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DETERMINATION OF THE $^{108\text{-}112}$Pd ISOTOPES IDENTITY USING INTERACTING BOSON MODEL

Energy levels, $B(E2)$ transitions and potential energy surface for palladium isotopes with proton number Z = 46 and neutron numbers (n) between 62 and 66 have been calculated through the interacting boson model. The set of parameters used in these calculations are the best approximation that has been carried out so far. The ratio of the excitation energies of the first 4_1^+ and the first 2_1^+ excited states, $R = E4_1^+ / E2_1^+$, is also calculated and an achievable degree of agreement has been investigated in O(6) symmetry for $^{108\text{-}112}$Pd nuclei. The comparison between the calculated energy levels and the transition probabilities $B(E2)$ with those of the experimental show that it is a good agreement. The contour plot of the potential energy surfaces shows all nuclei of interests are deformed and have γ-unstable-like characters.

Keywords: interacting boson model, Pd isotopes, energy levels, ground band, $B(E2)$.

Introduction

The quadrupole collectivity in atomic nucleus exhibits distinct regularities, where the nuclear shape can be spherical, deformed and the situation in between. Arima and Iachello had developed the interacting boson model (IBM), which was based on the well- known shell model and on geometrical collective model of the atomic nucleus [1]. In the other models and theories [1, 2], the Interacting Boson Model [3, 4] had been successful in reproducing the nuclear collective levels in terms of s and d bosons, which are essentially the collective s and d pairs of valence nucleons [5], respectively. As the s and d bosons span a six-dimensional Hilbert space, the Hamiltonian corresponding to the IBM-1 has a group structure U(6). The underlying U(6) group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specie limits of the collective structure vibrations U(5), rotational SU(3) and gamma unstable O(6) [6, 7]. The even-even palladium isotopes Pd (Z = 46) are one of the most important nuclei which characterized by shape changes between spherical and deformed. Many experimental and theoretical studies on the structure of energy level and electromagnetic transition properties of the even-even rare-earth isotopes had been investigated [8 - 17].

In this study, the calculations of energy levels of $^{108\text{-}112}$Pd isotopes have been done using interacting boson model. Positive parity state energies, the reduce probabilities of $E2$ and transitions ($B(E2)$ values) and potential energy surface are calculated and compared with the available experimental data.

Interacting boson model (IBM)

The original version of IBM-1 (no distinction between neutron and proton bosons) applied to even-even nuclei low-lying collective levels [7, 18] and the basic building blocks are s and d bosons with angular momentum L = 0 and L = 2. Furthermore, the model assumes that the structure of low-lying levels is dominated by excitations among the valence partials outside major closed shells. The IBM-1 Hamiltonian can be expressed as [1, 18, 19]:

$$H = \varepsilon_s (s^\dagger \cdot \tilde{s}) + \varepsilon_d (d^\dagger \cdot \tilde{d}) + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} c_L [(d^\dagger \cdot d^\dagger)^{(L)} \cdot (\tilde{d} \cdot \tilde{d})^{(L)}]^{(0)} +$$

$$+ \frac{1}{\sqrt{2}} v_2 [(d^\dagger \cdot d^\dagger)^{(2)} \cdot (\tilde{d} \cdot \tilde{s})^{(2)} + (d^\dagger \cdot s^\dagger)^{(2)} \cdot (\tilde{d} \cdot \tilde{d})^{(2)}]^{(0)} +$$

$$+ \frac{1}{2} v_0 [(d^\dagger \cdot d^\dagger)^{(0)} \cdot (\tilde{s} \cdot \tilde{s})^{(0)} + (s^\dagger \cdot s^\dagger)^{(0)} \cdot (\tilde{d} \cdot \tilde{d})^{(0)}]^{(0)} +$$

$$+ u_2 [(d^\dagger \cdot s^\dagger)^{(2)} \cdot (\tilde{d} \cdot \tilde{s})^{(2)}]^{(0)} + \frac{1}{2} u_0 [(s^\dagger \cdot s^\dagger)^{(0)} \cdot (\tilde{s} \cdot \tilde{s})^{(0)}]^{(0)}. \quad (1)$$

