УДК 539.161

ALPHA-DECAY OF DEFORMED NUCLEI HAVING NONZERO ORBITAL MOMENTUM GROUND STATE

V. V. Davydovskyy

Institute for Nuclear Research, National Academy of Sciences of Ukraine, Kyiv

The production of α-particles with nonzero orbital momentum in the decays of deformed nuclei is considered in the framework of a cluster model using the WKB approximation. The model is based on the α-nucleus potential with parameters derived by using the data for both the α-decay half-lives and the fusion cross sections around the barrier. The α-decay half-lives for a set of deformed nuclei having nonzero orbital momentum ground state are evaluated and compared with the experimental data. New expression for the formation probability of α-particle with nonzero orbital momentum is proposed.

1. Introduction

The phenomenological models (in contrast to microscopic models) of α-decay are based on the α-nucleus interaction potential, which is a key element for the description of various processes involving α–particles and nuclei.

The α-decay process involves sub-barrier penetration of α–particles through the barrier, caused by interactions between α–particles and nucleus. Therefore, α-decay half-lives depend strongly on the α-nucleus interaction potential. However, the same α-nucleus interaction potential is the principal factor in describing the fusion reaction between α particle and nucleus.

In ref. [1], for the first time, the data for both the α-decay half-lives and the sub-barrier fusion reactions were used for determination of the global α-nucleus interaction potential at the energy range from close to zero to around the barrier. The nucleus-nucleus interaction potential, obtained by Denisov and Ikezoe in [1] (the DI potential), consists of the Coulomb repulsion part, the nuclear attraction part and the centrifugal part. These parts form a barrier at small distances between α–particle and nucleus.

In order to describe extensive experimental information (see Refs. [2 - 10] and articles cited therein) on α-decay half-lives, a cluster approach to the α-decay, which is the most suitable for determining the interaction potential between α–particles and the nucleus, was used in [1]. Using this potential the available data for both the α-decay half-lives and the sub-barrier fusion reaction cross sections were simultaneously described.

The most of α-decay data used in [1] for the potential parameter calculation concerns the α-decay of even-even parent nuclei to even-even daughter nuclei both having 0^+ ground state, so the outgoing α–particle carries zero orbital momentum. For those data corresponding to nonzero momentum of α–particle, the momentum of outgoing α–particle was neglected in [1]. In present work, we mainly focus on the nonzero momentum case and take the momentum of outgoing α–particle into account.

It is well known that one-particle Gamow's approach with properly parameterized α–particle formation probability describes half-lives of the most even-even nuclei very well. On the other hand, a number of α-decaying nuclei have the ground states of both parent and daughter nuclei with nonzero total momentum. Direct application of the one-particle approach to such decays gives the half-lives a few orders of magnitude less than the experimental values. Here, we propose a modification to the one-particle Gamow's theory by introducing new expression for the α–particle formation probability and using the DI α-nucleus potential.

The advantage of the DI α-nucleus potential is its dependence on the angle θ between the direction of α emission and the axial-symmetry axis of the deformed nucleus and therefore it is suitable for deformed α emitters. It also takes the centrifugal effects into account. It belongs to the class of the so-called global phenomenological potentials. Therefore, it is a good choice of potential for calculations of realistic tunneling probabilities of α–particles through the barriers in a great variety of the deformed nuclei, in particular, having non-zero total momentum ground states.

Finally, we compare our calculations with available experimental data on half-lives using the model.

2. Brief review of the DI model of α-decay

Here we briefly summarize the main components of the DI model. The α-decay half-life $T_{1/2}$ is calculated as follows:

$$T_{1/2} = \hbar \ln(2) / \Gamma , \qquad (1)$$

where Γ is the total width of decay. It is evaluated by averaging partial widths. Therefore the total α-decay width is as follows:

$$\Gamma = \frac{1}{4\pi}\int \gamma(\theta,\phi)d\Omega\,, \tag{2}$$

where $\gamma(\theta,\phi)$ is the partial width of α emission in direction θ and ϕ and Ω is the space angle.

