Phys. - 1996. - Vol. A608. - P. 11; Nuovo Cim. - 1996. - Vol. 109A. - P. 1669.
32. *Jolos R.V., Palchikov Yu.V.* // Yad. Fiz. - 1997. - Vol. 60(7). - P. 1202; Phys. Atomic Nuclei. - 1997. - Vol. 60. - P. 1077.
33. *Minkov N., Drenska S.B., Raychev P.P. et al.* // Phys. Rev. - 2000. - Vol. C61. - P. 064301.
34. *Safarov A.R., Safarov R.Kh., Sitdikov A.S.* // Yad. Fiz. - 2001. - Vol. 64(8). - P. 1496; Phys. Atomic Nuclei. - 2001. - Vol. 64. - P. 1419.
35. *Tsvetkov A., Kvasil J., Nazmitdinov R.G.* // J. Phys. - 2002. - Vol. G28. - P. 2187.
36. *Raduta A.A., Ionescu D.* // Phys. Rev. - 2003. - Vol. C67. - P. 044312.
37. *Wu C.Y., Cline D.* // Phys. Rev. - 1996. - Vol. C54. - P. 2356.
38. *Meyer U., Raduta A.A., Faessler A.* // Nucl. Phys. - 1998. - Vol. A641. - P. 321.
39. *Cottle P.D., Zamfir N.V.* // Phys. Rev. - 1996. - Vol. C54. - P. 176.
40. *Cottle P.D., Zamfir N.V.* // Phys. Rev. - 1998. - Vol. C58. - P. 1500.
41. *Minkov N., Drenska S.B., Raychev P.P. et al.* // Phys. Rev. - 1999. - Vol. C60. - P. 034305.
42. *Diallo A.F., Barrett B.R., Navratil P., Gorrichategui C.* // Ann. Phys. - 2000. - Vol. 279. - P. 81.
43. *Zamfir N.V., Kusnezov D.* // Phys. Rev. - 2001. - Vol. C63. - P. 054306; Phys. Rev. - 2003. - Vol. C67. - P. 014305.
44. *Li S.C., Kuyucak S.* // Nucl.Phys. - 1996. - Vol. A604. - P. 305.
45. *Löffler M., Scheerer H.J., Vonach H.* // Nucl. Instum. Methods. - 1973. - Vol. 111. - P. 1.
46. *Zanotti E., Bisenberger M., Hertenberger R. et al.* // Nucl. Instrum. Methods. - 1991. - Vol. A310. - P. 706.
47. *Wirth H.-F., Angerer H., Von Egidy T. et. al.* // Annual Report, Be-schleunigerlaboratorium München, 2001. - P.71.
48. *Wirth H.-F.* Ph.D. Thesis, Techn. Univ. München, http://tumb1.biblio.tu-muenchen.de/publ/diss/ph/ 2001/wirth.html, 2001.
49. *Ardisson G., Hussonnois M., LeDu J.F. et al.* // Phys. Rev. - 1994. - Vol. C49(6). - P. 2963.
50. *Kunz P.D.* Computer code CHUCK3. - University of Colorado, unpublished.
51. *Perey C.M., Perey F.G.* // At. Data Nucl. Data Tables - 1976. - Vol. 17. - P. 1.
52. *Iudice N.Lo, Sushkov A.V., Shirikova N.Yu.* // Phys. Rev. - 2005. - Vol. C72. - P. 034303.

0^+ СТАНИ ТА КОЛЕКТИВНІ СМУГИ В ДЕФОРМОВАНИХ ЯДРАХ АКТИНІДІВ

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У реакції (p, t) вивчались 0^+-стани в деформованих ядрах ^{228}Th, ^{230}Th та ^{232}U, використовуючи Q3D магнітний спектрограф на Мюнхенському тандемному прискорювачі. Кутові розподіли передачі в 0^+-стани при малих кутах мають крутий підйом, що дозволяє ідентифікувати ці стани в оточенні дуже складних і густих спектрів. Для кожного з цих ядер було надійно ідентифіковано близько 10 збуджених 0^+-станів. Вивчались енергії збудження до 2.5, 2.7 та 2.3 МеВ відповідно. Виконано порівняння з розрахунками в рамках МВБ у *spdf*-бозонному просторі. Це надзвичайно схематичний модельний опис, що включає октупольну колективність, але нехтує іншими відповідними ступенями свободи, дає числа 0^+-станів у цих ядрах, дуже близькі до одержаних експериментально. Відібрані послідовнсті станів, які можна розглядати як ротаційні смуги. За допомогою підгонки енергій цих смуг визначено інерційні параметри, які порівнюються з розрахунками в моделі взаємодіючих бозонів.

0^+ СОСТОЯНИЯ И КОЛЛЕКТИВНЫЕ ПОЛОСЫ В ДЕФОРМИРОВАННЫХ ЯДРАХ АКТИНИДОВ

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В реакции (p, t) изучались возбужденные 0^+-состояния в деформированных ядрах ^{228}Th, ^{230}Th та ^{232}U, используя Q3D магнитный спектрограф на Мюнхенском тандемном ускорителе. Угловые распределения передачи в 0^+-состяния при малых углах имеют крутой подъем, что позволяет идентифицировать эти состояния в окружении очень сложных и плотных спектров. Для каждого из этих ядер были надежно идентифицированы около 10 возбужденных 0^+-состояний. Изучались энергии возбуждения вплоть до 2.5, 2.7 и 2.3 МэВ соответственно. Выполнено сравнение с расчетами в рамках МВБ в *spdf*-бозонном пространстве. Это чрезвычайно схематическое модельное описание, включающее октупольную коллективность, но пренебрегающее другими соответствующими степенями свободы, дает числа 0^+-состояний в этих ядрах, очень близкие к наблюдаемым экспериментально. Отобраны последовательности состояний, которые можно рассматривать как ротационные полосы. С помощью подгонки энергий этих полос определены инерционные параметры, которые сравниваются с расчетами в модели взаимодействующих бозонов.

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