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EFFECT OF SPECTRAL STATISTICS ON THE NUCLEAR DISSIPATION

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We investigate the effect of quantum mechanical diffusion in the space of adiabatic states of intrinsic nucleonic subsystem on the dissipative properties of macroscopic collective dynamics. By applying the cranking model approach to the nuclear macroscopic dynamics, we derive the Newtonian-like equation of motion for a single collective variable, where the dissipative character of the collective dynamics is due to both the Landau-Zener transitions and the Kubo mechanism. A diffusion equation is used to determine the time evolution of the occupation probabilities of the nuclear states. The transport characteristics are calculated for Gaussian orthogonal and unitary ensembles of the energy levels. We discuss under what conditions a time-irreversible energy exchange between the collective and nucleonic degrees of freedom is possible.

1. Introduction

The theoretical understanding of nuclear dissipation as a consequence of the energy exchange between the macroscopic collective and intrinsic nucleonic degrees of freedom is far from complete. On the quantum mechanical level, nuclear dissipation is treated within linear response model [1], dissipative diabatic dynamics model [2], Fermi-liquid model [3], path integral method [4, 5], and others.

As was proposed by Hill and Wheeler [6], the microscopic origin for the nuclear dissipation is transitions between the avoided crossings of adiabatic nuclear states. In Fig. 1, we show a typical behaviour of the instantenious adiabatic energies of the nuclear excited states as a function of collective deformation parameter. The energy levels fluctuate strongly with the collective deformation and the feature of interest here is the large number of the avoided crossings.

Deforming a nucleus slowly, the intrinsic nucleonic subsystem is excited due to the transitions from the lower-lying to higher-lying states at the avoided crossings in the same way as in the standard Landau-Zener picture [7, 8] illustrated in Fig. 2.

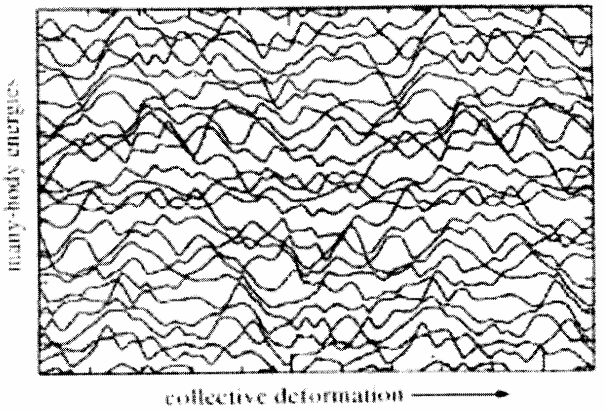

Fig. 1. Typical dependence of the eigenenergies of the nuclear many-body states on collective deformation parameter.

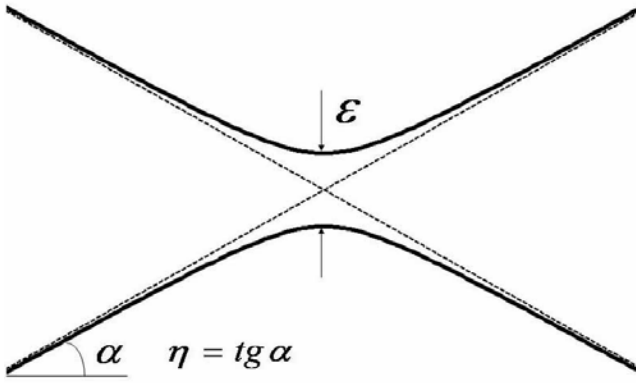

Fig. 2. Typical picture of the avoided level crossing considered in the Landau-Zener model [7, 8].

The probability $R^{LZ}(\varepsilon,\eta)$ of the Landau - Zener transition from the lower-lying initially occupied level to the higher-lying unoccupied level is given by

$$R^{LZ}(\varepsilon,\eta)=\exp\left(-\frac{\pi\varepsilon^2}{2\eta|\dot{q}|}\right), \qquad (1)$$

where ε is the gap and η is the slope of the avoided crossings of two nearest levels. One can see that quantum statistics of nuclear eigenenergies, i.e., the distribution of the slopes of nuclear levels and the distances between them, may strongly affect on the dissipative properties of the collective motion.

Besides the Landau - Zener transitions, there are direct interlevel transitions which change the occupancies of excited nuclear states. In fact, dissipation of the collective energy arises as a result of time-diffusion of the occupational probabilities of the intrinsic nucleonic subsystem. How such collective dissipation appears and what are the necessary conditions for that are the main purpose of the present work.

