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RESONANCES IN THREE-CLASTER CONTINUUM OF ^{5}H NUCLEUS

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The resonance structure of ^{5}H is investigated within a three-cluster microscopic model. Hyperspherical Harmonics are used to characterize the channels of the three-cluster continuum and to implement the appropriate boundary conditions. The model predicts the energy and width of the ^{5}H resonance states well and allows for a detailed channel analysis.

1. Introduction

The ^{5}H nucleus has a large neutron excess and lies beyond the neutron drip line (see [1] for a review of recent progress concerning the drip line problem). It has, in the last five years, been the object of quite a few experimental [2 - 10] investigations. The experimental investigations are aimed at finding a clear evidence of the existence of resonance structures in ^{5}H, while the theoretical research is focused on the interpretation of the resonance structure and the determination of its energy and width.

Different experimental approaches are used the study ^{5}H. For instance, in [9] the transfer reaction ^{1}H(^{6}He, ^{2}He)^{5}H was considered. A resonant peak was observed at energy 1,7 ± 0,3 MeV above the $t + n + n$ threshold with a width of 1,9 ± 0,4 MeV. In [5] triple coincidence experiments on the two-neutron transfer reaction t(t, p)^{5}H revealed a resonance at 1,8 ± 0,1 MeV with a small width $\Gamma \leq 0{,}5$ MeV. The two reactions ^{3}H(t, p)^{5}H and ^{2}H(^{6}He, ^{3}He)^{5}H were considered in the experiments in [7] that showed evidence of ^{5}H resonance states at 1,8 ± ± 0,1 MeV and 2,7 ± 0,1 MeV above the $t + n + n$ threshold. The width of these states is about 0,4 MeV. In [6] the one proton knockout reaction (^{6}He, nnt) on a carbon target uncovered a broad peak in the spectrum at 3 MeV with a width of 6 MeV. A recent report [3] of the same group confirms this result. In [2] the first results are reported on states of ^{5}H obtained from the reaction initiated by a ^{6}He + + ^{2}He collision. A resonance was observed at 1,8 ± + 0,2 MeV with a width of 1,3 ± 0,5 MeV. However, there is a contradiction between two sets of experimental results. On one hand [2, 7] there is a claim of observing a narrow resonance $1/2^+$ state in ^{5}H with $E \approx 1{,}8$ MeV and $\Gamma < 2$ MeV; on the other hand [4, 6] claim to observe a broad resonance with a large energy of $E \approx 2{,}5$ MeV and $\Gamma > 3$ MeV. We summarize all these experimental results in Table 1.

Table 1. **Experimental parameters of ^{5}H resonance states**

Experiments	References	E, MeV	Γ, MeV	E, MeV	Γ, MeV
^{1}H(^{6}He, ^{2}He)^{5}H	[8]	≈ 2	—	—	—
^{3}H(t,p)^{5}H, ^{2}H(^{6}He, ^{3}He)^{5}H	[5, 7]	1,8 ± 0,1	< 0,5	2,7 ± 0,1	< 0,5
C(^{6}He, nnt)	[3, 4, 6]	≈ 3	≈ 6	—	—
^{2}H(^{6}He, ^{3}He)^{5}H	[2]	1,8 ± 0,2	1,3 ± 0,5	—	—
^{1}H(^{6}He, ^{2}He)^{5}H	[9]	1,7±0,3	1,9 ± 0,4	—	—
^{1}H(^{6}He, ^{2}He)^{5}H	[15]	≈ 2	—	≈ 2,5	—

It is believed that the first resonance is a $J^\pi = 1/2^+$ state, while the second resonance peak has contributions from $J^\pi = 5/2^+$ and $J^\pi = 3/2^+$ states of ^{5}H.

Different theoretical models and methods have been used to calculate the energy and width of the resonances. In [11] ^{5}H was described as a three-cluster system $t + n + n$, with t treated as a structureless particle. The ^{3}H + n and $n + n$ potentials determine the overall interactions and generate a resonance at an energy of $E = 2{,}5$ - 3,0 MeV with a width of $\Gamma = 3$ - 4 MeV. In [12] was again described as a three-cluster system, but now using a full microscopic model. Resonance states were obtained by the method of analytic continuation in the coupling constant [16, 17]. A resonance state was found at an energy $E \approx 3$ MeV with a width of $\Gamma = 1$ - 5 MeV. In [13] the Complex Scaling Method was considered, also within a three-cluster microscopic model. Three resonance states were determined with total momenta $J^\pi = 1/2^+$, $J^\pi =$ $= 5/2^+$ and $J^\pi = 3/2^+$ respectively. The lowest in energy is the $1/2^+$ resonance with $E = 1{,}59$ MeV and width $\Gamma =$ $= 2{,}48$ MeV.

