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THE EQUATION OF STATE OF SYMMETRIC AND ASYMMETRIC NUCLEAR MATTER

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The equation of state (EOS) of nuclear matter is a very important ingredient in the study of nuclear properties, heavy ion collisions, neutron stars and supernova. Accurate assessment of the value of the incompressibility coefficient, K, of symmetric nuclear matter, which is directly related to the curvature of the EOS, is needed to extend our knowledge of the EOS in the vicinity of the saturation point. We review the current status of K as determined from experimental data on isoscalar giant monopole and dipole resonances (compression modes) in nuclei within the microscopic theory of mean-field-based random phase approximation.

1. Introduction

It is well-known that the EOS, $E/A = E(\rho)$, of symmetric nuclear matter (SNM) is a very important ingredient in the study of nuclear properties, heavy ion collisions, neutron stars and supernova. Experimentally, we have accurate data on the saturation point ($\rho_0, E(\rho_0)$) of the EOS. From electron and hadron scattering experiments on nuclei, one finds a constant central density of $\rho_0 = 0.16$ fm^{-3} and from the extrapolation of empirical mass formula, we have $E(\rho_0) = -16$ MeV for SNM. Since at saturation $\left.\frac{dE}{d\rho}\right|_{\rho_0} = 0$, one has,

$$E(\rho) = E(\rho_0) + \frac{1}{18}K\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots, \qquad (1)$$

where

$$K = 9\rho_0^2 \left.\frac{d^2(E/A)}{d\rho^2}\right|_{\rho_0} \qquad (2)$$

is the SNM incompressibility coefficient. Therefore, a very accurate value of K is needed to extend our knowledge of the EOS in the vicinity of the saturation point.

There have been many attempts over the years to determine the value of K by considering properties of nuclei which are sensitive to a certain extent to K (see Ref. [1]). In a macroscopic approach analysis of experimental data of a certain physical quantity, K appears in the expression for the physical quantity and the value of K is determined by a direct fit to the data. In a microscopic approach, one considers various effective two-body interactions which are associated with different value of K but reproduce experimental data of various properties of nuclei, such as binding energies and radii. One then determines the effective interaction which best fit the experimental data for a physical quantity which is sensitive to K. We mention, in particular, the attempts [1 - 4] of considering the physical quantities; nuclear masses, nuclear radii, nuclear scattering cross sections, supernova collapses, masses of neutron stars, heavy ion collisions and the interaction parameters F_0 and F_1 in Landau's Fermi liquid theory for nuclear matter. Here we examine the most sensitive method [5, 6] which is based on experimental data on the strength function distributions of the isoscalar giant monopole resonance (ISGMR), $T = 0, L = 0$, and the isoscalar giant dipole resonance (ISGDR), $T = 0, L = 1$, which are compression modes of nuclei, analyzed within microscopic theory of mean-field based random phase approximation (RPA) [7].

Over the last three decades, a significant amount of experimental work was carried out to identify strength distributions of the ISGMR and ISGDR in a wide range of nuclei [8 - 11]. The main experimental tool for studying isoscalar giant resonances is inelastic α-particle scattering since α-particles are selective as to exciting isoscalar modes. The current high precision experimental facilities make it possible to measure the centroid energy E_0 of the ISGMR with an error of $\delta E_0 \sim 0.1 - 0.3$ MeV [10, 11]. Using the approximate relation $(\delta K)/K = 2(\delta E_0)/E_0$ and, for example, the recent experimental value of $E_0 = 13.96 \pm 0.20$ MeV for the ISGMR in ^{208}Pb, one has an error of $\delta K = 6 - 9$ MeV for $K = 200 - 300$ MeV. This enhanced experimental precision calls for a critical accuracy check of the theoretical calculations.

The extraction of the incompressibility coefficient K from experimental data on ISGMR is not straightforward. The *static* incompressibility coefficient K of Eq. (2) describes the propagation of the *first sound* excitations in nuclear matter having the sound velocity

$$c = c_1 = \sqrt{K/9m}. \qquad (3)$$

However the propagation of the first sound implies the regime of frequent inter-particle collisions [12] which is not realized in cold and moderately heated nuclei, where the compression modes are related to the zero sound (rare inter-particle collisions) regime. In general, the sound velocity c is a complicated function of both the incompressibility coefficient K and the dimensionless collisional parameter $\omega\tau$, where τ is the relaxation time. This complicated dependence of the sound velocity of the compression modes (and thereby the eigenenergy of the ISGMR or ISGDR) on K is caused by the dynamic distortion of the Fermi surface which accompanies the collective motion in a Fermi liquid. In a cold nuclear matter, for the rare collision regime $\omega\tau \to \infty$, one has, instead of Eq. (3), the relation

