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THEORETICAL DESCRIPTION OF NUCLEONS PAIRED CORRELATIONS OF EVEN-EVEN NUCLEI IN THE ADIABATIC THREE-PARTICLE MODEL

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A hyperspherical adiabatic approach (HAA) has been suggested to find the energy spectrum of even-even atomic nuclei modelled by a spherically symmetric even-even core plus two valence nucleons in the external shell. The adiabatic three-particle model of nucleus for the case of the spherically symmetric and axially symmetric nucleus has been obtained. The so-called adiabatic three-particle model is based on the assumption on the separability of the motion of valence nucleons into the high-speed motion of nucleons over the angular variables and the adiabatic (slow-speed) motion of nucleons along the hyperradius *R*. The efficiency of the adiabatic approach is illustrated by the example of the numerical calculations of the energy spectrum of low-lying excited states of the even-even atomic nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}O, ^{18}Ne, ^{42}Ca, and ^{58}Ni, which possess two valence nucleons in the shell.

1. Introduction

The difficulties of the mathematical character, which arise in a process of the solution of the Schrodinger equation for stationary states of atomic nuclei, stimulates us to search the different approximate methods and the model approaches of its solution. The most familiar of them are: the shell model [1], the Hartree - Fock method [2], the superfluid nuclei model [3], the K-harmonic method [4], the interacting bosons model [5], the unitary correlation operator method (UCOM) [6], the Monte Carlo shell model [7] and others. It is well known that each of these methods has its characteristic advantages and accordingly some limitations.

The angular and radial correlations of nucleons and the pairing effects for nucleons of the same sort play an important role in the formation of excited states of nuclei and appear, particularly, in the presence of gaps in the energy spectra of excited states of even-even nuclei and in their absence in the spectra of odd and odd-odd nuclei. Thus, it is necessary to develop another method for the calculation of the wave functions and the energy spectra of stationary states of even-even nuclei, which would go beyond the limits of the one-nucleon Hartree - Fock approach [2]. As it is known the pair correlations of nucleons of the same sort, which result particularly in the existence of superfluid states of nuclei [8], are considered most logically and correctly in the superfluid model of nucleus [9, 10] within the secondary quantization formalism.

In the present paper, we suggest to consider the pair correlations between nucleons in the potential approach in the framework of the adiabatic three-particle model of nuclei [11 - 14], based on the assumption of separability of the motion of valence nucleons of a nucleus into the high-speed movement in angular variables, i.e. on the sphere $S^5(\Omega)$ and the adiabatic (low-speed) movement of nucleons along the hyperradius *R* and on the introduction of the notion of adiabatic potential term of nucleons $U_\mu(R)$ which is convenient for the description. We recall that the adiabatic three-particle model of nuclei is based on the assumption of the existence of an average self-consistent field in the model of independent particles with taking into account a short-range residual interaction of valence nucleons.

The further development and application of the adiabatic approach in nuclear theory to the investigation of the energy spectrum of both spherical and deformed even-even nuclei within the framework of the adiabatic three-particle model of nuclei and with consideration of the Coulomb interaction between valence protons besides the strong one are considered to be topical now.

2. Description of the Energy Spectrum of Stationary States of a Spherical Nucleus

A theoretical description of the energy spectrum of excited states of nuclei, which are modelled by a spherically symmetric even-even "core" plus two nucleons on the outer unfilled shell, was carried out in [11 - 14] within the HAA method. For the $^{A}_{Z}$X nucleus with two valence nucleons, the description of nucleus in HAA method is carried out in terms of collective variables, whose role is played by the hyperradius R, hyperangle α

$$R = (r_1^2 + r_2^2)^{1/2}, \qquad \alpha = \mathrm{arctg}(r_2 / r_1) \tag{1}$$

and usual spherical angles $\hat{r}_i = \{\varphi_i, \theta_i\}$, $i = 1,2$.

In the adiabatic three-particle model of nuclei, the effective self-consistent field is modelled by the static spherically symmetric Woods - Saxon potential [15]

$$U_i(r_i)=\left(-V_0+2V_1\frac{N-Z}{A}t_z\right)\left(1+\exp\left(\frac{r_i-R_0}{a_0}\right)\right)^{-1}+V_k\left(\frac{1}{2}-t_z\right),$$

$$i=1,2\,, \qquad (2)$$

where $R_0=r_0A^{1/3}$ and the projection of isotopic spin $t_z=\pm1/2$: "-" should be taken for a proton and "+" for a neutron.

If there are two valence protons on the external shell, then the Coulomb potential V_k can be modelled, for the sake of simplicity, as [15]

$$V_k=\sum_{i=1}^{2}V_k\left(r_i\right), \qquad (3)$$

where

$$V_k\left(r_i\right)=\begin{cases}\left[\frac{3}{2}-\frac{1}{2}\left(\frac{r_i}{R_0}\right)^2\right]\frac{e^2(z-2)}{R_0}, & r_i\le R_0\\ \frac{e^2(z-2)}{r_i}, & r_i>R_0\end{cases}. \qquad (4)$$

Here, $V_k\left(r_i\right)$ is the potential energy of interaction between the *i*-th proton and the Coulomb field of the uniformly charged sphere.