Algebraic Version of the Resonating Group Method with additional restriction of the Orthogonal Condition Method was used in [14] to consider ^{5}H as a three-cluster system $t + n + n$. Evaluated energy of $1/2^+$ resonance is more than 4 MeV and width of the resonance exceed 5 MeV.

The problem of an adequate interpretation of the theoretical predictions and the connection to the experimental data for resonances in the three-cluster continuum has been discussed in [18].

In this contribution we also present a microscopic model that considers ^{5}H as a three-cluster configuration $t + n + n$. This is only one of the possible three-cluster configurations for ^{5}H, but believed to be the prominent one. Our model is very close to the ones used in [12] and [13]. It allows us to obtain energies and widths of resonances, but also an explicit representation of the corresponding wave functions. Moreover, the model also allows determining the resonance decay rates with respect to the various channels. This in turn makes it possible to conjecture about possible ways for experimentally finding resonances. The current model uses Hyperspherical Harmonics to classify the states of the three-cluster system and to enumerate the channels of the three-cluster continuum. In this respect our method is close to the approach considered in [11].

2. The three-cluster model

We consider the microscopic three-cluster model formulated in the context of the Modified J-Matrix (MJM), or Algebraic, method described in [19], and applied in [20] for the calculation of three-cluster resonance states in ^{6}He and ^{6}Be.

In this contribution we apply it to the ^{5}H nucleus with a three-cluster configuration ^{3}H + n + n with total spin S = 1/2. Such a wave function contains a combination of Young tableaus [32] and [311], and the spin of the two-neutron subsystem can be $S^{(nn)} = 0$ or $S^{(nn)} = 1$. Here it will be applied to the three-cluster configuration ^{3}H + n + n for two values of the total spin, S = 1/2 and S = 3/2. In the first instance, the wave function has a combination of Young tableaus [32] and [311], while in the second instance there is only one Young tableau [311]. For total spin S = 1/2 case, the spin of the two neutrons can be $S^{(nn)} = 0$ or $S^{(nn)} = 1$. The relative behaviour of clusters can be described by two sets of Jacobi coordinates, $\{\vec{q}_1, \vec{q}_2\}$ to which we will refer as the "Y-tree" or "T-tree". If the T-tree of the Jacobi coordinates is considered, the angular momentum connected with the relative motion of the two neutrons will be even for S = 0 or odd for S = 1. For total spin S = 3/2, the spin of the two neutrons is necessarily $S^{(nn)} = 1$, and only odd values of the nn angular momentum are possible. The Young tableau classification ([32] and [311]) is diagonal for the overlap, or antisymmetrization operator, but will be coupled by nucleon-nucleon *(NN)* forces. The shell model wave function of ^{5}H for a configuration of three nucleons in the s-shell and two neutrons in the p-shell, has $SU(3)$ symmetry (λ,μ) = (2,0). The Elliott indexes $\lambda = 2$ and $\mu = 0$ imply that only two values of the total orbital angular momentum, $L = 0$ and $L = 2$ will be dominant in ^{5}H. They also indicate that the spin of the two-neutron subsystem $S^{(nn)} = 0$ would prevail for the low-lying states of ^{5}H.

The three-cluster wave function can be written as

$$\Psi_{LS;JM} = \hat{A}\left\{\left[\Phi_1(t)\Phi_2(n)\Phi_2(n)\right]_S \phi_L(\vec{q}_1,\vec{q}_2)\right\}_{JM}, \quad (1)$$

where $\Phi_1(t)$ is an antisymmetric shell-model wave function describing the internal structure of the triton with three nucleons in the s-shell. The neutron wave function $\Phi_2(n)$ only includes spin and isospin variables of the neutron. $\hat{A}$ stands for the overall antisymmetrization operator. The inter-cluster wave function $\phi_L(\vec{q}_1,\vec{q}_2)$ of relative three-cluster motion is to be determined by solving the Schrödinger equation with the proper boundary conditions. We therefore expand the wave function $\phi_L(\vec{q}_1,\vec{q}_2)$ onto a Hyperspherical Harmonic basis [19 - 24].

$$\phi_L(\vec{q}_1,\vec{q}_2) = \sum_{l_1,l_2} \phi_{l_1,l_2;L}(\rho,\vartheta)\left\{Y_{l_1}(\vec{q}_1)Y_{l_2}(\vec{q}_2)\right\}_{LM} = \\ = \sum_{K,l_1l_2} \phi_{K;l_1,l_2;L}(\rho)\chi_K^{(l_1,l_2)}(\vartheta)\left\{Y_{l_1}(\vec{q}_1)Y_{l_2}(\vec{q}_2)\right\}_{LM}. \quad (2)$$