2. Nuclear many-body system

We start by formulating the nuclear many-body problem and writing the total many-body Hamiltonian as

$$\hat{H}_{tot} = \frac{1}{2} B(q)\dot{q}^2 + U(q) + \hat{H}_{\text{int}}(q), \qquad (2)$$

where $q(t)$ is a single collective time-dependent variable, $B(q)$ is the adiabatic mass parameter, U is the collective potential energy and $\hat{H}_{\text{int}}$ represents the intrinsic nucleonic degrees of freedom. The dynamics of the nucleonic subsystem is determined in terms of the density matrix $\hat{\rho}$,

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = \left[\hat{H}_{\text{int}}(q), \hat{\rho}\right]. \qquad (3)$$

We introduce the moving basis functions $\Psi_n(q)$ which satisfy the stationary Schrödinger equation,

$$\hat{H}_{\text{int}}(q)\Psi_n(q) = E_n(q)\Psi_n(q). \qquad (4)$$

Using Zwanzig's projection technique [9], one can get the following equation for the diagonal elements of the density matrix [10]

$$\frac{\partial \rho_{nn}(t)}{\partial t} = \sum_m P_{nm}(t)\left[\rho_{nn}(t) - \rho_{mm}(t)\right], \qquad (5)$$

where $P_{mn}(t)$ is the transition rate

$$P_{nm}(t) = 2\,\text{Re}\left[\int_0^{+\infty} dt' D_{nm}(t) D_{mn}(t-t') \exp(it'\omega_{nm})\right], \qquad (6)$$

with

$$D_{nm}(t) = \left\langle \Psi_n \left| i\hbar \frac{\partial}{\partial t} \right| \Psi_m \right\rangle, \qquad (7)$$

and

$$\omega_{nm} = (E_n - E_m)/\hbar. \qquad (8)$$

3. Diffusion of occupation probabilities

The transition rate (6) is obtained as a result of second order perturbation theory in $\dot{q}$. To avoid the difficulty connecting with the divergence of the cranking model parameter $\dot{q}\left|\left\langle \Psi_n \left| \partial \hat{H}_{\text{int}} / \partial q \right| \Psi_m \right\rangle\right| / \omega_{mn}(q)$ for two neighboring levels, we assume that the density matrix varies with time only due to the Landau - Zener transitions between the adiabatic levels. We introduce the averaged transition rate of the Landau - Zener transitions $\left\langle P^{LZ} \right\rangle$, i.e., averaged over all possible gap sizes ε and slopes η:

$$\left\langle P^{LZ} \right\rangle = \dot{q} \int_0^{+\infty} d\eta \int_{-\infty}^{+\infty} d\varepsilon N(\eta,\varepsilon) R^{LZ}(\eta,\varepsilon), \qquad (9)$$

where $N(\eta,\varepsilon)d\eta d\varepsilon$ is the number of avoided crossings encountered per unit length with slopes in the interval $[\eta, \eta + d\eta]$ and gap sizes in the interval $[\varepsilon, \varepsilon + d\varepsilon]$. In the limit of small gap sizes ε and assuming that the matrix elements $\left\langle \Psi_n \left| \partial \hat{H}_{\text{int}} / \partial q \right| \Psi_m \right\rangle$ are independently Gaussian distributed, one can show that the distribution $N(\eta,\varepsilon)$ is given by [11]

$$N(\eta,\varepsilon) = W(E)\eta P(\varepsilon) P'(\eta), \qquad (10)$$

where $W(E)$ is the density of excited states. The functions $P(\varepsilon)$ and $P'(\eta)$ are distribution functions of gap sizes and slopes, respectively.

For special ensembles of levels, one can get analytical expressions for these functions. Thus, for systems with time-reversal invariance [*Gaussian orthogonal ensemble* (GOE)]:

$$P(\varepsilon) \approx \frac{\pi}{2} \varepsilon \cdot W(E), \qquad (11)$$

$$P'(\eta) = \frac{\eta}{4\sigma^2} \exp\left(-\frac{\eta^2}{8\sigma^2}\right), \qquad (12)$$

while for systems without time-reversal invariance [*Gaussian unitary ensemble* (GUE)]:

$$P(\varepsilon) \approx \frac{32}{\pi^2} \varepsilon^2 \cdot W^2(E), \qquad (13)$$

$$P'(\eta) = \frac{\eta}{2\sqrt{\pi}\sigma^3} \exp\left(-\frac{\eta^2}{4\sigma^2}\right). \qquad (14)$$