$$c = c_0 = \sqrt{K'/9m}\,, \tag{4}$$

where K' is a strongly renormalized incompressibility coefficient [13]

$$K' \approx 3K\,. \tag{5}$$

The Fermi-surface distortion effects increase the incompressibility coefficient. This increase of the incompressibility coefficient leads to an increase of the energy E_0 of the ISGMR. However, it can be shown [13] that the consistent presence of the same FSD effects in the boundary condition strongly suppresses the increase of E_0 and thus the centroid energy of the ISGMR, the lowest isoscalar giant monopole resonance in the Fermi-liquid drop, is close to the one obtained in the usual liquid drop model where the FSD effects are not taken into account. We point out that the Fermi-surface distortion effects are completely washed out from the dynamic incompressibility coefficient K' and from the corresponding boundary condition for the breathing mode in the case of the scaling assumption for the displacement field $\chi(\mathbf{r},t)$, taken in the form $\chi(\mathbf{r},t) = \alpha(t)\mathbf{r}$ (Tassie model). Note also that the effect of the Fermi surface distortion in the boundary condition is rather small for the overtone excitations. The dynamic and relaxation effects on the ISGMR and on the ISGDR are significantly different. In contrast to the ISGMR, which is the lowest breathing mode, the ISGDR appears as the *overtone* to the lowest isoscalar dipole excitation, which corresponds to a spurious center of mass motion. Due to this fact, the energy of the ISGDR, E_1, varies with τ much faster than the energy E_0 of the ISGMR. This feature can be used to improve the agreement between the theoretical values of the ratio E_1/E_0 to the experimental data.

There have been several attempts [8] in the past to determine K by a least square (LS) fit to the ISGMR data of various sets of nuclei using a semi-empirical expansion in power of $A^{-1/3}$ of the nucleus incompressibility coefficient, K_A, obtained from E_0 using, for example, the scaling model assumption. It was found [8] that the value deduced for K varied significantly, depending on the set of data of the ISGMR energies used in the fit. This is mainly due to the limited number of nuclei in which E_0 is known. We also point out that the scaling model assumption is not very reliable for medium and light nuclei.

The basic theory for the microscopic description of different modes of giant resonances is the Hartree-Fock(HF) based RPA [5, 7]. Self-consistent HF calculations using Skyrme type interactions [14], which are density and momentum dependent contact (delta) interactions, have been very successful in reproducing experimental data on ground state properties of nuclei. The nuclear response function is evaluated within the (continuum) RPA, i.e., small amplitude oscillation [7, 15]. We emphasize that the values of E_0 and E_1 are sensitive, although not directly related to the value of K which is associated with the effective nucleon-nucleon interaction adopted in the HF-RPA calculations, and thus can be used to extract an accurate value for K. It is important to point out that the HF-RPA method optimizes the collective modes in the space of one-particle-one-hole $(1ph)$ excitations. Correlations, associated with excitations of $2ph$ and higher structures, are not accounted for explicitly. The effects of these correlations have been discussed in the literature, see for example the reviews in Refs. [16 - 18]. The main effect is a collisional broadening of the strength distributions which can be accompanied by a certain shift of the resonance peak position. This shift grows with excitation energy and can be of the order of 1 MeV for the rather high lying isovector modes (in the range above 20 MeV). However, the Skyrme interactions, commonly employed in nuclear HF and RPA calculations, are effective forces which incorporate already a great deal of correlations [19]. This reduces the correlation effects on the peak positions of the collective modes [18, 20]. Therefore, for modes with moderate excitation energy around and below 15 MeV, such as the ISGMR, the net effect is of the order of a tenth of MeV [21].

The experimental identification of the ISGMR in ^{208}Pb at excitation energy of $E_0 = 13.7$ MeV [22] provided a very important source of information for K since the excitation energy of the ISGMR is sensitive to K. Random phase approximation (RPA) calculations using existing or modified effective interactions having $K = 210 \pm 30$ MeV were in agreement with experiment [23]. We point out, however, that in these early investigations (i) only a limited class of effective interactions were explored. (ii) For a certain interaction, calculations of the strength distribution of the ISGMR were carried out only for a limited number of nuclei in which experimental data on the ISGMR was available.

The study of isoscalar giant dipole resonance (ISGDR) is very important since this compression mode provides an independent source of information on K. Early experimental investigation of the ISGDR in ^{208}Pb resulted in a value of $E_1 \sim 21$ MeV for the centroid energy [24, 25]. It was first pointed out in [26] that corresponding HF-RPA results for E_1, obtained with interactions adjusted to reproduced experimental values of E_0, are higher than the experimental value by more than 3 MeV and thus this discrepancy between theory and experiment raises some doubts concerning the unambiguous extraction of K from energies of compression modes. A similar result for E_1 in ^{208}Pb was obtained in more recent experiments [27, 9]. Therefore, the value of K deduced from these early experimental data on ISGDR is significantly smaller than that deduced from ISGMR data.