This Hamiltonian contains two terms of one body interactions, (ε_s and ε_d), and seven terms of two-body interactions [c_L (L = 0, 2, 4), v_L (L = 0, 2), u_L (L = 0, 2)]. The ε_s and ε_d are the single-boson energies, and c_L, v_L and u_L describe the two boson interactions [18]. However, it turns out that for a fixed boson number N, only one of the one-body terms and five of the two bodies are termed independent, as it can be seen by noting $N = n_s + n_d$. The equation (1) wrote in terms of the Casimir operators of U (6) group. The O(6) symmetry of the IBM-1 is based on the chain U(6) ⊃ O(6) ⊃ O(5) ⊃ O(3) of nested sub-algebra with quantum numbers N, σ, τ and L, respectively [18, 19]. Then, the IBM-1 Hamiltonian in equation (1) can be written in general form as [19]

$$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_2\hat{Q}\cdot\hat{Q} + a_3\hat{T}_3\cdot\hat{T}_3 + a_4\hat{T}_4\cdot\hat{T}_4, \quad (2)$$

where $\hat{n}_d = (s^\dagger, d^\dagger)$ – total number of d boson operator; $\hat{p} = 1/2\left[\left(\tilde{d}\cdot\tilde{d}\right) - \left(\tilde{s}\cdot\tilde{s}\right)\right]$ – pairing operator; $\hat{L} = \sqrt{10}\left[d^\dagger\cdot\tilde{d}\right]^1$ – angular momentum operator; $\hat{Q} = \left[d^\dagger\cdot\tilde{s} + s^\dagger\cdot\tilde{d}\right]^2 + \chi\left[d^\dagger\cdot\tilde{d}\right]^2$ – quadrupole operator (χ is the quadrupole structure parameter and take the values 0 and $\pm\sqrt{7/2}$ [18, 20]), $T = \left[d^\dagger\cdot d\right]^{(r)}$ is the octoupole (r = 3) and hexadecapole (r = 4) operator, and $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy.

The parameters a_0, a_1, a_2, a_3 and a_4 designated the strength of the pairing, angular momentum, quadrupole, octoupole and hexadecapole interaction between the bosons.

Results and discussions

Energy Level

The Pd nuclei have particle boson neutron numbers 6, 7 and 8 and 2 hole boson proton number according to framework of interacting boson model-1 (IBM-1). The total numbers of boson number are 8, 9 and 10 of $^{108\text{-}112}$Pd nuclei, respectively. The symmetry shape of a nucleus can be predicted from the energy ratio $R = E4_1^+ / E2_1^+$, where $E4_1^+$ is the energy level at 4_1^+ and $E2_1^+$ is the energy level at 2_1^+. Actually, R has a limit value of ≈ 2 for the U(5) vibration nuclei, ≈2.5 for O(6) γ-unstable nuclei and ≈3.33 for SU(3) rotational nuclei [1, 21 - 25].

Table 1 shows that the experimental values of $R = E4_1^+ / E2_1^+$ of those nuclei, in even $^{108\text{-}112}$Pd isotopes $R_{4/2}$ attains to the O(6) value of ~ 2.5.

Table 1. **The $R_{4/2}$ [27 - 30] for Pd isotopes with proton numbers Z = 46**

Isotopes	^{108}Pd	^{110}Pd	^{112}Pd
$R_{4/2}$	2.420	2.466	2.534

The calculations have been performed with IBM-1 and hence, no distinction made between neutron and proton bosons. For the analysis of excitation energies in Pd isotopes it was trying to keep to minimum number of free parameters in the Hamiltonian. The explicit expression of Hamiltonian adopted in calculations is [19].

$$\hat{H} = a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_3\hat{T}_3\cdot\hat{T}_3. \quad (3)$$

Table 2 shows that the number of bosons and the best values of the parameters which give the best fit between the theoretical and the experimental energy levels of the above isotopes. All parameters are given in MeV except N. The energy levels of the ground states g-, γ- and β-bands for even-even Pd isotopes have calculated using IBM with PHINT code [26].

Table 2. **The parameters used for IBM-1 calculations for Pd isotopes**

A	N	PAIR	ELL	OCT
^{108}Pd	8	0.0584	0.0549	0.0382
^{110}Pd	9	0.0473	0.0481	0.0321
^{112}Pd	10	0.0511	0.0494	0.0286

PAIR = $a_0/2$, ELL = $2a_1$, OCT = $a_3/5$) [17].

