ISSN 1818-331X ЯДЕРНА ФІЗИКА ТА ЕНЕРГЕТИКА 2013 Т. 14 № 4

ЯДЕРНА ФІЗИКА

УДК 539.143

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PREDICTIVE POWER OF NUCLEAR-MASS MODELS

Ten different theoretical models are tested for their predictive power in the description of nuclear masses. Two sets of experimental masses are used for the test: the older set of 2003 and the newer one of 2011. The predictive power is studied in two regions of nuclei: the global region ($Z, N \geq 8$) and the heavy-nuclei region ($Z \geq 82$, $N \geq 126$). No clear correlation is found between the predictive power of a model and the accuracy of its description of the masses.

Keywords: nuclear mass, nuclear models, accuracy of a model, predictive power of a model, heavy nuclei, global region of nuclei.

Introduction

Mass of a nucleus is a fundamental property of it. It is decisive for its other properties and also for the properties of various nuclear processes. A realistic description of the mass is an important question for nuclear models.

The objective of this paper is to test the quality of the description of measured masses by various theoretical models and also to test the predictive power of the models in this description. An interesting question is also the relation between these two properties of a model.

Ten models of various nature are considered: semi-empirical, macroscopic-microscopic, purely microscopic (self-consistent) and others. The quality of the description is tested with the use of experimental masses evaluated recently [1]. The predictive power of a model is studied by comparing its description of the older mass data [2] with that of the new data [1], to which the model was not adjusted. Between the older evaluation [2] and the new one [1], masses of more than 140 nuclei have been measured. Also the accuracy of the newly measured masses has been improved for many nuclei. The present study is an extension of our discussion on the description of the heavy-nuclei masses by macroscopic-microscopic models [3].

Considered models

Ten various models are considered in the study. These are: one semi-empirical (LMZ) [4], five macroscopic-microscopic, two purely microscopic (self-consistent) and two models of other kind. The macroscopic-microscopic models are: the Finite-Range Droplet Model (FRDM) [5], the Finite-Range Liquid Drop Model (FRLDM) [5], the nuclear Thomas - Fermi (TF) [6], the Warsaw model for Heavy Nuclei (HN) [7] (see also [8]), and the Lublin - Strasbourg (LSD) model [9]. The purely microscopic models are: the most recent (21^{st}) version of the Hartree - Fock - Bogoliubov approach (HFB21) [10], which uses the Skyrme interactions, and the HFB approach exploiting the Gogny forces (GHFB) [11]. Two other models are the following: the model of Duflo and Zuker (DZ) [12] and that of Koura et al. (KTUY) [13].

Eight of the models are of a global character describing all nuclei with $Z, N \geq 8$. Two of the models (LMZ and HN) are of a local type, specially adapted to describe heavy nuclei with proton number $Z \geq 82$ and neutron number $N \geq 126$.

Quality of the description of masses

In this section, we illustrate the quality of the description of nuclear masses by the considered models in two regions of nuclei: the whole (global) region ($Z, N \geq 8$) and in its part corresponding to heavy nuclei ($Z \geq 82$, $N \geq 126$). Three quantities characterizing the quality are calculated: rout-mean-square (rms) of the discrepancies between theoretical and experimental masses, the average value of the discrepancies, $\bar{\delta}$, and the maximum of the absolute values of the discrepancies, max $|\delta|$. The experimental masses are taken from Ref. [1]. The results are given in Table 1, where the year of publication of each model and the number of nuclei with both measured and calculated masses in each of the considered regions, N_{nucl}, are also indicated. The most important quantities, rms, are also illustrated in a graphical form in Figs. 1 and 2.

One can see in Fig. 1 that the rms values may be divided into three groups. The lowest value is obtained for the DZ model. Medium values, close to each other, appear for the LSD, FRDM, TF and HFB21 approaches. The largest values are obtained for the three remaining models.