The relativistic mean field based RPA (to be referred to as RRPA) calculations, with the neglect of contributions from negative-energy sea, yielded for K a value in the range of $280 - 350$ MeV [28]. Recent RRPA calculations [29, 30], with the inclusion of negative-energy states in the response function, yield a value of $K = 250 - 270$ MeV. Note that since an uncertainty of about 20 % in the values of K is tantamount to an uncertainty of 10 % in the value of E_0, the discrepancy in the value of K obtained from relativistic and non-relativistic models is quite significant in view of the accuracy of about 2 % in the experimental data currently available on the ISGMR centroid energies. It has been claimed in recent studies [31, 32] that these significant differences are due to the model dependence of K.

On the theoretical side, the experimental data pose a challenge to theory [26] to understand the conflicting results for K deduced from the data on the ISGMR and the data on the ISGDR; (i) the discrepancy associated with the values of E_0 and E_1 and (ii) the discrepancy associated with the relativistic and non-relativistic models.

It was first pointed out in [33] that, although not always stated in the literature, self-consistency is commonly violated in actual implementation of HF based RPA theory for determining the strength functions $S(E)$ and transition densities ρ_t of giant resonances. Therefore it is important to assess the consequences of common violations of self-consistency. We also point out that it is quite common in theoretical work on giant resonances to calculate $S(E)$ for a certain simple scattering operator F whereas in the analysis of experimental data of the excitation cross section $\sigma(E)$ one carries out distorted-wave-Born-approximation (DWBA) calculations with a transition potential δU obtained from a collective model transition density ρ_{coll} using the folding model approximation. Therefore, it is important to examine the relation between $S(E)$ and the excitation cross section $\sigma(E)$ of the ISGMR and the ISGDR, obtained by α-scattering, using the folding model DWBA method with ρ_t obtained from HF based RPA.

In Section 2 we review the basic elements of microscopic HF based RPA theory for the strength function and the FM-DWBA method for the calculation of the excitation cross sections of giant resonances by inelastic α-scattering. In section 3, we provide some results of the consequences of violations of self-consistency on the calculated strength function $S(E)$, the excitation cross section $\sigma(E)$ and recent results of fully self consistent HF based RPA calculations of the centroid energies (E_0 and E_1) for the ISGMR and ISGDR. We also present simple explanations for the discrepancies in the value deduced for K. Our conclusions are given in section 4.

2. Formalism

2.1 Self-Consistent HF-RPA Approach

In the microscopic and self-consistent Hartree-Fock (HF) base random-phase-approximation (RPA) approach one starts by adopting specific effective nucleon-nucleon interaction, V_{12}, such as the Skyrme type interaction, with parameters determined by a fit to experimental data of a wide range of nuclei on nuclear masses, charge and mass density distributions, etc. The properties of the giant resonances are then calculated within the self-consistent HF-RPA approach by solving the RPA equation using the particle-hole (p-h) interaction V_{ph} which corresponds to V_{12}. Various numerical methods have been adopted in the literature to solve the RPA equations, see for example Refs. [7, 15, 23, 34, 35]. In particular, in the Green's function approach [7, 15] one evaluates the RPA Green's function G,

$$G = G_0(1+V_{ph}G_0)^{-1}, \tag{6}$$

where G_0 is the free *p-h* Green's function. Then, the strength function $S(E)$ and the transition density ρ_t, associated with the scattering operator

$$F = \sum_{i=1}^{A} f(\mathbf{r}_i), \tag{7}$$

are obtained from

$$S(E) = \sum_n \left|\langle 0|F|n\rangle\right|^2 \delta(E-E_n) = \frac{1}{\pi}\,\mathrm{Im}\,[\mathrm{Tr}(fGf)], \tag{8}$$

$$\rho_t(r,E) = \frac{\Delta E}{\sqrt{S(E)\Delta E}}\int f(r')\left[\frac{1}{\pi}\mathrm{Im}\,G(r',r,E)\right]dr'. \tag{9}$$

Note that $\rho_t(r,E)$, as defined in (9), is associated with the strength in the region of $E \pm \Delta E/2$ and is consistent with

$$S(E) = \left|\int \rho_t(r,E)\, f(r)dr\right|^2 / \Delta E. \tag{10}$$

In fully self-consistent HF-RPA calculations, the spurious state (associated with the center of mass motion) $T = 0$, $L = 1$ appears at zero excitation energy ($E = 0$) and no spurious state mixing (SSM) in the ISGDR occurs. However, although not always stated in the literature, many actual implementations of HF-RPA (and relativistic RPA) are not fully self-consistent [33], see however Refs. [34 - 40]. One usually makes the following approximations: (i) limiting the p-h space in a discretized calculation by a cut-off energy E_{ph}^{max}, (ii) introducing a smearing parameter (i.e., a Lorenzian with $\Gamma/2$), and (iii) neglecting the two-body Coulomb and spin-orbit interactions in V_{ph}. Each of these approximations may shift the centroid energies of giant resonances and introduce a SSM in the ISGDR.