The majority of the ground-state α emitters are spherical nuclei or axial-symmetric nuclei with moderate quadrupole deformation. Therefore, the expression for total width can be simplified as follows:

$$\Gamma = \int_0^{\pi/2} \gamma(\theta)\sin(\theta)d\theta\,, \tag{3}$$

where θ is the angle between the symmetry axis of axially symmetric deformed nuclei and the vector from the center of the deformed nucleus to the emission point on the nuclear surface. Because of the small or moderate values of the quadrupole deformation of nuclei we neglect the difference between the surface normal direction θ and ϕ. It is obvious that $\Gamma = \gamma(\theta) = \gamma(0)$ for spherical nuclei.

The width of α emission in direction θ is given by the following:

$$\gamma(\theta) = \hbar\,\xi\, t(Q,\theta,\ell)\,, \tag{4}$$

where $\xi = \nu S$; ν is the frequency of assaults of a α–particle on the barrier; S is the spectroscopic or preformation factor; $t(Q,\theta,\ell)$ is the transmission coefficient, which shows the probability of penetration through the barrier; and Q is the released energy at α decay. Note, that the above Gamow's style expression for ξ is very specific to the case of zero orbital momentum of α–particle and does not hold in the nonzero case due to the big contribution of collective effects into the α–particle formation mechanism.

The transmission coefficient can be obtained in the semiclassical WKB approximation

$$t(Q,\theta,\ell) =$$

$$= \left(1+\exp\left\{\frac{2}{\hbar}\int_{a(\theta)}^{b(\theta)} dr\sqrt{2\mu\left[v(r,\theta,\ell,Q)-Q\right]}\right\}\right)^{-1}, \tag{5}$$

where $a(\theta)$ and $b(\theta)$ are the inner and outer turning points determined from the equations $v(r,\theta,\ell,Q)\big|_{r=a(\theta),b(\theta)} = Q$ and μ is the reduced mass. The α-nucleus potential $v(r,\theta,\ell,Q)$ consists of Coulomb $v_C(r,\theta)$, nuclear $v_N(r,\theta,Q)$, and centrifugal $v_\ell(r)$ parts, i.e., $v(r,\theta,\ell,Q) = v_C(r,\theta)+v_N(r,\theta,Q)+v_\ell(r)$.

It is proposed in [1] that the parts of α-nucleus potential be written in the following form:

$$v_C(r,\theta) = \frac{2Ze^2}{r}\left[1+\frac{3R^2}{5r^2}\beta Y_{20}(\theta)\right], \tag{6}$$

if $r \ge r_m$,

$$v_C(r,\theta) \approx \frac{2Ze^2}{r_m}\left[\frac{3}{2}-\frac{r^2}{2r_m^2}+\frac{3R^2}{5r_m^2}\beta Y_{20}(\theta)\left(2-\frac{r^3}{r_m^3}\right)\right], \tag{7}$$

if $r < r_m$,

$$v_N(r,\theta,Q) = V(A,Z,Q)/(1+\exp\{[r-r_m(\theta)]/d\})\,, \tag{8}$$

$$v_\ell(r) = \hbar^2\ell(\ell+1)/(2\mu r^2)\,. \tag{9}$$

Here A, Z, R, and β are, respectively, the number of nucleons, the number of protons, the radius, and the quadrupole deformation parameter of the nucleus interacting with the α–particle; e is the charge of proton; $Y_{20}(\theta)$ is the spherical harmonic function; and $V(A,Z,Q,\theta)$ and $r_m(\theta)$ are, respectively, the strength and effective radius of the nuclear part of α-nucleus potential.

The obtained values of parameters are as follows:

$$V(A,Z,Q) = -[30{,}275 - 0{,}45838Z/A^{1/3} +$$

$$+ 58{,}270I - 0{,}24244Q]\,, \tag{10}$$

$$R = R_p(1+3{,}0909/R_p^2)+0{,}12430t\,, \tag{11}$$

$$R_p = 1{,}24A^{1/3}(1+1{,}646/A - 0{,}191I)\,, \tag{12}$$

$$t = I - 0{,}4A/(A+200)\,, \tag{13}$$

$$d = 0{,}49290\,, \tag{14}$$

$$r_m(\theta) = 1{,}55268 + R(\theta)\,, \tag{15}$$

$$R(\theta) = R[1+\beta Y_{20}(\theta)]\,, \tag{16}$$

where $I = (A - 2Z)/A$.