For the simplification of further calculations, we can represent the residual strong interaction of valence nucleons between themselves as the potential with zero interaction radius with regard to the repulsion of nucleons at short distances [15]

$$V_{\rm res}(\vec{r}_1,\vec{r}_2)=-V_{12}[1-g\rho(\frac{\vec{r}_1+\vec{r}_2}{2})]\delta(\vec{r}_1-\vec{r}_2)\,. \qquad (5)$$

The repulsion of nucleons is characterized by the term $\rho(\frac{\vec{r}_1+\vec{r}_2}{2})$ which denotes the total one-particle density of nucleons. The relative contribution of repulsion is defined by the g $(g>0)$. Such a choice of the residual interaction simplifies the algorithm of the energy spectrum computation, because it allows one to calculate, in the explicit analytic form, the matrix elements of this interaction and does not distort, possibly, the real situation. In the future, more realistic models of the interaction should be developed.

In the case of valence protons, their Coulomb interaction

$$V_{k12}=\frac{e^2}{\left|\vec{r}_1-\vec{r}_2\right|} \qquad (6)$$

must be added to (5).

The spin-orbital interaction of the *i*-th nucleon is given by

$$V_{l_is_i}(r_i)=W_i(r_i)(\vec{l}_i\cdot\vec{s}_i),\qquad W_i(r_i)=-\chi\frac{1}{r_i}\frac{\partial U_i(r_i)}{\partial\, r_i},$$

$$i=1,2\,. \qquad (7)$$

Thus, in the framework of the adiabatic three-particle model of nuclei in terms of collective variables (1), the potential energy $V\left(R,\Omega\right)$ of the system under study is given by

$$V\left(R,\Omega\right)=U_1\left(R\cos\alpha\right)+W_1\left(R\cos\alpha\right)\left(\vec{l}_1\cdot\vec{s}_1\right)+$$

$$+U_2\left(R\sin\alpha\right)+W_2\left(R\sin\alpha\right)\left(\vec{l}_2\cdot\vec{s}_2\right)+V_{\rm res}+V_{k12}. \qquad (8)$$

We note that using the Hamiltonian with central two-particle and spin-orbital one-particle interactions for a spherical nucleus corresponds to the so-called intermediate coupling approximation.

As was shown in works [11 - 14], the problem of determination of the energy spectrum for spherical atomic nuclei in the framework of the adiabatic three-particle model of nuclei is reduced to solving the two following problems.

In the first place, it is the problem of determination of adiabatic potential terms $U_\mu\left(R\right)$ of nucleons of the nucleus and the corresponding basis functions $\Phi_\mu\left(R,\Omega\right)$. This can be done by a numerical solution of the system of differential equations for the variable α ($\hbar=m_N=1$),

$$\left[\frac{d^2}{d\alpha^2}-\frac{l_1(l_1+1)}{\cos^2\alpha}-\frac{l_2(l_2+1)}{\sin^2\alpha}+U_\mu(R)\right]\varphi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha)+$$

$$+R^2\sum_{j_1'j_2'l_1'l_2'}V^{j_1j_2l_1l_2}_{j_1'j_2'l_1'l_2'}(R,\alpha)\varphi^{(\mu)}_{j_1'j_2'l_1'l_2'}(R,\alpha)=0, \qquad (9)$$

where the coefficients $\Phi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha)$ which appear in the decomposition [11] of the $\Phi_\mu\left(R,\Omega\right)$

$$\varphi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha)=\sin\alpha\cos\alpha\;\Phi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha)\,. \qquad (10)$$

System (9) is supplemented by the boundary conditions that ensure a boundedness of the function $\varphi_\mu(R,\alpha)$ at zero and the validity of the Pauli principle:

$$\varphi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha=0)=0, \qquad (11)$$

$$\varphi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha)\,|_{\alpha=\pi/4}=(-1)^{j-j_1-j_2+1}\varphi^{(\mu)}_{j_2j_1l_2l_1}(R,\pi/2-\alpha)\,|_{\alpha=\pi/4},$$

$$\partial\varphi^{(\mu)}_{j_1j_2l_1l_2}(R,\alpha)/\partial\alpha\,|_{\alpha=\pi/4}=(-1)^{j-j_1-j_2}\,\partial\varphi^{(\mu)}_{j_2j_1l_2l_1}(R,\alpha)/\partial\alpha\,|_{\alpha=\pi/4}\,.$$

Thus, we can find the adiabatic terms $U_\mu\left(R\right)$ and basis functions $\Phi_\mu\left(R,\Omega\right)$ by solving system