Fig. 1 show that the calculated g-, γ- and β-bands and the experimental data [27 - 30] for even-even Pd isotopes. From this Figure, the calculated energy levels are in good agreement with the experimental ones for all isotopes. Furthermore, The Figure shows that in all studied nuclei IBM-1 predicts gamma band 3+ and 4+ states to be too high in energy and very close each other, where the O(6) limit is analogous to the geometric model of a γ-unstable rotor, and thus has the γ-band typical 2+, (3+, 4+), (5+, 6+), . . . energy staggering of a γ-soft potential. This is opposite to the staggering in the rigidly asymmetric rotor model of Davydov and Filippov (1958) for γ ~30° [19]. Levels with "( )" in γ- and β-bands correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.

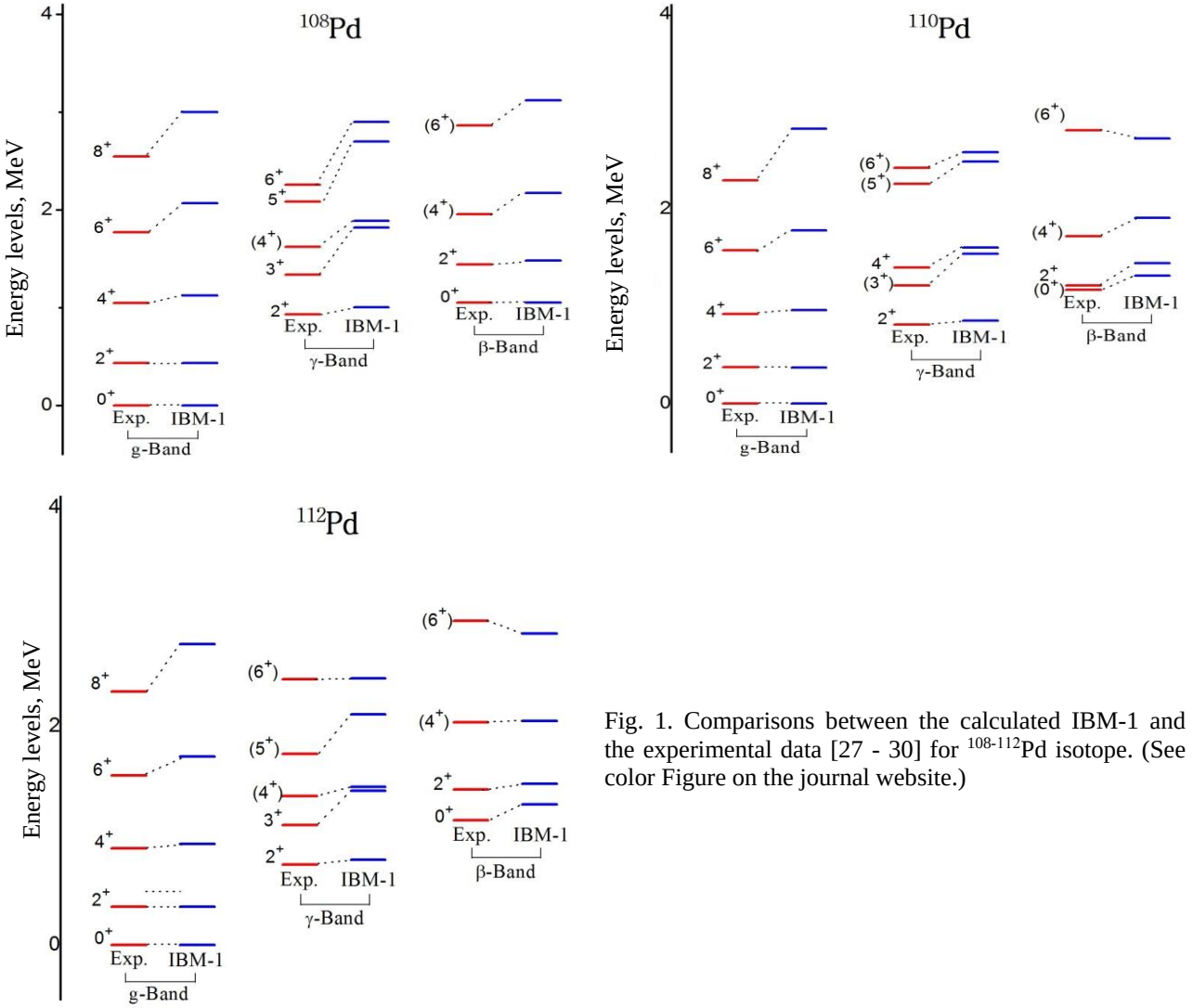

Fig. 1. Comparisons between the calculated IBM-1 and the experimental data [27 - 30] for $^{108\text{-}112}$Pd isotope. (See color Figure on the journal website.)