It was shown in Refs. [33, 41, 42] (see also [43, 44]) that in order to correct for the effects of the SSM on $S(E)$ and the transition density one should replace the scattering operator $F = \sum_{i=1}^{A} f(\mathbf{r}_i)$, by the projection operator

$$F_\eta = \sum_{i=1}^{A} f_\eta(\mathbf{r}_i) = F - \eta F_1, \tag{11}$$

with $f_\eta = f - \eta f_1$ where $f(\mathbf{r}) = f(r)Y_{1M}(\Omega)$ and $f_1(\mathbf{r}) = rY_{1M}(\Omega)$. The value of η is obtained from the coherent spurious state transition density [45]

$$\rho_{ss}(r) = \alpha_a \frac{\partial \rho_0}{\partial r} Y_{1M}(\Omega), \tag{12}$$

where ρ_0 is the ground state density of the nucleus, by

$$\eta = \langle f\rho_{ss}\rangle / \langle f_1\rho_{ss}\rangle. \tag{13}$$

To determine ρ_t for the ISGDR one first uses (9), with f_η and obtain $\rho_\eta(r)$ and then projects out the spurious contribution which is proportional to $\rho_{ss}(r)$

$$\rho_t(r) = \rho_\eta(r) - \alpha\rho_{ss}, \qquad \alpha = \langle f_1\rho_\eta\rangle / \langle f_1\rho_{ss}\rangle. \tag{14}$$

Using (12) and (13) with $f(r) = f_3(r) = r^3 Y_{1M}(\Omega)$, adopted in the calculations for the ISGDR, one has that $\eta = \frac{5}{3}\langle r^2\rangle$.

2.2 DWBA Calculations of Excitation Cross-Section

The folding model approach [46] to the evaluation of optical potentials appears to be quite successful and, at present, is extensively used in theoretical descriptions of αparticle scattering [47]. The main advantage of this approach is that it provides a direct link to the description of αparticle scattering reactions based on microscopic HF-RPA results.

The DWBA differential cross section for the excitation of a giant resonance by inelastic α-scattering, $\alpha + N \to \alpha + N^*$, is given by,

$$\frac{d\sigma^{DWBA}}{d\Omega} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i}\left|T_{fi}\right|^2, \tag{15}$$

where μ is the reduced mass and k_i and k_f are the initial and final linear momenta of the α-nucleus relative motion, respectively. The transition matrix element T_{fi} is given by,

$$T_{fi} = \left\langle \chi_f^{(-)} \Psi_f \left| V \right| \chi_i^{(+)} \Psi_i \right\rangle, \tag{16}$$

where V is the α-nucleon interaction, Ψ_i and Ψ_f are the initial and final states of the nucleus and $\chi_i^{(+)}$ and $\chi_f^{(-)}$ are the corresponding distorted wave functions of the relative α-nucleus relative motion, respectively. To calculate T_{fi}, Eq. (16), one can adopt the following approach which is usually employed by experimentalists. First, assuming that Ψ_i and Ψ_f are known, the integrals in Eq. (16) over the coordinates of the nucleons are carried out to obtain the transition potential

$$\delta U \sim \int \Psi_f^* V \Psi_i. \tag{17}$$

Second, the cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i} \left| \left\langle \chi_f^{(-)} \left| \delta U \right| \chi_i^{(+)} \right\rangle \right|^2 \tag{18}$$

is calculated using a certain DWBA code with δU and the optical potential $U(r)$ as input.

Within the FM approach, the optical potential $U(r)$ is given by

$$U(r) = \int dr' V(\left| r - r' \right|, \rho_0(r')) \rho_0(r'), \tag{19}$$

where $V(\left| r - r' \right|, \rho_0(r'))$ is the α-nucleon interaction and $\rho_0(r')$ is the ground state HF density of a spherical target nucleus. To obtain the results given in the following, both the real and imaginary parts of the α-nucleon interaction were chosen to have Gaussian forms with density dependence [47]

$$V(\left| \mathbf{r} - \mathbf{r}' \right|, \rho_0(r')) = -V(1 + \beta_V \rho_0^{2/3}(r')) e^{-\frac{\left|\mathbf{r}-\mathbf{r}'\right|^2}{\alpha_V}} -$$

$$iW(1 + \beta_W \rho_0^{2/3}(r')) e^{-\frac{\left|\mathbf{r}-\mathbf{r}'\right|^2}{\alpha_W}}. \tag{20}$$