Hypermomentum K and partial angular momenta l_1 (along $\vec{q}_1$) and l_2 (along $\vec{q}_2$) define the three-cluster geometry, and characterize the different scattering channels. These three quantum numbers will be collectively denoted as $c = \{K;\ l_1,\ l_2\}$. The hyperradial wave function will be expanded onto the basis of the 6-dimensional radial oscillator:

$$\phi_{c,l}(\rho) = \sum_{n_\rho} C_{n_\rho}^{(c,L)} \Phi_{n_\rho}^{(K)}(\rho). \quad (3)$$

We solve the Schrödinger equation by substituting (1) as an ansatz with (2), (3) and obtain a matrix equation in the expansion coefficients $C_{n_\rho}^{(c;L)}$ (see [19] and [20]). Because of the oscillator expansion, we can obtain the solution through the Modified J-Matrix approach, or Algebraic Model [19]. This is an implementation of the Resonating

Group Method in function space. As in coordinate space, we distinguish an interaction and an asymptotic region, but now in the space of expansion coefficients. The latter are split in two subsets: the first set represents the three-cluster wave function in the internal region and is determined by solving the Schrödinger matrix equation; the second set is connected with the asymptotic form of the wave functions and has to represent the proper boundary conditions. The asymptotic set for a scattering boundary condition is obtained from the solution of reference hamiltonians describing three non-interacting clusters. These hamiltonians are defined in [19] and [20]. There will be N_c (the number of channels) independent solutions, with an asymptotic behaviour for large hyperradius ρ

$$\Psi_c^{c_0} \to \Phi_1(A_1)\Phi_2(A_2)\Phi_3(A_3)\sum_c f_c^{(c_0)}(\rho)\chi_c(\Omega 5), \quad (4)$$

c_0 referring to the incoming channel, and $f_c^{(c_0)}$ denoting the asymptotic expansion coefficients, defined as

$$f_c^{(c_0)} \to \delta_{c_0,c}\psi_c^{(-)} - S_{c_0,c}\psi_c^{(+)}(k\rho). \quad (5)$$

Here $\psi_c^{(-)}(k\rho)$ $\left(\psi_c^{(+)}(k\rho)\right)$ is the incoming (outgoing) channel wave function and $S_{c_0,c}$ is the *S*-matrix describing the transition from the initial channel c_0 to the final channel c.

We obtain the resonance parameters from the eigenphase shifts. These are obtained by diagonalizing the *S*-matrix, so that in this eigenchannel representation one obtains (α-enumerates the uncoupled eigenchannels)

$$S_\alpha = \exp\{2i\delta\}, \quad \alpha = 1,2,...,N_c.$$

The relation between the original $\|S_{c,c'}\|$ and diagonal $\|S_\alpha\|$ forms of the *S*- matrix is

$$S_{c,c'} = \sum_\alpha U_\alpha^c S_\alpha U_\alpha^{c'}$$

with $\|U_\alpha^c\|$ an orthogonal matrix. The extraction of resonance position and width is done in the traditional way by

$$\left.\frac{d^2\delta_\alpha}{dE^2}\right|_{E=E_\alpha} = 0, \quad \Gamma = 2\left(\frac{d\delta_\alpha}{dE}\right)^{-1}\Bigg|_{E=E_\alpha} \quad (6)$$

The eigenchannel wave functions are then

$$\Psi_\alpha = \sum_{c_0} U_\alpha^{c_0}\Psi_{c_0} =$$

$$= \hat{A}\left\{\Phi_1(A_1)\Phi_2(A_2)\Phi_3(A_3)\sum_{c_0} U_c^{c_0}\sum_c f_c^{c_0}(\rho)\chi_c(\Omega_5)\right\} \quad (7)$$

so that in the asymptotic region following relation holds

$$f_\alpha(\rho)\chi_\alpha(\Omega_5) = \sum_{c_0} U_\alpha^{c_0}\sum_c f_c^{(c_0)}(\rho)\chi_c(\Omega_5) =$$

$$= \left[\sum_\sigma \psi_{\alpha\sigma}^{(-)}(k\rho) - S_\alpha\sum_\sigma \Psi_{\alpha\sigma}^{(+)}(k\rho)\right]\chi(\Omega_5), \quad (8)$$

where

$$\Psi_{\alpha\sigma}^{(\pm)}(k\rho) = \sum_\sigma\sum_c U_\alpha^c \Psi_c^{(\pm)}(k\rho)U_\sigma^c = \delta_{\alpha\sigma}\Psi_\alpha^{(\pm)}(k\rho).$$