(9) with boundary conditions (11). The expansion of the full wave function of the system $\Psi(R,\Omega)$ in the hyperspherical adiabatic basis $\{\Phi_\mu(R,\Omega)\}$ looks

$$\Psi(R,\Omega) = R^{-5/2}\sum_\mu F_\mu(R)\Phi_\mu(R,\Omega). \qquad (12)$$

Secondly, we must determine the radial functions $F_\mu(R)$ and the energy spectrum E of bound states of nucleons through a numerical solution of the system of differential equations for the variable R

$$\left\{-\frac{d^2}{dR^2}-\frac{1}{4R^2}+U_\mu(R)-2E\right\}F_\mu(R)+\sum_{\mu'}\left\{H_{\mu\mu'}(R)F_{\mu'}(R)+Q_{\mu\mu'}(R)\frac{d}{dR}F_{\mu'}(R)+\frac{d}{dR}\left[Q_{\mu\mu'}(R)F_{\mu'}(R)\right]\right\}=0. \qquad (13)$$

The radial functions $F_\mu(R)$ satisfy the boundary conditions

$$F_\mu(0) = F_\mu(\infty) = 0. \qquad (14)$$

In numerical calculations, we substitute the interval $(0,\infty]$ of R by a finite interval $[0,R_{\max}]$ and truncate system (13) to that with a finite number of equations. Using the adiabatic approximation, in which expansion (12) contains only diagonal matrix elements, reduces system (13) to a single equation.

The explicit form of potentials (2) - (7) is given in [12, 13].

The efficiency of the HAA method in the framework of the adiabatic three-particle model of nuclei has been illustrated by the example of numerical calculations of the energy spectra of even-even nuclei under assumption of spherical symmetry of the field of nucleus. The calculated energies of excited states of nucleons for the certain studied nuclei indicate the necessity to take into account the polarization effects for the even-even core, i.e. to consider a deformation of the core field by nucleons from the external unfilled shell.

3. Description of the Energy Spectrum of Stationary States of a Deformed Nucleus

In the calculations of stationary states of deformed nuclei, the Nilsson potential was used for a long time as an effective potential of the average nuclear field of a core [16]. With the help of the Nilsson potential, a rather simple scheme for the determination of the one-particle levels and corresponding wave functions of states of deformed nuclei was developed. However, the Nilsson potential has a number of essential limitations. For example, it has infinite depth, which yields the improper behaviour of wave functions on the nucleus boundary and outside it. Moreover, the spin-orbital interaction in the Nilsson's scheme is independent of the mass number A and the deformation parameters.

Therefore, a more realistic finite anisotropic Woods - Saxon potential becomes recently to be widely used in calculations of the energy spectrum of deformed nuclei [17, 18]. For the first time, the problem of determination of one-particle levels and wave functions of states in a deformed Woods - Saxon potential was investigated by Nemirovskii and Chepurnov in [17]. Later on, other methods of solving the Schrodinger equation with anisotropic Woods - Saxon potential were proposed in the one-particle approximation [18].

It is necessary to note that the integral of motion for deformed nuclei with the form of an ellipsoid of revolution is the projection K of the total angular momentum of a nucleon on the nucleus symmetry axis, i.e. one-particle nucleon states are characterized by energy, parity, and projection K.

In the adiabatic three-particle model of nuclei, the stationary states of two valence nucleons in the deformed nucleus field, which is simulated by the anisotropic Woods - Saxon potential, are determined [19] from the Schrodinger's equation

$$\left(-\frac{\hbar^2}{2\mu_1}\Delta_1-\frac{\hbar^2}{2\mu_2}\Delta_2+\hat{V}-E\right)\Psi=0, \qquad (15)$$

where the potential energy operator of the system is given by

$$\hat{V} = U_1(\vec{r}_1,\beta)+V_{so}(\vec{r}_1,\vec{\sigma}_1,\beta)+U_2(\vec{r}_2,\beta)+$$
$$+V_{so}(\vec{r}_2,\vec{\sigma}_2,\beta)+V_{\text{res}}(\vec{r}_1,\vec{r}_2)+V_{k12}(\vec{r}_1,\vec{r}_2). \qquad (16)$$

Here, $U_i(\vec{r}_i,\beta)$ is purely the nuclear potential energy of the *i*-th nucleon at the point $\vec{r}_i$ in the deformed axially symmetric Woods - Saxon field:

$$U_i(r_i) = \left(-V_0+2V_1\frac{N-Z}{A}t_z\right)\times$$
$$\times\left(1+\exp\left(\frac{r_i-R(\theta_i,\beta)}{a_0}\right)\right)^{-1}+V_k\left(\frac{1}{2}-t_z\right). \qquad (17)$$