B(*E*2)Transition

The reduced matrix elements of the *E*2 operator *T*(*E*2) have the form [18, 19]

$$\hat{T}(E2) = \alpha_2 \left[d^{\dagger} s + s^{\dagger} d \right]^{(2)} + \beta_2 \left[d^{\dagger} s + s^{\dagger} d \right]^{(2)}, \quad (4)$$

where $(s^{\dagger}, d)$ and (s, d) are creation and annihilation operators for *s* and *d* bosons, respectively, while α_2 and β_2 are two parameters, and $\beta_2 = \chi\alpha_2$, $\alpha_2 = e_B$ – effective charge boson.

The reduced transition rates can be obtained from the matrix elements of the *T*(*E*2) operator as [3, 19, 31]

$$B((E2)\, I_i \to I_f) = \frac{1}{2L_i + 1} \left| \langle I_f \| T(E2) \| I_i \rangle \right|^2. \quad (5)$$

The effective charge $e_B = \alpha_2$, was determined by normalizing the predictions to the experimental $B(E2; 2_1^+ \to 0_1^+)$ values and tabulated in Table 3. The calculated *B*(*E*2) values compared with the experimental data [27 - 30] for all isotopes under study, and as given in Table 4. This table shows that the *B*(*E*2) transitions β→g and γ→g are smaller and weekly than the *B*(*E*2) transitions between g→g and γ→γ and in general, most of the calculated results in IBM-1 reasonably consistent with the available experimental data, except for the few cases that deviate from the experimental data.

Table 3. **Parameters (in eb) used to reproduce *B*(*E*2) values for $^{108\text{-}112}$Pd isotopes.**

Isotopes	N	e_B
^{108}Pd	8	0.0895
^{110}Pd	9	0.0862
^{112}Pd	10	0.0685

Table 4. **The IBM-1 and experimental [27 - 30] values of $B(E2)$ (in $e^2\,b^2$)**

$I_i \to I_f$	^{108}Pd		^{110}Pd		^{112}Pd	
	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1
$2_1^+ \to 0_1^+$	0.151	0.153	0.171	0.173	0.126	0.131
$2_2^+ \to 0_1^+$	0.002	0.002	0.022	0.024	-	0.018
$2_3^+ \to 0_2^+$	0.105	0.096	-	0.114	-	0.090
$2_2^+ \to 2_1^+$	0.216	0.380	0.135	0.237	-	0.181
$2_3^+ \to 2_1^+$	0.005	0.002	-	0.002	-	0.001
$0_2^+ \to 2_2^+$	0.118	0.026	-	0.025	-	0.016
$4_1^+ \to 2_1^+$	0.228	0.208	0.277	0.237	-	0.181
$4_1^+ \to 2_2^+$	0.003	0.008	-	0.009	-	0.006
$4_2^+ \to 2_2^+$	-	0.117	0.104	0.210	-	0.104
$4_2^+ \to 4_1^+$	<0.005	0.106	-	0.123	-	0.095
$6_1^+ \to 4_1^+$	0.321	0.224	0.332	0.260	-	0.200
$6_2^+ \to 4_2^+$	-	0.149	-	0.176	-	0.138
$8_1^+ \to 6_1^+$	-	0.218	-	0.259	-	0.203
$8_2^+ \to 6_2^+$	-	0.0003	-	0.184	-	0.148

Potential Energy Surface

The potential energy surface gives a final shape to the nucleus that corresponds to the function of Hamiltonian [32], as the equation [1, 19]

$$E(\mathrm{N},\beta,\gamma)=\frac{\langle \mathrm{N},\beta,\gamma|H|\mathrm{N},\beta,\gamma\rangle}{\langle \mathrm{N},\beta,\gamma|\mathrm{N},\beta,\gamma\rangle}. \qquad (6)$$

The expectation value of the IBM-1 Hamiltonian with the coherent state $|\mathrm{N},\beta,\gamma\rangle$ is used to create the IBM energy surface [1, 19].