The eigenphases can be analyze by considering the Breit - Wigner resonance formula in each eigenchannel

$$\delta_\alpha = \phi_\alpha + \frac{1}{2}\arctan\left(\frac{\Gamma_\alpha}{E - E_\alpha}\right), \quad (9)$$

where Γ_α is the total width of the α-labelled resonance and ϕ_α a background phase shift. The *S*-matrix for eigenchannels α around α-resonance then has the form

$$S_\alpha = \exp\{2i\delta_\alpha\} = \exp\{2i\phi_\alpha\}\left[\frac{E - E_\alpha - \frac{i}{2}\Gamma_\alpha}{E - E_\alpha + \frac{i}{2}\Gamma_\alpha}\right]. \quad (10)$$

The *S*-matrix in original representation can be expressed in the eigenchannel representation as

$$S_{cc'} = \sum_\alpha U_\alpha^c S_\alpha U_\alpha^{c'} =$$

$$= \sum_\alpha U_\alpha^c \exp\{2i\phi_\alpha\}\left[\frac{E - E_\alpha - \frac{i}{2}\Gamma_\alpha}{E - E_\alpha + \frac{i}{2}\Gamma_\alpha}\right]U_\alpha^{c'}. \quad (11)$$

The following expression for a many-channel *S*-matrix is commonly used

$$S_{cc'} = S_{cc'}^{(bg)} - i\frac{\sqrt{\Gamma_c\Gamma_{c'}}}{E - E_r + \frac{i}{2}\Gamma_r}, \quad (12)$$

where Γ_r is the total width of the resonance state and Γ_c (c = 1, 2, … , N_c) are the partial widths, indicating the decay rate of the compound resonance into the partial channels c. The sum of the partial

widths equals the total width. $S_{cc'}^{(bg)}$ stands for the background *S*-matrix and is assumed to be a monotonic function of energy. It is sometimes assumed that background scattering only occurs on the diagonal of the *S*-matrix, i.e. $S_{cc'}^{(bg)} = \delta_{cc'} S_{cc'}^{(bg)}$

We now show that the eigenchannel representation suggests a new expression for the many-channel resonance form of the *S*-matrix, and a simple way to calculate partial widths. We assume that resonance with a label α manifests itself in the eigenchannel with the same label only. Other eigenchannels don't exhibit resonance behaviour in the energy range around resonance energy E_α. This assumption is based on author's experience (see, for instance [20, 30]) and means that there are no so-called shadow resonances in other eigenchannels. With this assumption we rewrite (11) as

$$S_{cc'} = \left[S_{cc'}^{(bg)} - i\sum_{\alpha} \exp\{i\phi_\alpha\} \frac{U_\alpha^c \Gamma_\alpha U_\alpha^{c'}}{E - E_\alpha + \frac{i}{2}\Gamma_\alpha} \exp\{i\phi_\alpha\} \right] = \tag{13}$$

$$= \left[S_{cc'}^{(bg)} - i\sum_{\alpha} \exp\{i\phi_\alpha\} \frac{\sqrt{\Gamma_{c\alpha}\Gamma_{c\alpha'}}}{E - E_\alpha + \frac{i}{2}\Gamma_\alpha} \exp\{i\phi_\alpha\} \right],$$

where the background *S*-matrix is

$$S_{cc'}^{(bg)} = \sum_{\alpha} U_\alpha^c \exp\{2i\phi_\alpha\} U_\alpha^{c'} \tag{14}$$

and the partial widths $\Gamma_{c\alpha}$ are related to the total width Γ_α by

$$\Gamma_{c\alpha} = \left| U_\alpha^c \right|^2 \Gamma_\alpha . \tag{15}$$

As $\left\| U_\alpha^c \right\|$ is an orthogonal matrix one easily verifies that the total width is indeed the sum of partial widths. Equation (15) suggests a simple way for calculating the partial widths for eigenchannel α. One notices from (13) that the background phase shifts create the background *S*-matrix, and also affect the resonance part.

3. Calculations

In this contribution we consider two *NN*-interactions: the Minnesota potential (MP) [25] and the Modified Hasegawa-Nagata potential (MHN) [26, 27]. We choose the oscillator radius *b* to minimize the ground state energy of the ^{3}H nucleus. This leads to *b* = 1,489 fm for MP, and *b* = 1,470 fm for MHN. We have also (slightly) modified the parameters of the potentials to reproduce the resonance structure of ^{4}H, as well as the scattering phase shift of a neutron from ^{3}H. This was achieved by reducing the *u* parameter of the central part of the MP to 0,98 and by reducing the intensity of the *LS* components of the MHN to *0,5*. Note that the parameters *u* = 0,98 and *b* = 1,470 fm for MP coincide with those selected by Arai [13], but differ with those determined in [12] (*b* = 1,58 fm and *u* = 1,12).

The calculation of the potential part of the energy matrices is a computationally intensive problem. It requires the evaluation of a large number of matrices, characterized by the quantum numbers *c* (as in (3) and an additional expansion parameter coming from the decomposition in terms of two-particle operators (ranging from 1 to 45 in the case of ^{5}H). All these matrices have then to be recomposed using Raynal-Revai coefficients to obtain the final result (for details see [19]). In order to achieve a reasonable computing time, the calculations have been distributed on a grid infrastructure. The description of this methodology can be found elsewhere [29].