As is shown in [10], we can reduce the master equation (5) into the diffusion equation,

$$W(E)\frac{\partial \rho(E,t)}{\partial t} = \frac{\partial}{\partial E} W(E) D(E) \frac{\partial \rho(E,t)}{\partial E}, \qquad (15)$$

where E is the continious energy variable measuring the excitation of the nucleonic subsystem and $D(E)$ is the diffusion coefficient

$$D(E) = \dot{q} W(E) \int_{-\infty}^{+\infty} d\varepsilon \varepsilon^2 P(\varepsilon) \int_0^{+\infty} d\eta \eta P'(\eta) \exp\left(-\frac{\pi\varepsilon^2}{2\eta|\dot{q}|}\right). \qquad (16)$$

We calculated the diffusion coefficient (16) separately for the GOE and GUE ensembles and found that

$$\text{GOE:} \quad D(E) \propto W(E)\frac{dW(E)}{dE}\sigma^{5/2}\left|\dot{q}\right|^{5/2}, \quad (17)$$

$$\text{GUE:} \quad D(E) \propto W^3(E)\frac{dW(E)}{dE}\sigma^{3}\left|\dot{q}\right|^{3}. \quad (18)$$

We proceed by considering the process of diffusion in the space of the intrinsic degrees of freedom and introduce a new function $F(E,t)$

$$F(E,t) \equiv W(E)\rho(E,t), \quad (19)$$

which has a meaning of the density of probability to find the intrinsic nucleonic subsystem in the state with an energy lying in the interval $[E, E+dE]$.

We rewrite the diffusion equation (15) in terms of the new distribution function F,

$$\frac{\partial F}{\partial t} = -\frac{\partial}{\partial E}[r(E,t)F] + \frac{\partial^2}{\partial E^2}[D(E,t)F], \quad (20)$$

with the drift coefficient

$$r(E,t) = \frac{dD(E,t)}{dE} + \frac{D(E,t)}{W(E)}\cdot\frac{dW(E)}{dE}. \quad (21)$$

The excitation of the nucleonic subsystem implies that the distribution function F grows with time. The necessary condition for that is the positively defined the drift coefficient (21). We see that there are two different contributions to the drift r. The first one is caused by the growth of the nuclear level-density with the excitation energy. In view of Eqs. (17) and (18), one can say that the second contribution is due to possible energy increase of the ensemble averaged off-diagonal matrix elements

$$\sigma^2 = \left|\left\langle \Psi_n \left| \frac{\partial \hat{H}_{\text{int}}}{\partial q} \right| \Psi_m \right\rangle\right|^2. \quad (22)$$

We are going to find an analytical solution to the diffusion equation (20) by assuming that the drift and diffusion coefficients varies slowly with the energy E and putting them at $E = E_F$, where E_F is the Fermi energy. In that case, Eq. (20) corresponds to a standard Brownian motion with time-dependent drift and diffusion coefficients. One can get the analytical expression for the distribution function F, or for the density matrix ρ (see Eq. (19)),

$$\rho(E,t) = \frac{1}{W(E)}\frac{1}{\sqrt{2\pi\Delta^2(t)}}\exp\left(-\frac{[E - < E(t) >}{2\Delta^2(t)}\right), \quad (23)$$

where the excitation energy of the nucleonic subsystem $< E >$ reads

$$< E(t) > = < E(t=0) > + \int_0^t r(E_F, t')dt', \quad (24)$$

and the variance Δ^2 is given by

$$\Delta^2(t) = \Delta^2(t=0) + 2\int_0^t r(E_F,t')dt' \times \int_0^{t'} r(E_F,t'')dt'' + 2\int_0^t D(E_F,t')dt' - \left[\int_0^t r(E_F,t')dt'\right]^2. \quad (25)$$

4. Coupled dynamics of the collective and nucleonic subsystems

Now we turn to the consideration of time evolution of the classical collective variable q. We derive equation of motion for q by requiring that the total energy of the nuclear many-body system is conserved with time,

$$E(t) \equiv \left\langle \hat{H}_{tot} \right\rangle = \frac{1}{2}B(q)\dot{q}^2 + U(q) + \left\langle \hat{H}_{\text{int}} \right\rangle = const. \quad (26)$$

Differentiating over time both sides of the last equation, we obtain

$$B(q)\ddot{q} = -\frac{1}{2}\frac{\partial B(q)}{\partial q}\dot{q}^2 - \frac{\partial U(q)}{\partial q} - \frac{1}{\dot{q}}\frac{d\left\langle \hat{H}_{\text{int}} \right\rangle}{dt}, \quad (27)$$

where the rate of change of the nucleonic subsystem's energy can be represented as

$$\frac{d\left\langle \hat{H}_{\text{int}} \right\rangle}{dt} = Tr\left(\hat{H}_{\text{int}}\dot{\hat{\rho}}\right) = \dot{\Xi}_1 + \dot{\Xi}_2. \quad (28)$$

