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FOKKER - PLANCK EQUATION SOLVER FOR STUDY STOCHASTIC COOLING IN STORAGES RINGS

In this paper so-called the PDE-method for solution of the Fokker - Planck Equation is proposed to study the beam dynamic in the storage ring, where the stochastic cooling is used. This method has been implemented in the new FOPLEQ code. The results of numerical calculations obtained by this code are presented. Calculated results by PDE-method are compared with other numerical algorithms. Application, stability, convergence and precision of the proposed method are discussed.

Keywords: Fokker - Planck equation, stochastic cooling, evolution of the particle distribution.

1. Introduction

Fokker - Planck Equation (FPE) is used for a large variety of physical phenomena. Formally, the FPE equilibrium solution can be easily determined, while its time evolution can be analytically obtained only in few particular cases. Numerical solution of the FPE in general is interesting for a number of stochastic physics problems. In this paper, a several algorithms are described for numerical solution of the FPE. This problem, especially if variable coefficients are included in the model, is computationally very expensive. The solution of FPE often takes a long time even with today's high-speed computers.

One important problem of accelerator physics is to investigate the particle motion under the influence of noise [1]. There are various sources of noise: rf noise, random power supply ripple, random ground motion, rest gas scattering, and quantum fluctuations due to radiation. The physical questions one wants to answer are: what is the long time behaviour of the dynamics, what is the probability for the particle to hit the vacuum chamber (and then be lost) (mean first passage time), what are the average fluctuations of the particle around the periodic design orbit of the accelerator (moments), and what is the time evolution of the probability density (transient and stationary behaviour). Mathematically stochastic systems can be modelled by stochastic maps (in the time discrete case) and by stochastic differential equations (s.d.e.) in the time continuous case. In the following we will restrict our considerations to s.d.e. with Gaussian white noise. Gaussian white noise is a very good approximation in many accelerator problems [1]. The solution of these s.d.e. are Markovian diffusion processes which can be described by the FPE [2]. The FPE is a Partial Differential Equation (PDE) for the probability density and the transition probability of these stochastic processes. In general, the stochastic equations of motion of a particle in an accelerator are very complicated and cannot be solved analytically, therefore one has to use numerical schemes. One way is to consider the s.d.e. directly. An alternative way is to investigate and solve the FPE.

The algorithms described in this article involve numerical solution of the parabolic PDE. Solving PDEs numerically is a well-established topic both in mathematics and in applied areas. There exists a vast body of literature and there are also many 'black box' PDE solvers available (e.g. the PDE toolbox of Matlab or Mathematic packages). Nevertheless, in order to keep the text as self-contained as possible, we give here a rough sketch of the numerical method used new implementation of "fast-FP" (FP – Fokker - Planck). For a more detailed description we refer to the following text books [2 - 7]. Theoretical background is described in [3, 4] for ordinary and parabolic differential equations. How to solve these equations numerically is described.

Stochastic momentum cooling is operated to obtain a high-density beam within a small momentum spread for experiments. A FPE is used as a powerful tool for investigating the stochastic momentum cooling process.

2. Simple method

The FPE is a second order partial differential equation, which can be put in the form

$$\frac{\partial \psi(t,z)}{\partial t} = -\left(\frac{\partial}{\partial z}F(t,z) - \frac{1}{2}\frac{\partial^2}{\partial z^2}D(t,z)\right)\psi(t,z), \quad (2.1)$$

where $F(t, z)$ and $D(t, z)$ are known functions which may depend, in principle, on time, and $\psi(z, t)$ represents the unknown solution. It can be easily shown that this solution corresponds to the z-coordinate probability distribution of a mass less particle whose dynamics is described by the Langevin equation.

The key to the numerical solution of FPE such as Eq. (2.1) by using finite difference methods is to

discrete 'space' z and time t. We only consider the discrete set of z-values {0, Δz, 2Δz, ..., NΔz} where $\Delta z = a/N$ for some positive integer N and the t - values {0, Δt, 2Δt, nΔt}. Then the partial derivatives present in Eq. (2.1) can be numerically approximated.