The radius $R(\theta_i,\beta)$ of the deformed axially symmetric field of a nucleus depends on the deformation parameter β and the angle θ_i relative to the symmetry axis of a nucleus and is chosen as

$$R(\theta_i,\beta)=R_0[1+\beta Y_{20}(\theta_i)]. \quad (18)$$

As is well known, spin-orbital interaction operators in the case of the nuclear potential $U_i(\vec{r}_i,\beta)$ have the form [17]

$$V_{so}(\vec{r}_i,\vec{\sigma}_i,\beta)=-\chi[\vec{p}_i\times\vec{\sigma}_i]\cdot \mathrm{grad}U_i(\vec{r}_i,\beta). \quad (19)$$

In (15), we separate the spherically symmetric part of interaction and the additional term which sets a deviation of the interaction symmetry from the spherical one. As a result, we obtain the equation

$$(-\frac{\hbar^2}{2\mu_1}\Delta_1-\frac{\hbar^2}{2\mu_2}\Delta_2+V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)-V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0)+V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0)+V_{\mathrm{res}}(\vec{r}_1,\vec{r}_2)-E)\Psi=0. \quad (20)$$

It is convenient to seek for solutions of Eq. (20) in hyperspherical coordinates (1) in the form of a superposition of solutions Ψ_{nJK},

$$\Psi_K(R,\Omega)=\sum_n\sum_J C_{nJK}\Psi_{nJK}(R,\Omega), \quad (21)$$

of the stationary Schrodinger's equation

$$(-\frac{\hbar^2}{2\mu_1}\Delta_1-\frac{\hbar^2}{2\mu_2}\Delta_2+V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0)+$$

$$+V_{\mathrm{res}}(\vec{r}_1,\vec{r}_2)-\varepsilon_{nJ})\Psi_{nJK}=0 \quad (22)$$

with the spherically symmetric potential $V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0)$.

The stationary states

$$\Psi_{nJK}\equiv\Psi_{nJK}(R,\Omega)=F_{nJK}(R)\Phi_{nJK}(R,\Omega) \quad (23)$$

of the corresponding spherical nucleus can be obtained from (22) according to the scheme introduced in [11 - 14] and briefly given in Section 2.

After the substitution of (21) to (20), multiplication of all terms of the equation by $\Psi^*_{n'J'K'}(R,\Omega)$, and integration over the whole region of hyperspherical coordinates, we obtain

$$\sum_n\sum_J(\varepsilon_{nJ}-E)C_{nJK}\delta_{nn'}\delta_{JJ'}+$$

$$+\sum_n\sum_J C_{nJK}\left\langle\Psi_{n'J'K'}\middle|\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)\middle|\Psi_{nJK}\right\rangle=0, \quad (24)$$

where

$$\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)=V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)-$$

$$-V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0)=$$

$$=\sum_{i=1}^{2}[\tilde{U}_i(\vec{r}_i,\beta)+\tilde{V}_{iso}(\vec{r}_i,\vec{\sigma}_i,\beta)], \quad (25)$$

$$\tilde{U}_i(\vec{r}_i,\beta)=U_i(\vec{r}_i,\beta)-U_i(\vec{r}_i,\beta=0), \quad (26)$$

$$\tilde{V}_{iso}(\vec{r}_i,\vec{\sigma}_i,\beta)=V_{iso}(\vec{r}_i,\vec{\sigma}_i,\beta)-V_{iso}(\vec{r}_i,\vec{\sigma}_i,\beta=0). \quad (27)$$

We can represent the spin-orbital addition $\tilde{V}_{so}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$ (27) in potential (25) as [17, 18]

$$\tilde{V}_{so}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)=W_1+W_2+W_3, \quad (28)$$

where

$$W_1=-\chi\sum_{i=1}^{2}\frac{1}{r_i}\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial r_i}(p_{\theta_i}\sigma_{\varphi_i}-\frac{1}{\sin\theta_i}p_{\varphi_i}\sigma_{\theta_i}), \quad (29)$$

$$W_2=-\chi\sum_{i=1}^{2}\frac{1}{r_i^2\sin\theta_i}\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial\theta_i}p_{\varphi_i}\sigma_{r_i}, \quad (30)$$

$$W_3=\chi\sum_{i=1}^{2}\frac{1}{r_i}\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial\theta_i}p_{r_i}\sigma_{\varphi_i}, \quad (31)$$

and

$$p_{r_i}=-i\hbar\frac{\partial}{\partial r_i},\ p_{\theta_i}=-i\hbar\frac{\partial}{\partial\theta_i},\ p_{\varphi_i}=-i\hbar\frac{\partial}{\partial\varphi_i}. \quad (32)$$

In (29) - (31), $\sigma_{r_i},\sigma_{\theta_i},\sigma_{\varphi_i}$ is the Pauli matrices which are given explicitly in [17].

In order to solve the system of equations (24) numerically, we need to know the matrix elements of the potentials of both purely nuclear and spin-orbital interactions.