An important issue in microscopic calculations using a discrete basis lies in the convergence of the results. In the current report our calculations are limited to a strict maximum of Hyperspherical Harmonics with hypermomentum $L \le K \le 10$, within which a sufficiently convergent set will be considered. As a result, the full set of three-cluster channels consists of 21 channels for total angular momentum *L* = 0, and 45 channels for *L* = 2, representing all possible hyperspherical three-cluster decay configurations of the ^{5}H compound system. The main parameters to be considered in the convergence behavior are: the number of Hyperspherical Harmonics in the internal region (K_i), respectively asymptotic region K_a and the number of hyperradial oscillator shells for each *K* in the internal region (N_i).

4. Results

The ^{5}H nucleus belongs to the so-called Borromean nuclei, i.e. three-cluster systems which have no bound state in any of the two-cluster subsystems. Still, there are features of the two-cluster subsystems that can affect the resonance states of ^{5}H. For instance, while there is no bound state in ^4H, it has two resonances states, created by the ^{3}H+*n* channel with total momentum $J^\pi = 3^-$, $J^\pi = 2^-$ and angular momentum *L* = 1.

There are two important subsystems in ^{5}H that determine its structure. The first one is the two-cluster subsystem consisting of a triton and a

neutron. It is responsible for a set of broad resonance states in ^{4}H [28]. The properties of the ^{4}H resonances, obtained in our approach for a two-cluster description with MP, are listed in Table 2. They are compared to other theoretical descriptions and to experiment. Note that Arai [13] used the Complex Scaling Method to obtain resonance energy and widths, while Descouvemont and Kharbach [12] employed *R*-matrix theory to extract the background phase shift and obtain the resonance properties. Our results are obtained from the calculated phase shifts by (6).

Table 2. **Parameters of the ^{4}H resonance states obtained by Arai, Descouvemont and Kharbach (DK) and the current work**

L, S, J^π	This work	Arai [13]	DK [12]	Experiment [28]
1, 1, 2^-	1,65 + *i*5,60	1,52 + *i*4,11	3,05 + *i*5,1	3,19 + *i*5,42
1, 1, 1^-	1,74 + *i*9,54	1,23 + *i*5,80	3,89 + *i*7,6	3,50 + *i*6,73
1, 1, 0^-	1,82 + *i*11,26	1,19 + *i*6,17	—	5,27 + *i*8,92
1, 0, 1^-	1,51 + *i*8,01	1,32 + *i*4,72	—	6,02 + *i*12,98

The second important subsystem is the neutron-neutron two-cluster component. The MP gives evidence of a virtual state for $L^\pi = 0^+$, $S^{(nn)} = 0$ state, which could be important in the formation of ^{5}H resonance states.

In [20] it was shown that the resonance properties in ^{6}He and ^{6}Be are very sensitive to the number of Hyperspherical Harmonics in the expansion of the internal part of the resonance wave function(s), but less sensitive to those in the asymptotic part of the function(s), and consequently the decay of the three-cluster system. It merely shows that the compound system requires an extensive description, whereas lower Hyperspherical Harmonics describe the decay configurations well. The current calculations indicate the same behaviour, i.e. the need for an involved description of the internal part of the solution, but a less stringent content for the asymptotic part.

We present a convergence study for $L = 0$, $J^\pi = 1/2^+$ resonances and MP. In Table 3 we display the results for increasing $K_i = K_a$ at $N_i = 25$. While full convergence has not been reached yet, reasonable stable results are obtained.

Table 3. **Convergence of resonance properties obtained with MP for $1/2^+$ with varying $K_i = K_a$ and $N_i = 25$**

$K_a = K_i$	E, MeV	Γ, MeV	E, MeV	Γ, MeV
0	—	—	—	—
2	1,750	3,371	3,700	15,165
4	1,770	2,875	3,640	15,900
6	1,680	2,413	4,310	9,356
8	1,550	1,934	4,250	8,784
10	1,480	1,763	4,230	7,847

Table 4 displays, for fixed $K_i = 10$ the convergence as a function of K_a, with $N_i = 25$. One notices only a limited effect of the higher K_a-channels.