The state is a product of boson creation operators ($b_c^\dagger$), with

$$|N,\beta,\gamma\rangle=\frac{1}{\sqrt{N!}}(b_c^\dagger)^N|0\rangle, \qquad (7)$$

$$b_c^\dagger=(1+\beta^2)^{-1/2}\{s^\dagger+\beta[\cos\gamma(d_0^\dagger)+\sqrt{1/2}\sin\gamma(d_2^\dagger+d_{-2}^\dagger)]\}. \qquad (8)$$

The energy surface, as a function of β and γ, has been given by [19]

$$E(N,\beta,\gamma)=\frac{N\varepsilon_d\beta^2}{1+\beta^2}+\frac{N(N-1)}{(1+\beta^2)^2}\times$$

$$\times(\alpha_1\beta^4+\alpha_2\beta^3\cos 3\gamma+\alpha_3\beta^2+\alpha_4), \qquad (9)$$

where the α_i's are related to the coefficients C_L, ν_2, ν_0, u_2 and u_0 of the equation (1) as follows [1]:

$$\alpha_1=\frac{1}{10}C_0+\frac{1}{7}C_2+\frac{9}{35}C_4, \qquad \alpha_2=-2\left(\frac{1}{35}\right)^{\frac{1}{2}}\nu_2,$$

$$\alpha_3=\left(\frac{1}{5}\right)^{\frac{1}{2}}(\nu_0+u_2), \qquad \alpha_4=\frac{1}{2}u_0.$$

The β is a measure of the total deformation of the nucleus, where β = 0 the shape is spherical, and is distorted when β ≠ 0, and γ is the amount of deviation from the focus symmetry and correlates with the nucleus, if γ = 0 the shape is prolate, and if γ = 60 the shape becomes oblate.

In the Fig. 2, the contour plots in the γ-β plane, resulting from the $E(N,\beta,\gamma)$ are shown for $^{108\text{-}112}$Pd isotopes. For most of the considered Pd nuclei, the mapped IBM energy surfaces are triaxial shape. Triaxial shape is associated with intermediate values $0 < \gamma < \pi/3$. The triaxial deformation helps to understand the prolate-to-oblate shape transition that occurs in the considered Pd isotopes. Fig. 2 shows the even-even Pd nuclei under study are deformed and have γ-unstable-like characters.

Conclusion

The low-lying positive parity states, β- and γ-bands (energy levels), electric transition probabilities $B(E2)$ values have been calculated using Interacting Boson Model (IBM).

These nuclei assumed to be as medium mass nuclei. All the results were compared with experimental data and acceptable agreement obtained. Electric quadruple transitions probability $B(E2)$ were calculated and compared with available experimental data. The potential energy surfaces show that all nuclei are deformed and have γ-unstable-like characters.