Table 1. **Results for all (global) and heavy nuclei**

Model (Year)	LMZ (2000)	HN (2001)	LSD (2003)	FRDM (1995)	TF (1996)	FRLDM (1995)	HFB21 (2010)	GHFB (2009)	DZ (1995)	KTUY (2005)
GLOBAL										
N_{nucl}	-	-	2267	2294	2293	2294	2294	2294	2294	2294
Rms	-	-	0.600	0.645	0.629	0.768	0.573	0.784	0.373	0.690
$\bar{\delta}$	-	-	-0.029	-0.062	0.027	0.057	0.030	-0.108	-0.030	-0.048
Max $\lvert\delta\rvert$	-	-	4.34	3.64	4.61	4.17	3.20	3.23	3.01	2.63
HEAVY										
N_{nucl}	297	297	289	297	296	297	297	297	297	297
Rms	0.202	0.358	0.352	0.455	0.476	0.731	0.484	1.057	0.333	0.986
$\bar{\delta}$	0.028	-0.133	0.163	0.131	0.340	0.562	0.132	-0.118	-0.011	-0.307
Max $\lvert\delta\rvert$	1.12	1.13	1.43	1.95	1.75	1.92	1.33	3.23	3.01	2.38

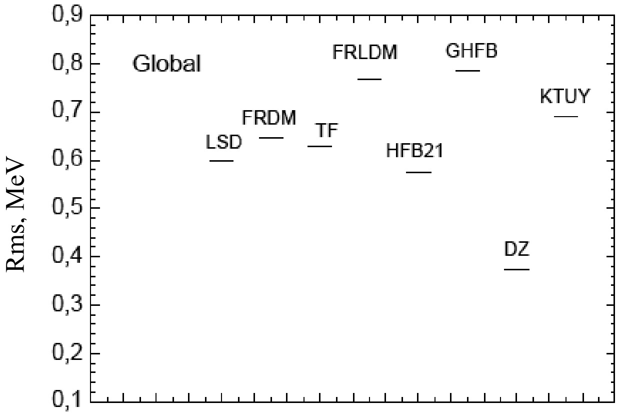

Fig. 1. Rms values of the discrepancies between the mass values calculated with 8 global models (see text for the notation of the models) and the experimental ones.

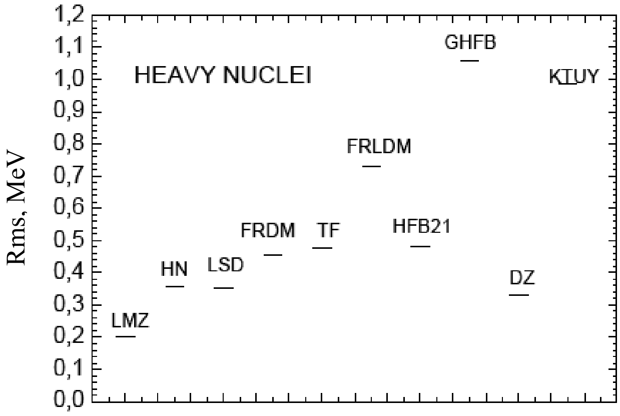

Fig. 2. Same as in Fig. 1, but for the heavy-nuclei region. Results for the two local models (LMZ and HN) are also shown.

The results obtained for the heavy nuclei (see Fig. 2) differ significantly from those of Fig. 1. Rms of the LSD, FRDM, TF and HFB21 models decrease significantly, while those of the GHFB and KTUY approaches significantly increase, with respect to the rms values of Fig. 1. The rms values of the LMZ and HN approaches are small, as could be expected for these local models, specially adapted for heavy nuclei.

The results presented in this Section show that the accuracy of the description of nuclear masses by a given model significantly depends on the region of nuclei to which the model is applied.

Predictive power of the models

Let us test the predictive power of the considered models in description of masses in both studied regions of nuclei.

Table 2 shows the results for the global region. The first row gives the number of nuclei, the masses of which are described by each model in the case of data evaluated in Ref. [2]. The second row specifies the same quantity in the case of using Ref. [1]. In the third row, the difference, δN_{nucl}, between the number of nuclei with measured masses in the later evaluation of Ref. [1] and the earlier one of Ref. [2], is shown. The respective difference in the rms, δRms, given in the last row, is also illustrated in a graphical form in Fig. 3.