Table 4. **Convergence of resonance properties obtained with MP for $1/2^+$ with varying K_a and $K_i = 10$, $N_i = 25$**

K_a	E, MeV	Γ, MeV	E, MeV	Γ, MeV
0	2,240	0,865	—	—
2	1,530	1,797	3,380	18,813
4	1,520	1,787	3,840	11,709
6	1,500	1,780	4,260	8,051
8	1,490	1,783	4,220	7,841
10	1,480	1,777	4,180	7,746

The matter of convergence in terms of N_i is less clear. The results obtained with $N_i = 25$ are stable and reasonably converged. Increasing N_i leads to slightly modified values for the resonance shape, possibly due to numerical instabilities from accumulation errors when calculating matrix elements for high hyperradial quantum numbers. Consequently, and also to limit the computational burden, we will limit ourselves to the calculation of all following results with $K_i = K_a = 8$, $N_i = 25$. We consider these to be sufficiently converged and numerically stable.

Fig. 1 shows the eigenphases obtained with MP for $L = 0$ $J^\pi = 1/2^+$ and Fig. 2 shows those for $L = 2$, $J^\pi = 3/2^+$ and $J^\pi = 5/2^+$. The figures clearly indicate a number of resonances, whose properties can be obtained using (6). In the Table 5 we compare energies and widths of the lowest resonances of ^{5}H for each J^π from the present calculations to other theoretical results. The current results are close to those of Arai [13], but are different from the ones of [12] obtained in a microscopic model, and of [11] obtained in a semi-microscopic model. Our results are very close to Arai's for the energy of the ^{5}H "$1/2^+$ ground state", while the energy and width for the $3/2^+$ and $5/2^+$ states are 1,5 - 2,5 times smaller than Arai's. This may be due to small differences in the model spaces involved in both calculations, and to different methods used for extracting the poles of the *S*-matrix.

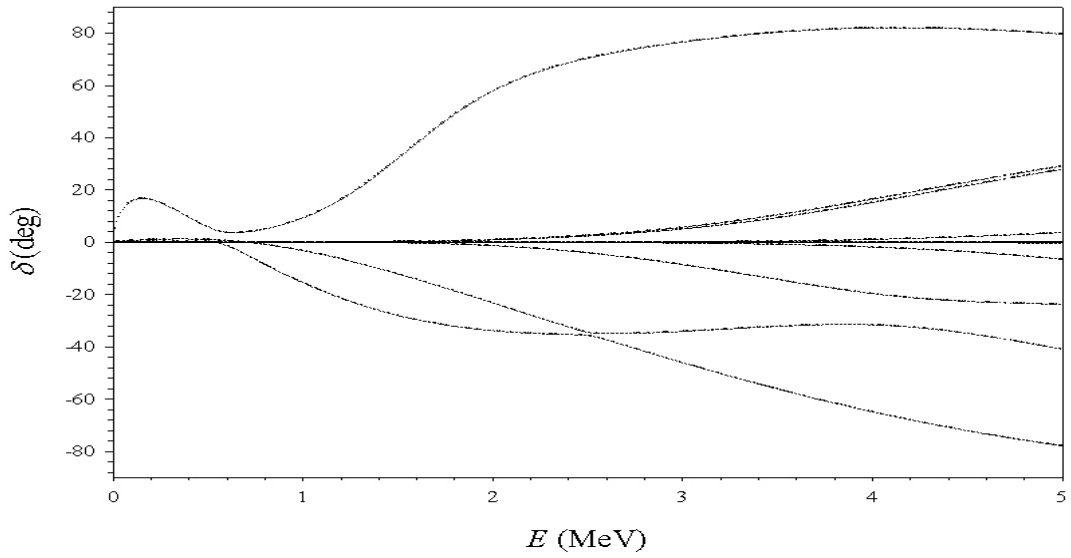

Fig. 1. Eigenphase obtained for $L = 0$, $J^{\pi} = 1/2^{+}$ with MP.