The first term in Eq. (28) gives the rate of change of the adiabatic potential energy,

$$\dot{\Xi}_1 = \dot{q}\sum_n \frac{\partial E_n}{\partial q}\rho_{nn} \quad (29)$$

and it will be dropped in the sequel as far as we are interested in the averaged dynamics of the collective variable, i.e., averaged over all random realizations of the slopes $\partial E_n / \partial q$. The second term in (28)

describes the time-irreversible change of the intrinsic nucleonic energy and gives rise to dissipative effects in the collective motion. It can be written as

$$\dot{\Xi}_2 = \dot{q}(t)\sum_n \int_0^t ds \dot{q}(s) N_n(t,s)\rho_{nn}(s)\,, \qquad (30)$$

where we introduced the memory kernel

$$N_n(t,s) = \sum_{m\neq n} \frac{1}{\omega_{mn}(s)} \left\langle \Psi_n \left| \frac{\partial \hat{H}_{\text{int}}}{\partial q} \right| \Psi_m \right\rangle (t) \times$$

$$\times \left\langle \Psi_m \left| \frac{\partial \hat{H}_{\text{int}}}{\partial q} \right| \Psi_n \right\rangle (s) \left[\exp\left(-i \int_s^t dt' \omega_{mn} \right) + c.c. \right]. \qquad (31)$$

We assume that the matrix elements in the expression (31) fluctuate with time much weaker than the frequencies ω_{mn} and approximately write that

$$N_n(t,s) \approx \sum_{m\neq n} \frac{2\cos\left(\omega_{mn}[t=0](t-s)\right)}{\omega_{mn}(t=0)} \left| \left\langle \Psi_n \left| \frac{\partial \hat{H}_{\text{int}}}{\partial q} \right| \Psi_m \right\rangle \right|^2 (t=0). \qquad (32)$$

To simplify the expression for the memory kernel $N_n(t,s)$, we average it over all random realizations of off-diagonal matrix elements and level spacing:

$$N_E(t,s) = 2\int_{-\infty}^{+\infty} de\, W(E+e) R(s) \int_{-\infty}^{+\infty} dh P(h) \cos\left(e[t-s]\right) \frac{h^2}{e}, \qquad (33)$$

where $s = W(E+e/2)e$ is the level spacing normalized to unit spacing, $P(h)$ is the distribution of the off-diagonal matrix elements $h = \left\langle \partial \hat{H}_{\text{int}} / \partial q \right\rangle_{nm}$,

$$P(h) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{h^2}{2\sigma^2} \right), \qquad (34)$$

and $R(s)$ is the level spacing distribution function; see, for example Ref. [12]. The main contribution to the integral in (33) gives the region of e where $R(s) \approx 1$ and therefore, we find

$$N_E(t,s) = 2\int_{-\infty}^{+\infty} W(E+e)\sigma^2(E,e) \frac{\cos\left(e[t-s]\right)}{e} de\,. \qquad (35)$$

For the variance of the distribution of the off-diagonal matrix elements σ^2 we use the result of Ref. [13], where it is shown that it has a Gaussian shape characterized by the spreading width Γ_G [14]

$$\sigma^2(E,e) = \sigma_0{}^2 \frac{1}{\sqrt{2\pi} W(E) \Gamma_G} \exp\left(-\frac{e^2}{2\Gamma_G{}^2} \right). \qquad (36)$$

Here, $\sigma^2{}_0$ is the variance of the distribution of energy slopes of nuclear mean-field. The spreading width Γ_G is defined by strength of the residual interaction between nucleons $< V_{res} >$ and is given by

$$\Gamma_G = 2\pi W < V_{res} >. \qquad (37)$$

Since we consider the limit of highly excited states of the nucleonic subsystem where $e << E$, one can expand the level density in the integral (35) in Taylor series and find that

$$N_E(t,s) \approx 2\sigma_0{}^2 \frac{1}{W(E)} \frac{dW(E)}{dE} \exp\left(-\frac{[t-s]^2}{[\hbar/\Gamma_G]^2} \right). \qquad (38)$$