Table 2. **Predictive power of the models in description of global masses**

Model	LSD	FRDM	TF	FRLDM	HFB-21	GHFB	DZ	KTUY
N_{nucl} (03)	2141	2149	2149	2149	2149	2149	2149	2149
N_{nucl} (11)	2267	2294	2293	2294	2294	2294	2294	2294
δN_{nucl}	126	145	144	145	145	145	145	145
Rms (03)	0.621	0.655	0.637	0.769	0.577	0.798	0.360	0.653
Rms (11)	0.600	0.645	0.629	0.768	0.574	0.784	0.374	0.690
δRms	-0.021	-0.010	-0.008	-0.001	-0.003	-0.014	0.014	0.037

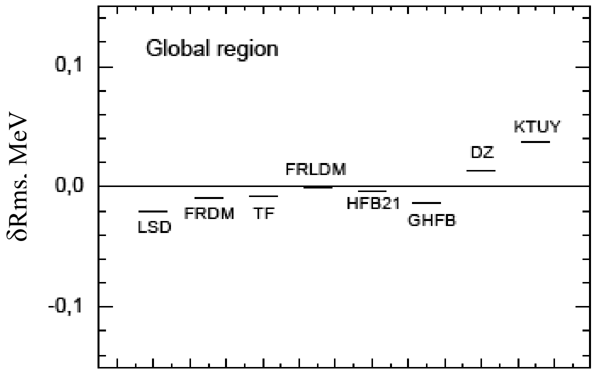

Fig. 3. Difference, δRms, between the rms values obtained with the larger set of experimental masses [1] and the smaller one [2], for the global region of nuclei.

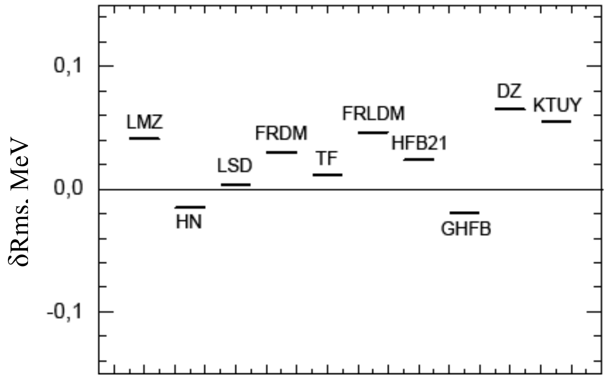

Fig. 4. Same as in Fig. 3, but for the region of heavy nuclei.

Table 3. **Predictive power of the models in description of the heavy-nuclei masses**

Model	LMZ	HN	LSD	FRDM	TF	FRLDM	HFB-21	GHFB	DZ	KTUY
N_{nucl} (03)	264	264	262	264	264	264	264	264	264	264
N_{nucl} (11)	297	297	289	297	296	297	297	297	297	297
δN_{nucl}	33	33	27	33	32	33	33	33	33	33
Rms (03)	0.161	0.373	0.348	0.425	0.464	0.685	0.460	1.076	0.268	0.931
Rms (11)	0.202	0.358	0.352	0.455	0.476	0.731	0.484	1.057	0.333	0.986
δRms	0.041	-0.015	0.004	0.030	0.012	0.046	0.024	-0.019	0.065	0.055

Respective results for the region of the heavy nuclei are presented in Table 3 and Fig. 4.

One can see in Fig. 3 that δRms is negative for five models (this means that the models better describe the larger set of nuclear masses, which includes masses unknown in the time when the model was elaborated), one model (FRLDM) describes equally well the larger and the smaller sets of masses, and two models (DZ and KTUY) have higher Rms for the larger set than for the smaller one (smaller predictive power).

For the heavy-nuclei region (see Table 3 and Fig. 4), the results are much different: most of the models show a poorer predictive power in the heavy-nuclei region than in the global one.