Equations (6) - (16) fully define the DI potential, so the integral in (5) can be calculated for a specific daughter nucleus and the released energy Q. In order to calculate the half-lives the following parameterization of the α–particle formation probability was proposed in [1]:

$$\xi = (6{,}1814 + 0{,}2988A^{-1/6})10^{19}\,\mathrm{s}^{-1}\,. \tag{17}$$

In the next section we will test this model and propose its extension to the processes of α–particle production with nonzero orbital momentum.

3. Extension of the DI model and data evaluation

As we mentioned in the introduction, our approach is based on two-stage mechanism of α-decay. We believe that the tunneling stage is well described by the full DI potential (6) – (9), including deformation and orbital momentum dependent terms.

The α–particle formation probability $\xi(\ell)$ has no clear obvious interpretation for $\ell > 0$ (in contrast to the original Gamow's model), but we propose that the expression for $\xi(\ell)$ should become the expression (17) at $\ell = 0$. We choose one of possible forms for $\xi(\ell)$ as follows:

$$\xi(\ell) = (6{,}1814 + 0{,}2988A^{-1/6})10^{19} \times \frac{\exp(-12\beta\,\ell/(\ell+1))}{(2\ell+1)^{1/2}}\ \mathrm{s}^{-1}. \quad (18)$$

The expression (18) will be used for calculation of half-lives.

The Fig. 1 demonstrates how good the DI potential (1) - (5), (6) - (16) with α–particle formation probability (17) fits experimental data, which represent logarithm of α-decay half-lives of the nuclei having 0+ ground state. Such nuclei emit α-particles with zero orbital momentum. The data we evaluate in this work are taken from [2].

In order to test the extended DI model, we selected 42 nuclei from [2] (see, the table below), which decay with the emission of α–particles with nonzero orbital momentum ($\ell = 1 \div 5$).

Nuclei selected for the analysis [2]

The results of our calculations are presented in Fig. 2.

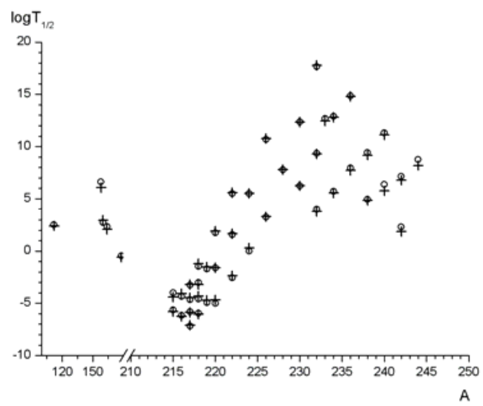

Fig. 1. Logarithm of half-lives for the α-decaying nuclei that emits α-particles with zero orbital momentum: circles are experimental data; crosses are calculation with DI potential for $\ell = 0$.

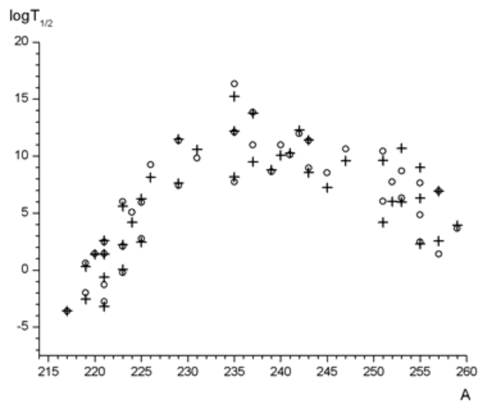

Fig. 2. Logarithm of half-lives for the α-decaying nuclei that emit α-particles with nonzero orbital momentum: circles are experimental data; crosses are calculation within our approach.