The parameters V, β_V, α_V, W, β_W, α_W in Eq. (20) were determined by a fit to the elastic scattering data. The radial form $\delta U_L(r, E)$ of the transition potential, for a state with the multi-polarity L and excitation energy E, is obtained from

$$\delta U(r, E) = \int dr' \delta\rho_L(r', E) \times$$

$$\times \left[V(\left| r - r' \right|, \rho_0(r')) + \rho_0(r') \frac{\partial V(\left| r - r' \right|, \rho_0(r'))}{\partial \rho_0(r')} \right], \tag{21}$$

where $\delta\rho_L(r', E)$ is the transition density for the considered state.

We point out that within the "microscopic" folding model approach to the α-nucleus scattering, both ρ_0 and ρ_L, which enter Eqs. (19) and (21), are obtained from the self-consistent HF-RPA calculations. Within the "macroscopic" approach, one adopts collective transition densities, ρ_{coll}, which are assumed to have energy-independent radial shapes and are obtained from the ground state density using a collective model.

Another approach for evaluating T_{fi}, usually employed in theoretical calculations, is to first integrate over the relative α-nucleus coordinates to obtain the scattering operator

$$O \sim \int \chi_f^{(-)*} V \chi_i^{(+)} \tag{22}$$

and then calculate the matrix element $< \Psi_f \mid O \mid \Psi_i >$ within a theoretical model for Ψ_i and Ψ_f. We emphasize that it is quite common in theoretical work to adopt for (22) the operator of Eq. (7), with a very simple scattering operator such as $f(r) \sim r^L Y_{LM}(\theta, \phi)$, where L is the multi-polarity of the excitation, and evaluate the strength function $S(E)$. Therefore, for a proper comparison between experimental and theoretical results for $S(E)$, one should adopt the "microscopic" folding model approach in the DWBA calculations of $\sigma(E)$.

3. Results and Discussion

3.1 Consequences of Violation of Self-Consistency

Very recently, the effects of common violations [33] of self consistency in HF based RPA calculations of $S(E)$ and ρ_t of various giant resonances were investigated in detail, see for example Refs. [37 - 39, 48 - 50]. In the following we present some results of these investigations to demonstrate the importance of carrying out fully self consistent calculations. In Table 1, we show results for isoscalar giant resonances ($L = 0$, 1 and 2) obtained within the HF based discretized and continuum RPA frameworks for ^{80}Zr ($N = Z = 40$) [38]. The two-body interaction V_{12} was taken to be of a simplified Skyrme type

$$V_{12} = \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[t_0 + \frac{1}{6} t_3 \rho^\alpha \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \right], \tag{23}$$

where $\alpha = 1/3$, $t_0 = -1800$ MeV $\cdot$ fm^3 and $t_3 = 12871$ MeV $\cdot$ fm$^{3(\alpha+1)}$. For these values of the interaction parameters the symmetric nuclear matter equation of state has a minimum at $E/A = -15.99$ MeV, $\rho_0 = 0.157$ fm^{-3} with $K = 226$ MeV. It is seen from Table 1 that for an accuracy of ~ 0.1 MeV in the values of the centroid energies, the maximum particle-hole excitation energy, E_{ph}^{max}, should be larger than 200 MeV.

*Table 1**. **Dependence of the spurious state ($T = 0, L = 1$) energy E_{ss} and the centroid energies E_L of isoscalar multipole giant resonances ($L = 0$, 1 and 2), in MeV, on the value of E_{ph}^{max} (in MeV) adopted in HF – Discretized RPA calculations for ^{80}Zr using the interaction of Eq. (23)**

E_{ph}^{max}	E_{ss}	E_0	E_1	E_2
50	4.7	23.92	35.34	16.11
75	3.3	23.51	35.76	15.51
100	2.9	23.25	35.66	15.14
200	1.5	23.09	35.55	14.82
400	1.0	23.02	35.51	14.73
600	0.9	23.02	35.51	14.72
∞	0.7	23.01	35.46	14.70

* The corresponding HF – Continuum RPA results are placed in the last row (taken from [38]).

In Fig. 1, we present results of recent calculations of $S(E)$ for isoscalar giant resonances of ^{208}Pb with multi-polarities $L = 0 - 3$ using the fully self-consistent method described in Ref. [34, 51]. The realistic SGII interactions [52] were used. It is seen (see also Ref. [48]) from Figure 1 that the effects of violation of self-consistency due to the neglect of the particle-hole (p-h) spin-orbit (LS) or p-h Coulomb interactions (CO) in the RPA calculations are most significant for the ISGMR. For the ISGMR in ^{208}Pb the shift in the centroid energy E_0 is about 0.8 MeV, which is 3 times larger than the experimental uncertainty. We note that a shift of 0.8 MeV in E_0 correspond to a shift of about 25 MeV in K.