Comparing Fig. 1 with Fig. 3 and Fig. 2 with Fig. 4, one can hardly see a clear correlation between the quality of the description of masses of a model and its predictive power.

Detailed description of the discrepancy

Fig. 5 shows a detailed map of the discrepancy δ (Z, N) in the heavy-nuclei region for the DZ model. This is the model which gives relatively small rms in both the global and the heavy-nuclei regions.

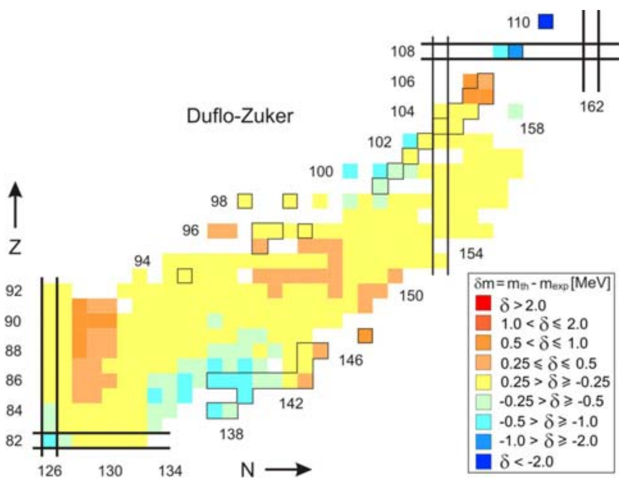

Fig. 5. Detailed map of the discrepancies obtained for the DZ model in the heavy-nuclei region. New masses of Ref. [1], which are absent in Ref. [2], are indicated by thin black contours. (See color Figure online.)

Conclusions

Two main conclusions may be drawn from our study:

1. The quality of the description of nuclear masses by a given model as well as its predictive power depends significantly on the region of nuclei for which they are calculated.
2. No clear correlation between these two quantities is observed.

The authors would like to thank Dieter Ackermann, Michael Block, Fritz Bosch, Hans Geissel, Stephane Goriely, Fritz Hessberger, Sigurd Hofmann, Christophor Kozhuharov, and Christoph Scheidenberger for helpful discussions and correspondence. Support by the Helmholtzinstitut Mainz (HIM), the Polish National Centre of Science (within the research project No. NN-202 204938), the European Science Foundation (within the EuroGenesis programme), the Helmholtz-CAS Joint Research Group (Group No. HCJRG-108), and the Polish-JINR (Dubna) Cooperation Programme is gratefully acknowledged.

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ПЕРЕДБАЧУВАЛЬНА СИЛА МОДЕЛЕЙ МАС ЯДЕР

Протестовано передбачувальну силу 10 різних теоретичних моделей мас ядер. Для тесту використано два набори експериментальних мас ядер 2003 і 2011 рр. Передбачувальну силу вивчено як для глобального набору ядер (Z, N ≥ 8), так і для набору важких ядер (Z ≥ 82, N ≥ 126). Показано відсутність чіткої кореляції між передбачувальною силою моделі і точністю опису мас у моделі.

Ключові слова: маса ядра, ядерні моделі, точність моделі, передбачувальна сила моделі, важкі ядра, глобальна область ядер.

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ПРЕДСКАЗАТЕЛЬНАЯ СИЛА МОДЕЛЕЙ МАСС ЯДЕР

Протестирована предсказательная сила 10 различных теоретических моделей масс ядер. Для теста использовано два набора экспериментальных масс ядер 2003 и 2011 гг. Предсказательная сила изучена как для глобального набора ядер (Z, N ≥ 8), так и для набора тяжелых ядер (Z ≥ 82, N ≥ 126). Показано отсутствие четкой корреляции между предсказательной силой модели и точностью описания масс в модели.

Ключевые слова: масса ядра, ядерные модели, точность модели, предсказательная сила модели, тяжелые ядра, глобальная область ядер.

Надійшла 25.11.2013
Received 25.11.2013