Finally, we get the coupled dynamics of the collective and the nucleonic subsystems:

$$B(q)\ddot{q} = -\frac{1}{2}\frac{\partial B(q)}{\partial q}\dot{q}^2 - \frac{\partial U(q)}{\partial q} -$$

$$-2\sigma_0{}^2 \frac{1}{W(E)} \frac{dW(E)}{dE} \int_0^t \exp\left(-\frac{[t-s]^2}{[\hbar/\Gamma_G]^2} \right) \dot{q}(s)\rho_E(s) ds, \qquad (39)$$

$$\rho_E(s) = \frac{1}{W(E)} \frac{1}{\sqrt{2\pi\Delta^2(\dot{q}[t])}} \exp\left(-\frac{(E - < E(\dot{q}[t])^2}{2\Delta^2(\dot{q}[t])} \right). \qquad (40)$$

5. Summary

Using Zwanzig's projection technique, we derived the master equation for the occupation probabilities, ρ_{nn}, of the energy levels in the moving frame. We introduced then the Landau - Zener transition rates, averaged over all possible gap sizes ε and slopes η of avoided level crossings. Due to the use of the averaging procedure, we have reduced the basic master equation for ρ_{nn} to the diffusion equation. Finally, we have established the coupled equations of motion for the collective and nucleonic subsystems. The main feature of the obtained equation of motion for the collective variable is the presence of the memory integral which gives rise to both the dissipative effect and the additional time-reversible force in the collective motion.

We have studied the structure of the memory integral. Averaging the corresponding memory kernel over all random realizations of the off-diagonal matrix elements and the level spacing, we have reduced significantly the memory integral. The key element of its structure is the variance of the

distribution of the off-diagonal matrix elements σ^2. We show that σ^2 has the Gaussian shape and depends on the spreading width Γ_G, which is derived by the strength of the residual interaction between nucleons, see Eq. (36).

Our approach gives a consistent description of nuclear large amplitude dynamics, including the motion along a collective path and along the internal excitation of the nucleus. The averaging over the spectral statistics of avoided level crossings in the moving frame allows us to extract the smeared macroscopic transport parameters. In particular, the memory integral and the diffusion coefficient can be derived along the collective path. An essential advantage of such a smearing procedure is the elimination of the level quasi-crossing problem, which occurs for the quantum cranking model in a moving frame [2, 6]. The result of Eq. (17) for the GOE and Eq. (18) for the GUE statistics shows the dependence of the memory term in Eq. (39) and thereby the dissipation energy on the derivative $dW(E)/dE$ of the level density. This means that the quantum diffusion within the energy level space and the time irreversible exchange between the collective and internal degrees of freedom is possible if the phase space volume is increasing with the excitation energy of the system. Using the structure of the memory integral in Eq. (39), one can see that the classical friction force which is proportional to the collective velocity appears here in the case of the GOE statistics for the motion close to the ground state. For other cases the velocity dependence of the friction force is much more complicated.

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ВПЛИВ СПЕКТРАЛЬНОЇ СТАТИСТИКИ НА ЯДЕРНУ ДИСИПАЦІЮ

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Досліджується вплив квантової дифузії у просторі адіабатичних рівнів внутрішньої нуклонної підсистеми на процеси дисипації при макроскопічному колективному русі ядра. Застосовуючи кренкінг-модельний підхід до опису макроскопічної ядерної динаміки, отримано ньютонівське рівняння руху для однієї колективної змінної, в якому дисипативний характер колективного руху обумовлений переходами Ландау - Зенера та механізмом дисипації Кубо. Рівняння дифузії застосовується при цьому для визначення часової еволюції ймовірності заповнення ядерних рівнів. При обчисленні транспортних коефіцієнтів використано гауссівський ортогональний та унітарний ансамблі рівнів енергії ядра. Досліджуються також умови, за яких можливий обмін енергією між колективними та внутрішніми ступенями волі ядра.

ВЛИЯНИЕ СПЕКТРАЛЬНОЙ СТАТИСТИКИ НА ЯДЕРНУЮ ДИССИПАЦИЮ

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Исследуется влияние квантовой диффузии в пространстве адиабатических уровней внутренней нуклонной подсистемы на процессы диссипации при макроскопическом коллективном возбуждении ядра. Используя кренкинг-модельный подход к описанию макроскопической ядерной динамики, получено ньютоновское уравнение для одной коллективной переменной, в котором диссипативный характер коллективного движения обусловлен переходами Ландау - Зенера и механизмом диссипации Кубо. Уравнение диффузии используется при этом для определения временной эволюции вероятности заполнения ядерных уровней. При вычислении транспортных коэффициентов использованы гауссовский ортогональный и унитарный ансамбли уровней энергии ядра. Исследуются также условия, при которых возможен обмен энергией между коллективными и внутренними степенями свободы ядра.

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