3.2 Nuclear Compressibility from ISGMR and ISGDR

In [53, 33], numerical calculations were carried out for the $S(E)$, $\rho_t(r)$ within the HF-RPA theory and for $\sigma(E)$ using the FM-DWBA method. The SL1 Skyrme interaction [54], which is associated with $K = 230$ MeV, was employed. The corrections for the effects of SSM in the ISGDR was carried out as described in section 2.1. The density dependent Gaussian α-nucleon interaction of Eq. (20) was used with parameters adjusted to reproduce the elastic cross section, with ρ_0 taken from the HF calculations, see Refs. [53, 33] for details.

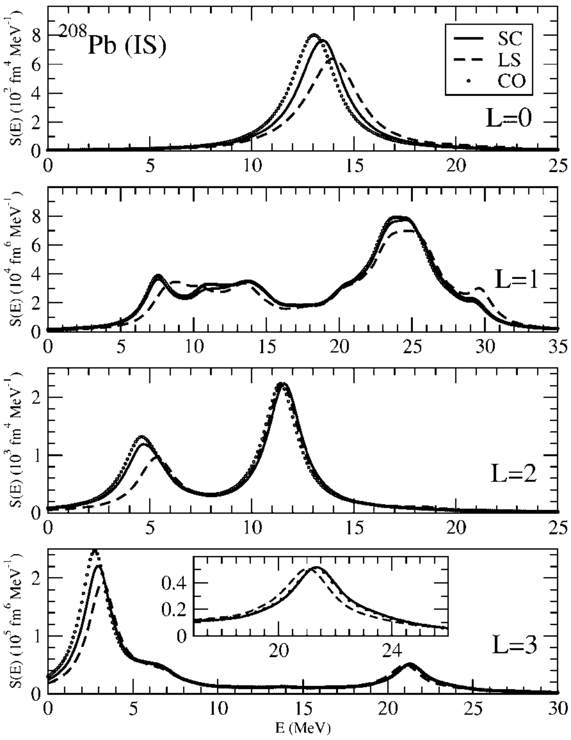

Fig. 1. Isoscalar strength functions of ^{208}Pb for $L = 0 - 3$ multi-polarities are displayed. SC (full line) corresponds to the fully self-consistent calculation where LS (dashed line) and CO (open circle) represent the calculations without the ph spin-orbit and Coulomb interactions in the RPA, respectively. The Skyrme interaction SGII [52] was used (taken from [48]).

In Fig. 2 we present results of microscopic calculations of the fraction of the energy weighted sum rule, ES(E)/EWSR, and the excitation cross section $\sigma(E)$ of the ISGDR in ^{116}Sn by 240 MeV α-particle scattering, carried out within the microscopic HF based RPA and the FM-DWBA theory. It is seen from the upper panel that the use of the collective model transition densities ρ_{coll} increases the EWSR by about 15 %. However, the shifts in the centroid energies are small (a few percents), similar in magnitude to the current experimental uncertainties. It was first pointed out in [33] that an important result of the calculation is that the maximum cross section for the ISGDR decreases strongly at high energy and may drops below the experimental sensitivity for excitation energy above 30 MeV. This high excitation energy region contains about 20 % of the EWSR. This missing strength leads to a reduction of about 3.0 MeV in the ISGDR energy which significantly reduces the discrepancy between theory and experiment.

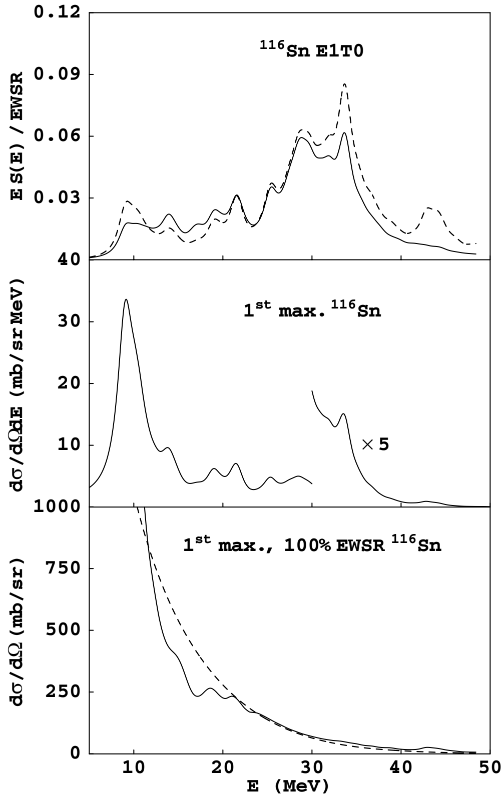

Fig. 2. Reconstruction of the ISGDR EWSR in ^{116}Sn from the inelastic α-particle cross sections. Middle panel: maximum double differential cross section obtained from ρ_t (RPA). Lowest panel: maximum cross section (0° degrees) obtained with ρ_{coll} (dashed line) and ρ_t (solid line) normalized to 100 % of the EWSR. Upper panel: The solid and dashed lines are the ratios of the middle panel curve with the solid and dashed lines of the lower panel, respectively (taken from [33]).