	Parent nucleus			Daughter nucleus				l_α (min)	Q, MeV	$\log_{10}T_{1/2exp}$
	Z	A	J^π	Z	A	J^π	β			
even-odd	88	221	5/2+	86	217	9/2+	0,111	2	6,89	1,45
	88	223	3/2+	86	219	5/2+	0,156	2	5,98	5,99
	92	235	7/2–	90	231	5/2+	0,215	1	4,69	16,35
	86	219	5/2+	84	215	9/2+	0,103	2	6,95	0,6
	88	219	(7/2+)	86	215	9/2+	0,077	2	8,13	–2
	90	217	(9/2+)	88	213	1/2–	0,0	5	9,43	–3,6
	90	221	(7/2+)	88	217	(9/2+)	0,102	2	8,64	–2,77
	90	223	(5/2+)	88	219	(7/2+)	0,138	2	7,58	–0,22
	90	225	(3/2+)	88	221	5/2+	0,165	2	6,92	2,76
	90	229	5/2+	88	225	1/2+	0,19	2	5,17	11,36
	92	231	(5/2–)	90	227	(1/2+)	0,198	3	5,56	9,82
	94	235	(5/2+)	92	231	(5/2–)	0,215	1	5,96	7,75
	94	237	7/2–	92	233	5/2+	0,215	1	5,75	10,97
	96	243	5/2+	94	239	1/2+	0,234	2	6,18	8,96
	98	251	1/2+	96	247	9/2–	0,236	5	6,18	10,45
	98	253	(7/2+)	96	249	(1/2+)	0,226	4	6,12	8,7
	100	251	(9/2–)	98	247	(7/2+)	0,245	1	7,43	6,03

Continuation of the table

	Parent nucleus			Daughter nucleus				l_α (min)	Q, MeV	$\log_{10}T_{1/2exp}$
	Z	A	J^π	Z	A	J^π	β			
even-odd	100	253	1/2+	98	249	9/2–	0,236	5	7,2	6,33
	100	255	7/2+	98	251	1/2+	0,237	4	7,24	4,86
	100	257	(9/2+)	98	253	(7/2+)	0,227	2	6,87	6,94
	102	255	(1/2+)	100	251	(9/2–)	0,237	5	8,45	2,48
	102	257	(7/2+)	100	253	1/2+	0,238	4	8,46	1,4
	102	259	(9/2+)	100	255	7/2+	0,228	2	7,81	3,67
odd-even	87	221	5/2–	85	217	9/2–	0,12	2	6,47	2,47
	89	225	(3/2–)	87	221	5/2–	0,164	2	5,94	5,94
	93	237	5/2+	91	233	3/2–	0,215	1	4,96	13,83
	95	241	5/2–	93	237	5/2+	0,223	1	5,64	10,13
	89	221	(3/2–)	87	217	9/2–	0,111	4	7,79	–1,28
	89	223	(5/2–)	87	219	9/2–	0,147	2	6,79	2,1
	91	229	(5/2+)	89	225	(3/2–)	0,19	1	5,84	7,43
	93	235	5/2+	91	231	3/2–	0,215	1	5,2	12,12
	95	239	(5/2–)	93	235	5/2+	0,215	1	5,92	8,63
	95	243	5/2–	93	239	5/2+	0,224	1	5,44	11,37
	97	245	3/2–	95	241	5/2–	0,234	2	6,46	8,55
	97	247	(3/2–)	95	243	5/2–	0,235	1	5,89	10,64
	99	255	(7/2+)	97	251	(3/2–)	0,226	3	6,44	7,63
odd-odd	87	220	1+	85	216	(1–)	0,111	1	6,81	1,44
	89	224	0–	87	220	1+	0,165	1	6,32	5,06
	89	226	(1)	87	222	2–	0,164	1	5,5	9,24
	95	240	(3–)	93	236	(6–)	0,223	4	5,71	10,98
	95	242	5–	93	238	2+	0,224	3	5,6	11,99
	99	252	(5–)	97	248	(6+,1–)	0,236	1	6,76	7,73

4. Results

The results of our evaluations of the data are presented in Figs. 1 - 2.