For the determination of matrix elements in (24), it is convenient to expand $\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$ in a series in terms of spherical functions. For the nuclear terms of potential (25), we obtain

$$\tilde{U}_i(\vec{r}_i,\beta)=\sum_{\lambda_i m_i}A_{\lambda_i m_i}(r_i,\alpha,\beta)Y_{\lambda_i m_i}(\theta_i,\varphi_i). \quad (33)$$

Respectively

$$\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial r_i}=\sum_{\lambda_i m_i}B_{\lambda_i m_i}(r_i,\alpha,\beta)Y_{\lambda_i m_i}(\theta_i,\varphi_i). \quad (34)$$

where the expansion coefficients $A_{\lambda_i m_i}(r_i,\alpha,\beta)$ and $B_{\lambda_i m_i}(r_i,\alpha,\beta)$ should be obtained numerically. In the examined case of an axisymmetric nucleus, $m_1=m_2=0$.

The system of homogeneous equations (24) has nonzero solutions if the determinant composed from

the coefficients of the unknowns C_{nJK} equals zero. Expanding this determinant, we obtain an algebraic equation for the determination of E.

By solving system (24) in the standard way, we can find the energy spectrum E of the deformed nucleus, coefficients C_{nJK}, and, hence, the corresponding wave functions of stationary states of the deformed nucleus. We can obtain the unknown energy E of the deformed nucleus for $\beta << 1$ by the method of perturbation theory with respect to the deformation parameter β.

For the case of minor deformations $\beta << 1$, we can consider the operator $\tilde{V}(\vec{r}_1, \vec{\sigma}_1, \vec{r}_2, \vec{\sigma}_2, \beta)$ in Eq. (24) as an operator of perturbation which represents the difference between a weakly deformed Woods - Saxon potential with deformation parameter $\beta << 1$ and a spherical Woods - Saxon potential. The energy of an arbitrary level E_{nJK} of a deformed nucleus in the first approximation of perturbation theory is given by

$$E = E^{(1)}_{nJK} = \varepsilon_{nJ} + V_{nJK,nJK}, \qquad (35)$$

where ε_{nJ} is the energy of the j-th level of a spherically symmetric nucleus and $V_{nJK,nJK}$ is the unknown matrix element of the operator $\tilde{V}(\vec{r}_1, \vec{\sigma}_1, \vec{r}_2, \vec{\sigma}_2, \beta)$. The explicit type of matrix elements $V_{nJK,nJK}$ (35) of the operator $\tilde{V}(\vec{r}_1, \vec{\sigma}_1, \vec{r}_2, \vec{\sigma}_2, \beta)$ was given in [19].

As it is seen from formula (35), due to the axial symmetry of the Woods - Saxon potential, the energy levels ε_{nJ} that were found for a spherically symmetric Woods - Saxon field split in the axially deformed Woods - Saxon field into the energy levels that correspond to different values of the quantum number K of the angular momentum projection J on the nucleus symmetry axis. That is, the degeneration $2J+1$ in $|K|$ is removed, but the twofold degeneration of levels in the sign of K remains.

Thus, to determine the energy spectrum of a deformed nucleus ${}^{A}_{Z}\mathrm{X}$ in the framework of the considered adiabatic three-particle model of nuclei, it is necessary, following works [11 - 14], to obtain the spectra of levels ε_{nJ} and the corresponding wave functions of stationary states in the assumption of the spherical symmetry of the field of a nucleus, and then to take into account the deformation of the nucleus field while numericaly solving system (24).

4. Numerical Calculations of the Energy Spectra of Even-Even Nuclei

Below we illustrate the main points of the numerical calculation of the energy spectrum of the nuclei in the framework of the adiabatic three-particle model of nuclei. It will be done on the example of low-lying excited states of even-even nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni which possess two valence nucleons in the external shell. For the simplification of calculations, we simulate the strong interaction of valence nucleons by the spherically symmetric Woods - Saxon potential. For the valence protons, we also consider the Coulomb interaction in addition to the strong one.

Accordingly to the asymptotic behavior of the terms $U_{\mu}(R)/R^2$ investigated in [14] in detail, the calculations of the energy spectra of nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}0, ^{42}Ca, and ^{58}Ni under assumption of the spherically symmetric field of a nucleus core were carried out as follows. Parameters of the Woods - Saxon potential were selected in such a way that the potential terms $U_{\mu}(R)/R^2$ of nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni tend to the corresponding levels of isotopes with mass numbers less by unit as $R \to \infty$. The values of the Woods - Saxon potential parameters for nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni defined in such a way are shown in Table 1. Then, following works [11 - 14], the spectra of levels and the corresponding wave functions of stationary states were determined by using the determined parameters of potentials. As the reference zero point, we took the energy when both valence nucleons were in the ground state, i.e. zero reference point, we took the energy of the state where both valence nucleons were in the ground state.