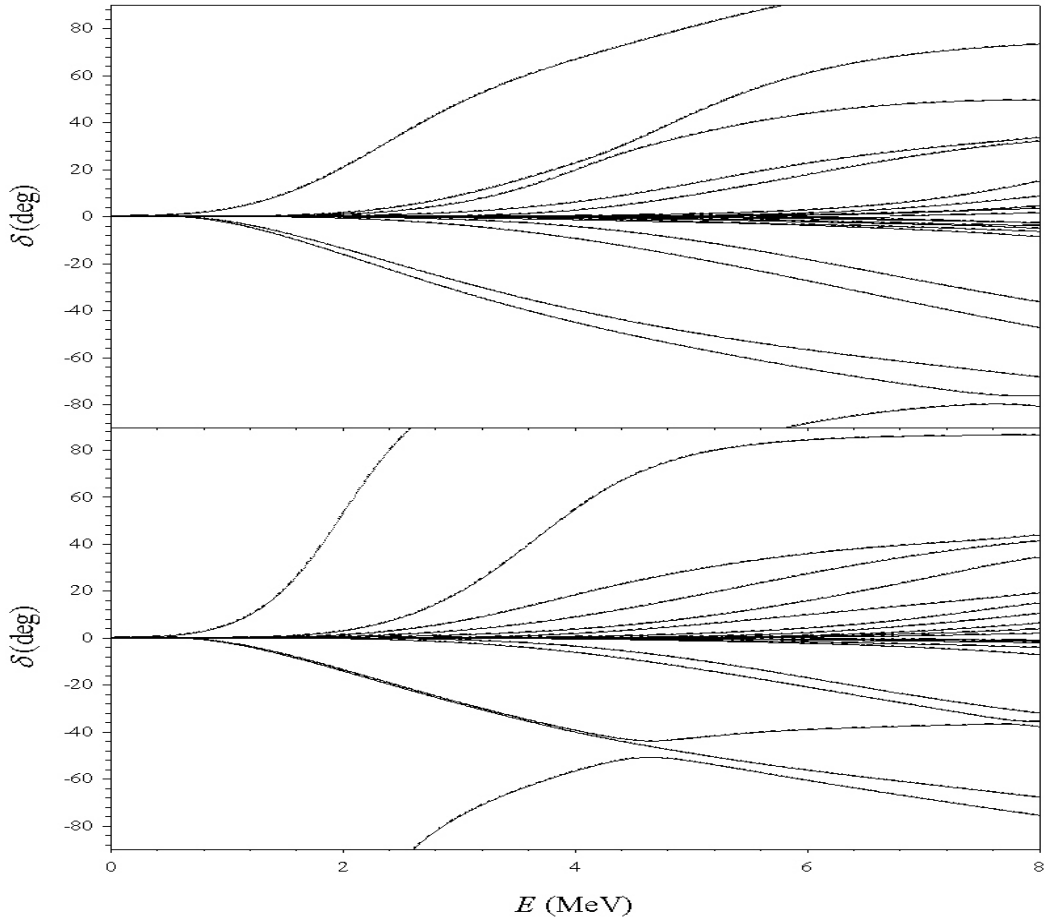

Fig. 2. Eigenphases obtained for $L = 2$, $J = 3/2^{+}$ (top) and $J = 5/2^{+}$ (bottom) with MP.

The effects of different NN-forces are shown in Table 6. Despite the quite different features of the MP and MHN potentials, the energy and width of the lowest resonance states for the $L = 0$ and $L = 2$ momenta are comparable to each other. The differences are more important for the second resonance states. It indicates that the position of the lowest resonance states is determined mainly by the $^{3}H + n$ subsystem, and that there is a small influence of the *nn* subsystem.

Table 5. **Resonance data of ^{5}H, obtained with MP, and results of other theoretical approaches. *E* and Γ in MeV units**

J^π	$½^+$		$3/2^+$		$5/2^+$	
	E	Γ	*E*	Γ	*E*	Γ
This work	1,48	1,78	2,00	1,55	2,38	3,98
Arai [13]	1,59	2,48	2,90	4,10	3,00	4,80
Shul'gina [11]	2,5-3,0	3 - 4	4,6-5,0	5,00	6,4 - 6,9	8,00
DK [12]	2,8-3,0	1 - 2	—	—	—	—

Table 6. **Results of our calculations with the MP and MHN**

L, J^π	MP	MNP	MP	MNP
0, $½^+$	1,55 + i1,93	1,46 + i1,27	2,61 + i528	4,36 + i8,17
2, $5/2^+$	2,00 + i1,55	2,28 + i1,82	3,69 + i2,98	3,88 + i4,23
2, $3/2^+$	2,38 + i3,98	2,71 + i3,81	4,05 + i6,21	4,65 + i7,62

We use equation (15) to determine the partial width of the ^{5}H resonances in order to analyze the resonance content. In Table 7 we display for MP the total width and the three largest values of partial widths, with explicit indication of the corresponding channel $c = \{K; l_1, l_2\}$. For MHN, analogous results are obtained. One finds that only two or three channels of the three-cluster continuum participate in creating the resonances of ^{5}H. More than 70 % of the total width corresponds to the decay of the compound system into channels with hyper-momentum $K = 2$, with a main contribution coming from zero angular momentum l_2 of the *nn* subsystem. It is a strong indication that the *nn* state with $S^{(nn)} = 0$ is dominant in the resonances we are investigating.