The DI model based on the global phenomenological potential describes the α-decay of nuclei that emit α-particles with nonzero orbital very well. However, the application of the DI model to the case of nonzero orbital momentum requires the modification of the α–particle formation probability. The modified model is in considerably good agreement with the experimental data.

Acknowledgments

The author thanks V. Yu. Denisov for the fruitful discussions.

REFERENCES

1. *Denisov V. Yu., Ikezoe H.* Alpha-nucleus potential for alpha-decay and sub-barrier fusion // Phys. Rev. C. - 2005. - Vol. 72. - P. 064613.
2. *Basu D. N.* Role of effective interaction in nuclear disintegration processes // Phys. Lett. - 2003. - Vol. B566. - P. 90.
3. *Buck B., Merchant A. C., Perez S. M.* Alpha-decay calculations with a realistic potential // Phys. Rev. C. - 1992. - Vol. 45. - P. 2247; At. Data Nucl. Data Tables. - 1993. - Vol. 54. - P. 53.
4. *Moustabchir R., Royer G.* Analytic expressions for the proximity energy, the fusion process and the α emission // Nucl. Phys. - 2001. - Vol. A683. - P. 266.
5. *Duarte S. B., Tavares O. A. P., Guzman F. et al.* Half-lives for proton emission, alpha decay, cluster radioactivity, and cold fission processes calculated in a unified theoretical framework // At. Data Nucl. Data Tables. - 2002. - Vol. 80. - P. 235.
6. *Danevich F. A., Georgadze A. S., Kobychev V. V. et al.* α activity of natural tungsten isotopes // Phys. Rev. C. - 2003. - Vol. 67. - P. 014310.
7. *Nishio K., Ikezoe H., Mitsuoka S. et al.* Half-life of ^{228}Pu and α decay of ^{228}Np // Phys. Rev. C. - 2003. - Vol. 68. - P. 064305.
8. *Hofmann S., Munzenberg G.* The discovery of the heaviest elements // Rev. Mod. Phys. - 2000. - Vol. 72. - P. 733.
9. *Armbruster P.* On the production of superheavy elements // Annu. Rev. Nucl. Part. Sci. - 2000. - Vol. 50. - P. 411.
10. *Xu C., Ren Z.* Systematical calculation of α decay half-lives by density-dependent cluster model // Nucl. Phys. - 2005. - Vol. A753. - P. 174.

АЛЬФА-РОЗПАД ДЕФОРМОВАНИХ ЯДЕР, ЩО МАЮТЬ ОСНОВНИЙ СТАН З НЕНУЛЬОВИМ ОРБІТАЛЬНИМ МОМЕНТОМ

В. В. Давидовський

У рамках кластерної моделі з використанням наближення ВКБ розглянуто народження α-частинок із ненульовим орбітальним моментом у розпадах деформованих ядер. Модель основана на α-ядерному потенціалі з параметрами, одержаними з використанням даних як для періодів α-напіврозпаду, так і для перерізів підбар'єрного злиття. Обчислено періоди α-напіврозпаду для ряду деформованих ядер, що мають основний стан з ненульовим орбітальним моментом, і проведено порівняння з експериментальними даними. Запропоновано новий вираз для ймовірності формування α-частинки з ненульовим орбітальним моментом.

АЛЬФА-РАСПАД ДЕФОРМИРОВАННЫХ ЯДЕР, ИМЕЮЩИХ ОСНОВНОЕ СОСТОЯНИЕ С НЕНУЛЕВЫМ ОРБИТАЛЬНЫМ МОМЕНТОМ

В. В. Давидовский

В рамках кластерной модели с использованием приближения ВКБ рассмотрено рождение α-частиц с ненулевым орбитальным моментом в распадах деформированных ядер. Модель основана на α-ядерном потенциале с параметрами, полученными с использованием данных как для периодов α-полураспада, так и для сечений подбарьерного слияния. Вычислены периоды α-полураспада для ряда деформированных ядер, имеющих основное состояние с ненулевым орбитальным моментом, и проведено сравнение с экспериментальными данными. Предложено новое выражение для вероятности формирования α-частицы с ненулевым орбитальным моментом.

Received 17.12.07,
revised - 28.12.07.