Table 1. **Parameters of the Woods - Saxon potential for nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca and ^{58}Ni**

Nucleus	V_0, MeV	V_1, MeV	R_0, fm	a_0, fm	χ, fm^2
^{6}He	28.0	14.0	1.27	0.625	0.415
^{10}Be	54.0	30.0	1.27	0.625	0.415
^{14}C	45.5	26.0	1.27	0.625	0.415
^{16}C	52.6	31.0	1.27	0.6	0.6
^{18}Ne	38.0	20.0	1.27	0.625	0.415
^{58}Ni	51.5	32.5	1.27	0.6	0.6

The results of calculations of the energy spectrum ε_{nJ} of low-lying excited states of nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni under assumption of a spherically symmetric field are given in Table 2, and their positions on the adiabatic potential terms $U_{\mu}(R)/R^2$ of the nuclei are presented, respectively,

by straight lines in Figure. As a null, we took the energy of separation of two nucleons from nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni, respectively.

Table 2. **Results of calculations of the energy of nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni states under assumption of the spherically symmetric Woods - Saxon potential**

Nucleus ^{A}X	Configuration of nucleons	J^{π}	ε_{nJ} , MeV	$\varepsilon_{\exp}$ [20], MeV	$U_{\mu}(R)/R^2$ for R = 15 fm, MeV	ε_{nJ} [20], for $^{A-1}$X, MeV
^{6}He	$1p_{3/2}\ 1p_{3/2}$	0^+	0	0	0.5575	0.8862
	$1p_{3/2}\ 1p_{3/2}$	2^+	1.7985	1.797	1.1052	0.8862
	$1p_{1/2}\ 1p_{1/2}$	0^+	2.3114	-	1.2786	2.1426
^{10}Be	$1p_{3/2}\ 1p_{3/2}$	0+	0	0	-26.1048	-13.97
	$1p_{3/2}\ 1p_{3/2}$	2+	3.4706	3.368	-26.1048	-13.97
	$1p_{1/2}\ 1p_{1/2}$	0+	6.1797	6.1793	-20.4006	-13.279
	$1d_{5/2}\ 1d_{5/2}$	2+	7.5396	7.542	-18.0282	-9.66
	$1d_{5/2}\ 1d_{5/2}$	4+	9.2702	9.27	-18.0282	-9.66
^{14}C	$1p_{1/2}\ 1p_{1/2}$	0+	0	0	-8.3113	-4.9464
	$1d_{5/2}\ 1d_{5/2}$	0+	6.59	6.5894	-1.7639	-1.0926
	$1d_{5/2}\ 1d_{5/2}$	2+	7.0124	7.012	-1.7638	-1.0926
	$1d_{5/2}\ 1d_{5/2}$	4+	10.7367	10.736	-1.7633	-1.0926
	$1d_{3/2}\ 1d_{3/2}$	0+	9.7461	9.746	0.6392	2.7396
	$1d_{3/2}\ 1d_{3/2}$	2+	10.4261	10.425	0.6399	2.7396
^{16}C	$1d_{5/2}\ 1d_{5/2}$	0^+	0	0	-1.5252	-1.2177
	$1d_{5/2}\ 1d_{5/2}$	2^+	1.7667	1.766	-1.5252	-1.2177
	$1d_{5/2}\ 1d_{5/2}$	4^+	4.1329	4.142	-1.5252	-1.2177
	$2s_{1/2}\ 2s_{1/2}$	0^+	3.0297	3.027	-1.8009	-1.9577
	$1d_{3/2}\ 1d_{3/2}$	0^+	5.1212	-	2.0938	3.4393
	$1d_{3/2}\ 1d_{3/2}$	2^+	6.1071	6.109	2.0938	3.4393
^{18}Ne	$1d_{5/2}\ 1d_{5/2}$	0^+	0	0	-0.5921	-0.6
	$1d_{5/2}\ 1d_{5/2}$	2^+	1.8875	1.8873	-0.5912	-0.6
	$1d_{5/2}\ 1d_{5/2}$	4^+	3.3765	3.3762	-0.5901	-0.6
	$1d_{3/2}\ 1d_{3/2}$	0^+	3.5766	3.5763	1.9102	4.04
	$1d_{3/2}\ 1d_{3/2}$	2^+	3.6165	3.6164	1.9106	4.04
	$2s_{1/2}\ 2s_{1/2}$	0^+	4.5889	4.59	-0.1446	-0.11
^{58}Ni	$2p_{3/2}\ 2p_{3/2}$	0^+	0	0	-11.096	-10.265
	$2p_{3/2}\ 2p_{3/2}$	2^+	1.4563	1.4545	-11.096	-10.265
	$2p_{1/2}\ 2p_{1/2}$	0^+	2.9426	2.9424	-9.8904	-9.1524
	$1f_{5/2}\ 1f_{5/2}$	0^+	3.531	3.5309	-9.9832	-9.4965
	$1f_{5/2}\ 1f_{5/2}$	2^+	3.9018	3.8983	-9.9832	-9.4965
	$1f_{5/2}\ 1f_{5/2}$	4^+	4.3043	4.299	-9.9832	-9.4965

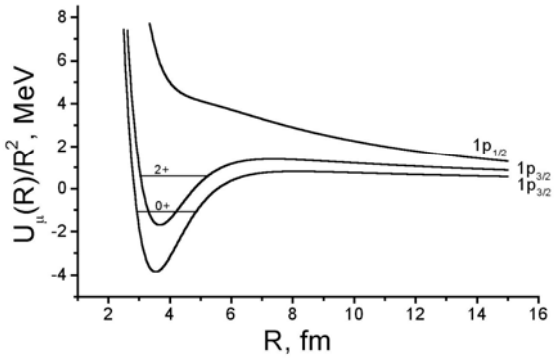
The comparison foregoing of the adiabatic calculations of energy spectrum of nuclei with results of calculations [21] another authors in frame diverse methods and models are evidence on good their coincidence.