In Table 2, we give the results of fully self-consistent HF-RPA calculations for the ISGDR centroid energy (E_1) obtained (see [48]) using the SGII [52] and KDE0 [55] interactions and compare them with the RMF based RPA results of Ref. [56] for the NL3 interaction [57] and with the experimental data. Note that the HF-RPA values for E_1 are larger than the corresponding experimental values of the early measurements of [9, 24, 25, 27] by more than 3 MeV. The more recent results of [10, 11, 58, 59], seems to confirm the prediction of [33]. The recent experimental values of E_1 are significantly closer to the theoretical values (within 2 MeV).

3.3 Compressibility in Relativistic and Nonrelativistic Models

To properly compare between the predictions of the relativistic and the non-relativistic models, parameter sets for Skyrme interaction were generated in [60] by a least square fitting procedure using exactly the same experimental data for the bulk properties of nuclei considered in [57] for determining the NL3 parameterization of an effective Lagrangian used in the relativistic mean field (RMF) models. The center of mass correction to the total binding energy, finite size effects of the protons and Coulomb energy were calculated in a way similar to that employed in determining the NL3 parameter set in [57]. Further, the values of the symmetry energy coefficient J and the charge rms radius of the ^{208}Pb nucleus were constrained to be very close to 37.4 MeV and 5.50 fm, respectively, as obtained with the NL3 interaction, and K was fixed in the vicinity of NL3 value of $K = 271.76$ MeV. In particular, the Skyrme interactions SK272 and SK255, having $K = 272$ and 255 MeV, respectively, were generated in [60].

Table 2[*]. **Fully self-consistent HF-RPA results [48] for ISGDR centroid energy (in MeV) obtained using the interactions SGII [52] and KDE0 [55] and compared with the RRPA results obtained [56] with the NL3 interaction [57]**

Nucleus	$\omega_1 - \omega_2$	Experiment	NL3	SGII	KDE0
^{90}Zr	18 - 50	25.7 ± 0.7^a	32.	28.8	29.1
		26.7 ± 0.5^b			
		26.9 ± 0.7^d			
^{116}Sn	18 - 45	23.0 ± 0.6^a	29.	27.4	28.0
		25.5 ± 0.6^c			
		25.4 ± 0.5^d			
^{144}Sm	18 - 45		26.5	26.4	27.3
		24.5 ± 0.4^c			
		25.0 ± 0.3^e			
^{208}Pb	16 - 40	19.9 ± 0.8^a	26.0	24.1	24.7
		22.2 ± 0.5^c			
		22.7 ± 0.2^d			
K (MeV)			272	215	229
J (MeV)			37.4	26.8	33.0

[*] Also given are the corresponding values of the nuclear matter incompressibility, K, and the symmetry energy, J, coefficients. The range of integration $\omega_1 - \omega_2$ is given in the second column. The experimental data are from a) [9], b) [10], c) [11], d) [58], and e) [59].

In Table 3, we present the results of fully self-consistent HF-RPA calculations for the ISGMR centroid energy (E_0) obtained [48] using the SGII [52], KDE0 [55] and the SK255 [60] interactions and compare them with the RMF based RPA results [31] for the NL3 interaction and with the experimental data. Within the non relativistic microscopic HF based RPA theory, the new Skyrme interaction SK255, yields for the ISGMR centroid energies values which are quite close to the RRMF

Table 3[*]. **Fully self-consistent HF-RPA results [48] for the ISGMR centroid energy (in MeV) obtained using the interactions SK255 [60], SGII [52] and KDE0 [55] and compared with the RRPA results obtained [31] with the NL3 interaction [57]**

Nucleus	$\omega_1 - \omega_2$	Experiment	NL3	SK255	SGII	KDE0
^{90}Zr	0 - 60		18.7	18.90	17.89	18.03
	10 - 35	17.81 ± 0.30		18.85	17.87	17.98
^{116}Sn	0 - 60		17.1	17.31	16.36	16.58
	10 - 35	15.85 ± 0.20		17.33	16.38	16.61
^{144}Sm	0 - 60		16.1	16.21	15.26	15.46
	10 - 35	15.40 ± 0.40		16.19	15.22	15.44
^{208}Pb	0 - 60		14.2	14.34	13.57	13.79
	10 - 35	13.96 ± 0.20		14.38	13.58	13.84
K (MeV)			272	255	215	229
J (MeV)			37.4	37.4	26.8	33.0