Table 7. **Results of our calculations. Total and partial width**

L, J^π	$E + i\Gamma$	Γ_1, c_1	Γ_2, c_2	Γ_3, c_3
0, $½^+$	1,550 + *i*1,934	1,376, {2,0,0}	0,552, {0,0,0}	0,005, {4,0,0}
2, $5/2^+$	2,000 + *i*1,551	1,116, {2,2,0}	0,384, {2,0,2}	0,016, {2,1,1}
2, $3/2^+$	2,380 + *i*3,982	3,162, {2,2,0}	0,612, {2,0,2}	0,093, {4,2,0}

A further analysis can be made directly on the resonance wave functions by showing its density $\left|\Psi_\alpha\left(E_\alpha; r_1, r_2\right)\right|^2$, and corresponding correlation function $r_1^2 r_2^2 \left|\Psi_\alpha\left(E_\alpha; r_1, r_2\right)\right|^2$, as a function of the r_1 and r_2 coordinates where $r_i = q_i \Big/ \sqrt{\mu_i}$, and μ_i is the reduced mass. Fig. 3 shows a visualization for these quantities for the lowest $L = 0$, $J^\pi = 1/2^+$ and $L = 2$, $J^\pi = 3/2^+$ ($L = 2$, $J^\pi = 5/2^+$ being quite identical) resonance states of MP.

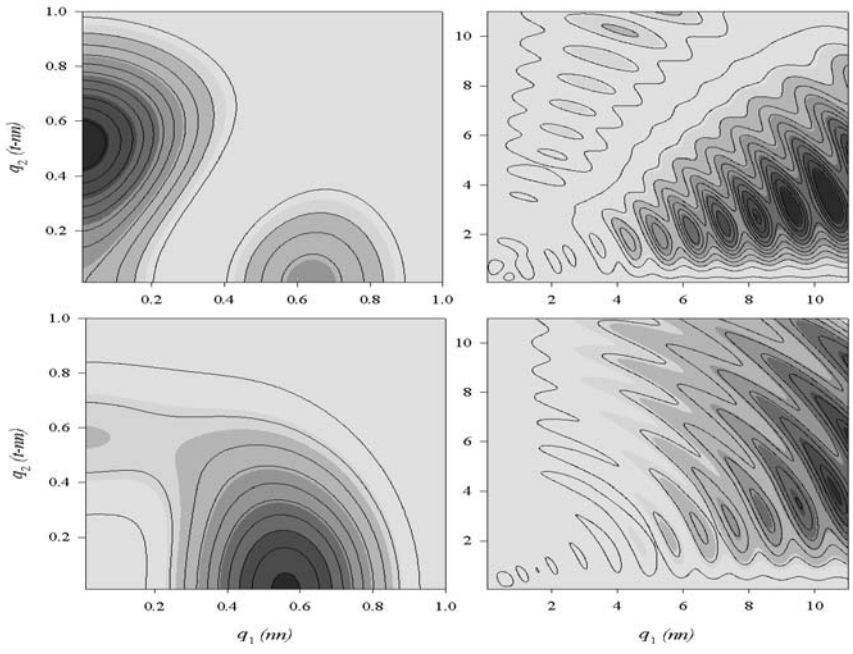

Fig. 3. Wave function density and correlation function (see text) for the lowest $L = 0$, $J^\pi = 1/2^+$ (top row) and $L = 2$, $J^\pi = 3/2^+$ (bottom row) obtained with MP.

5. Conclusions

We have considered the ^{5}H nucleus as a three-cluster configuration $t + n + n$ within a microscopic model. It treats the Pauli principle exactly and takes the proper boundary conditions for a three-body continuum into account. For this aim the hyperspherical harmonics have been used. It was demonstrated that the set of the hyperspherical harmonics, used in the calculations, is extensive enough to provide reliable and convergent results. A number of resonances for $J^\pi = 1/2^+$, $J^\pi = 3/2^+$ and $J^\pi = 5/2^+$ states of ^{5}H are obtained. The energies and widths of the resonances are in a reasonable agreement with the experimental data. We have calculated total and partial widths to identify the most prominent channels for decay of the three-cluster resonances in ^{5}H.

We have found that there is a weak coupling between different three-cluster channels. This is confirmed by the fact that only two hyperspherical harmonics dominate in all resonance states. Moreover, one of these harmonics gives more than seventy percent contribution to the asymptotic part of resonance wave function.

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РЕЗОНАНСИ В ТРИКЛАСТЕРНОМУ КОНТИНУУМІ ЯДЕР ^{5}H

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У рамках трикластерної мікроскопічної моделі розглянуто структуру резонансів ядра ^{5}H. Для класифікації станів та задання граничних умов у трикластерному континуумі використовувався метод гіперсферичних функцій. Модель дає розумні енергії та ширини резонансних станів ^{5}H і дозволяє провести детальний аналіз вірогідностей розпаду по різних каналах.

РЕЗОНАНСЫ В ТРЕХКЛАСТЕРНОМ КОНТИНУУМЕ ЯДЕР ^{5}H

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В рамках трехкластерной микроскопической модели рассмотрена структура резонансов ядра ^{5}H. Для классификации состояний и задания граничных условий в трехкластерном континууме использовался метод гиперсферических функций. Модель дает разумные энергии и ширины резонансных состояний ^{5}H и позволяет провести детальный анализ вероятностей распада по различным каналам.

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