5. Conclusions

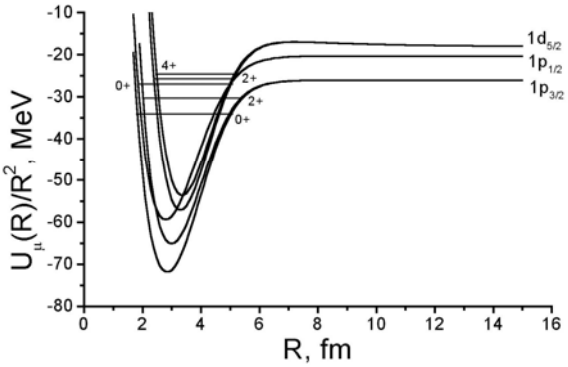
It should be said that in the K-harmonic method the basic functions formulate in obvious look with the help Jakobi polynoms and therefore the decomposition of wave function of stationary state with the help of basic K-harmonics converges very slowly when R increases and it is necessary to take into account a large number of K-harmonics in calculations. In our approach the basic functions $\Phi_{\mu}(R,\Omega)$ are obtained with the help of the numerical solution of the system of differential equations for all values of parameter R and so we can expect a fast convergence of the decomposition (12).

The investigation of a high-speed of the convergence of the adiabatic decomposition of a wave function $\Psi(R,\Omega)$ of arbitrary stationary nuclear state and also the study of a contribution of channel coupling into the energy levels E, it means that the dependence of values of levels from the number of N_R equations of the system (13), is the subject of our future investigations.

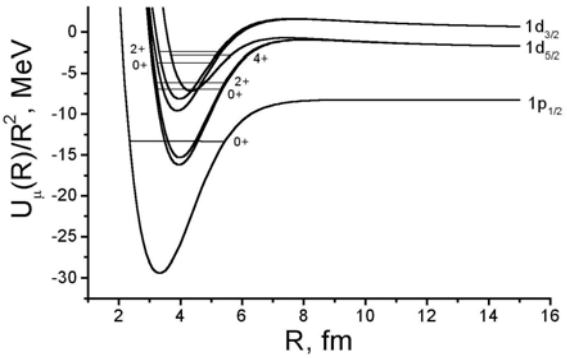
In future, for the numerical calculation of the energy spectrum of the stationary states of deformed nuclei, it is necessary to develop a package of applied computer programs, which would give us a possibility to use more realistic interaction potentials.