[*]Also given are the corresponding values of the nuclear matter incompressibility, K, and the symmetry energy, J, coefficients. The range of integration $\omega_1 - \omega_2$ is given in the second column. The experimental data are from [10, 11].

results obtained for the NL3 interaction. Moreover, for the SK255 interaction one finds a good agreement with experimental data for E_0 for all the nuclei considered, provided that the corresponding excitation energy ranges used in determining E_0 are the same as those used in obtaining the experimental data. One thus concludes that contrary to the statements made in [31, 32] the difference in the values of K obtained in the relativistic and non-relativistic models is not due to model dependence. It is mainly due to the differences in the values of the symmetry energy coefficient (J) and its slope associated with these models (see also [61]).

4. Conclusions

We have presented results of microscopic calculations of the strength function, $S(E)$, within the fully self consistent HF-RPA approach, and of α-particle excitation cross sections $\sigma(E)$, using the folding model DWBA, for the isoscalar giant monopole resonance (ISGMR) and the isoscalar giant dipole resonance (ISGDR). An accurate and a general method to eliminate the contributions of spurious state mixing in the ISGDR was employed in the calculations. Considering the status of determining the value of the nuclear matter incompressibility coefficient, K, from data on the compression modes ISGMR and ISGDR of nuclei, we conclude that:

(i) Recent improvement in experimental technique led to the identification of the ISGMR in light and medium nuclei and the observation of the ISGDR in nuclei. Currently the centroid energy E_0 of the ISGMR can be deduced experimentaly with very small uncertainty of about 0.2 MeV, which corresponds to an uncertainty of about 7 MeV in the extracted value of K.

(ii) The maximum cross section for the ISGDR decreases very strongly at high excitation energy and may drops below the current experimental sensitivity for excitation energies above 30 and 26 MeV for ^{116}Sn and ^{208}Pb, respectively. This leads to a missing experimental strength for the ISGDR at high energy and explained the discrepancy between theory and early experimental data concerning the centroid energy E_1 of the ISGDR. This prediction was confirmed in recent experiments [10, 11]. However, more accurate experimental data on the ISGDR is very much needed.

(iii) Common violations of self-consistency in HF based RPA calculations of the strength functions of giant resonances may result in shifts in the calculated values of the centroid energies which are significantly larger in magnitude than the current experimental uncertainties. Thus, it is important to carry out fully self-consistent HF-RPA calculations in order to extract an accurate value of K from experimental data on the ISGMR and ISGDR.

(iv) It is possible to build *bona fide* Skyrme forces so that the incompressibility is close to the relativistic value.

(v) Fully self-consistent calculations of the ISGMR using Skyrme forces lead to the conclusion that $K = 240 \pm 20$ MeV. The uncertainty of about 20 MeV in the value of K is mainly due to the uncertainty in the value of the nuclear matter symmetry energy J and its slope.

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РІВНЯННЯ СТАНУ СИМЕТРИЧНОЇ ТА АСИМЕТРИЧНОЇ ЯДЕРНОЇ МАТЕРІЇ

Ш. Шломо

Рівняння стану (РС) ядерної матерії є дуже важливою складовою у вивченні властивостей ядер, зіткнень важких іонів, нейтронних зірок та наднових. Акуратна оцінка величини модуля стискання симетричної ядерної матерії K , що напряму пов’язана з кривизною РС, необхідна для розширення наших уявлень про РС поблизу точки насичення. Розглядається сучасний стан задачі визначення K з експериментальних даних по гігантських монопольних та дипольних резонансах (моди стиснення) в ядрах у рамках наближення випадкових фаз, побудованого на основі мікроскопічної теорії середнього поля.

УРАВНЕНИЕ СОСТОЯНИЯ СИММЕТРИЧНОЙ И АСИММЕТРИЧНОЙ ЯДЕРНОЙ МАТЕРИИ

Ш. Шломо

Уравнение состояния (УС) ядерной материи является очень важным ингредиентом в изучении свойств ядер, столкновений тяжелых ионов, нейтронных звезд и сверхновых. Аккуратная оценка величины модуля сжатия симметричной ядерной материи K , которая напрямую связана с кривизной УС, необходима для расширения наших представлений об УС вблизи точки насыщения. Рассматривается современное состояние задачи определения K из экспериментальных данных по гигантским монопольным и дипольным резонансам (моды сжатия) в ядрах в рамках приближения случайных фаз, построенного на базе микроскопической теории среднего поля.

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