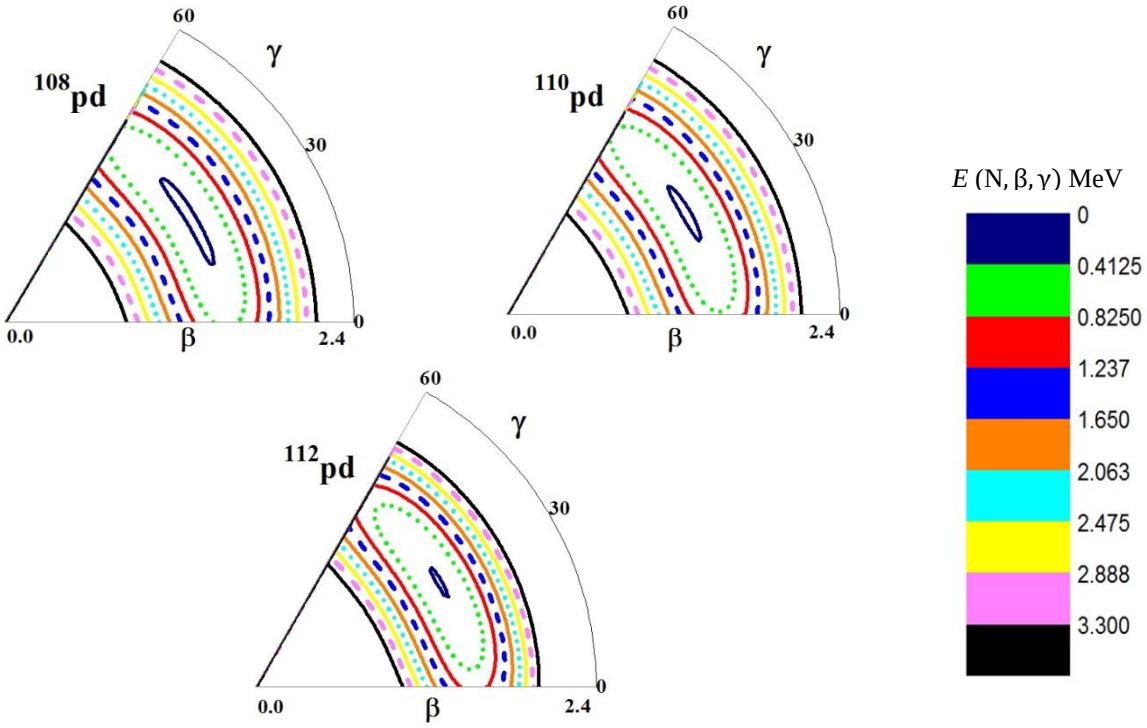

Fig. 2. The potential energy surface in γ-β plane for $^{108\text{-}112}$Pd nuclei.
(See color Figure on the journal website.)

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ВИЗНАЧЕННЯ ІДЕНТИЧНОСТІ ІЗОТОПІВ $^{108\text{-}112}$Pd З ВИКОРИСТАННЯМ МОДЕЛІ ВЗАЄМОДІЮЧИХ БОЗОНІВ

Енергетичні рівні, $B(E2)$ величини та поверхні потенційної енергії для ізотопів паладію з числом протонів Z = 46 та числом нейтронів (n) від 62 до 66 були розраховані за допомогою моделі взаємодіючих бозонів. Набір параметрів у цих розрахунках є найкращим наближенням, яке було використано до цього часу. Розраховано також співвідношення енергій збудження першого 4_1^+ та першого 2_1^+ збуджених станів, $R = E4_1^+ / E2_1^+$ та досліджено ступінь узгодження, досягнуту в O(6) симетрії для $^{108\text{-}112}$Pd ядер. Порівняння розрахованих енергетичних рівнів та ймовірностей переходу $B(E2)$ з експериментальними показало їхню хорошу згоду. Контур поверхонь потенційних енергій показує, що всі розглянуті ядра є деформованими і мають γ-нестабільний характер.

Ключові слова: модель взаємодіючих бозонів, ізотопи паладію, рівні енергії, основна смуга, $B(E2)$.

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ОПРЕДЕЛЕНИЕ ИДЕНТИЧНОСТИ ИЗОТОПОВ $^{108\text{-}112}$Pd С ИСПОЛЬЗОВАНИЕМ МОДЕЛИ ВЗАИМОДЕЙСТВУЮЩИХ БОЗОНОВ

Энергетические уровни, $B(E2)$ величины и поверхности потенциальной энергии для изотопов палладия с числом протонов Z = 46 и числом нейтронов (n) от 62 до 66 были вычислены с помощью модели взаимодействующих бозонов. Набор параметров в этих расчетах был наилучшим приближением, использованным до сих пор. Вычислено также отношение энергий возбуждения первого 4_1^+ и первого 2_1^+ возбужденных состояний, $R = E4_1^+ / E2_1^+$ и исследована степень согласования, достигнутая в O(6) симметрии для $^{108\text{-}112}$Pd ядер. Сравнение рассчитанных энергетических уровней и вероятностей перехода $B(E2)$ с экспериментальными показало их хорошее согласие. Контур поверхностей потенциальных энергий показывает, что все рассмотренные ядра являются деформированными и имеют γ-нестабильный характер.

Ключевые слова: модель взаимодействующих бозонов, изотопы палладия, уровни энергии, основная полоса, $B(E2)$.

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