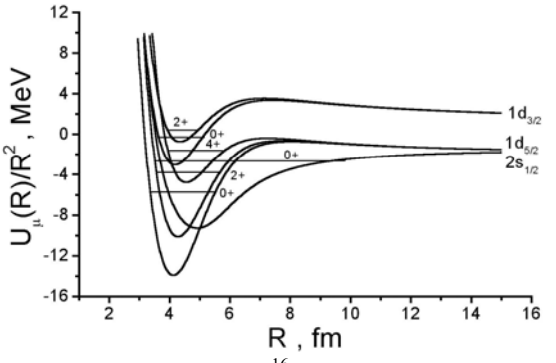
^{6}He



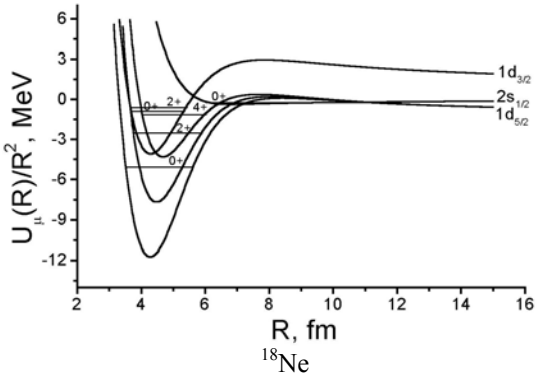
^{10}Be



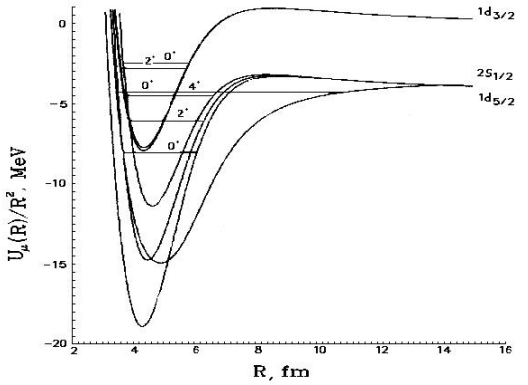
^{14}C



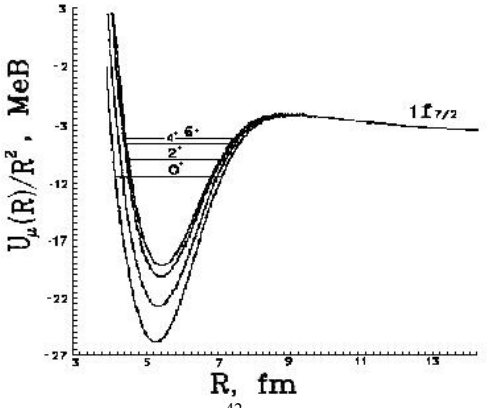
^{16}C



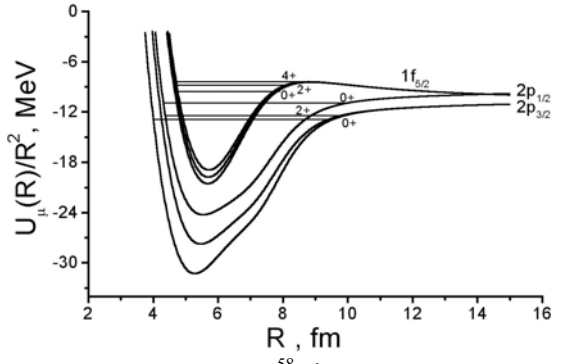
^{18}Ne



^{18}O



^{42}Ca



^{58}Ni

Behavior of the potential curves (terms) $U_\mu(R)/R^2$ and the energy levels of nucleus ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni under assumption of the spherically symmetric Woods - Saxon potential.

With regard for the deformation of the nucleus core field and the spin-orbital interaction (19), we hope to improve the accuracy of calculations of the energy spectra of deformed nuclei.

Numerical calculations of the energy spectra of deformed nuclei in the framework of the adiabatic three-particle model of nuclei are actual for further investigations. Thus, the adiabatic three-particle model of nuclei developed by us allows one to carry out, in the potential approach, the adequate theoretical description of pairing effects of nucleons and their angular and radial correlations which result, in particular, in the creation of superfluid nuclear states.

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ТЕОРЕТИЧНИЙ ОПИС ПАРНИХ КОРЕЛЯЦІЙ НУКЛОНІВ ПАРНО-ПАРНИХ ЯДЕР В АДІАБАТИЧНІЙ ТРИЧАСТИНКОВІЙ МОДЕЛІ

Р. М. Плекан, В. Ю. Пойда, І. В. Хіміч

Запропоновано гіперсферичний адіабатичний підхід для знаходження енергетичного спектра парно-парних атомних ядер, що моделюються відповідним сферично-симетричним парно-парним остовом плюс два валентні нуклони в зовнішній оболонці. Сформульовано адіабатичну тричастинкову модель ядра для випадку сферично-симетричного та аксіально-симетричного деформованого ядра, яка базується на припущенні про відокремлення руху валентних нуклонів на швидкий рух по кутових змінних та адіабатичний (повільний) рух уздовж гіперрадіуса R. Ефективність адіабатичного підходу ілюструється на прикладах чисельного розрахунку енергетичного спектра низьколежачих збуджених станів парно-парних атомних ядер ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}O, ^{18}Ne, ^{42}Ca,^{58}Ni, які містять два валентні нуклони в зовнішній оболонці.

ТЕОРЕТИЧЕСКОЕ ОПИСАНИЕ ПАРНЫХ КОРРЕЛЯЦИЙ НУКЛОНОВ ЧЕТНО-ЧЕТНЫХ ЯДЕР В АДИАБАТИЧЕСКОЙ ТРЕХЧАСТИЧНОЙ МОДЕЛИ

Р. М. Плекан, В. Ю. Пойда, И. В. Химич

Предложен гиперсферический адиабатический подход для нахождения энергетического спектра четно-четных атомных ядер, которые моделируются соответствующим сферическо-симметричным четно-четным остовом плюс два валентных нуклона во внешней оболочке. Сформулирована адиабатическая трехчастичная модель ядра на случай сферическо-симметрического и аксиально-симметрического деформированного ядра, которая базируется на предположении об отделении движения валентных нуклонов на быстрое движение по угловым переменным и адиабатическое (медленное) движение вдоль гиперрадиуса R. Эффективность адиабатического подхода иллюстрируется на примерах численного расчета энергетического спектра низколежащих возбужденных состояний четно-четных атомных ядер ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}O, ^{18}Ne, ^{42}Ca,^{58}Ni, которые содержат два валентных нуклона во внешней оболочке.

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