Using numerical approximations the function $\psi(t, z)$ at time $(n + 1)$ can be calculated by

$$\psi_i^{n+1} = \psi_i^n - \Delta t\left(\frac{F_{i+1}\psi^n_{i+1} - F_{i-1}\psi^n_{i-1}}{2\Delta z} - \frac{1}{2}\frac{D_{i+1}\psi^n{}_{i+1} - 2D_i\psi^n{}_i + D_{i-1}\psi^n{}_{i-1}}{\Delta z^2}\right). \tag{2.2}$$

3. Finite difference method

The simple expression (2.2) does not give a good approximation of the real solution ψ. To have more accurate approximation of the real solution of FPE V. Palleschi [7] proposes to express the solutions of Eq. (2.2) as a weighted average (weight θ) of its values calculated at times n and n+1 respectively, so that Eq. (2.2) reads

$$\psi_i^{n+1} = \psi_i^n + (1-\theta)\Delta t\left(-\frac{F_{i+1}\psi^n_{i+1} - F_{i-1}\psi^n_{i-1}}{2\Delta z} + \frac{1}{2}\frac{D_{i+1}\psi^n_{i+1} - 2D_i\psi^n_i + D_{i-1}\psi^n_{i-1}}{\Delta z^2}\right) +$$

$$+\theta\Delta t\left(-\frac{F_{i+1}\psi^{n+1}_{i+1} - F_{i-1}\psi^{n+1}_{i-1}}{2\Delta z} + \frac{1}{2}\frac{D_{i+1}\psi^{n+1}_{i+1} - 2D_i\psi^{n+1}_i + D_{i-1}\psi^{n+1}_{i-1}}{\Delta z^2}\right). \tag{3.1}$$

Eq. (3.1) is not directly solvable for ψ if not in the trivial case $\theta = 0$, which coincides with Eq. (2.2).

Rearranging terms, Eq. (3.1) can be transformed into a matrix equation

$$A_{ik}\psi_k^{n+1} = \psi_i^*, \tag{3.2}$$

where A_{ik} is an N x N tri-diagonal matrix and ψ^* is presented through the ψ and parameter θ [7]. Note that in the case $\theta = 1$ we have $\psi^* = \psi^n$.

Eq. (3.2) can be formally solved for ψ^{n+1}

$$\psi_k^{n+1} = A_{ik}^{-1}\psi_i^*, \tag{3.3}$$

where A_{ik}^{-1} is the inverse of the A_{ik} matrix.

To have a good spatial resolution for the solution ψ a fine spatial mesh more then 100 is needed. So the possibility of using fast computer routines is particularly important, especially when a large number of iterations of Eq. (3.3) are needed to follow the time evolution of ψ_k.

However, due to the particular tri-diagonal form of the A_{ik}^{-1} matrix (Eq. (3.3), the use of highly CPU time consuming matrix inversion routines is not necessary. Faster routines are indeed available for solving tri-diagonal systems. In the next Section the extended numerical algorithm is described.

4. PDE algorithm

The PDE algorithm computes values ψ_{ij} which approximate the true solution ψ by ψ_{ij} $\psi(i\Delta t, j\Delta z)$ for $i = 0, 1, 2, ...$ and $j = 0, 1, ..., N$. The accuracy of this approximation depends on the step sizes Δt and Δz. The algorithm works by considering a grid row with fixed t at a time, starting with an approximation of the initial condition: (ψ_{00}, ..., ψ_{0N}). Then, in each step, the algorithm uses the approximation (ψ_{i0}, ..., ψ_{iN}) for time $i\Delta t$ to compute an approximation for time $(i + 1)\Delta t$. A simple calculation shows that ψ solves with homogeneous boundary conditions $\psi(t, 0) = \psi(t, a) = 0$. We denote the computed solution for time $n\Delta t$ by $\psi_n = (\psi_{n1}, ..., \psi_{iN\text{-}1})$. We do not include the outermost points ψ_{n0} and ψ_{nN} since these are always zero due to the boundary conditions.

Using approximations given by formulae (2.1) one can then write the application of the differential operator

$$L = -\frac{\partial}{\partial z}F(z,t) + \frac{1}{2}\frac{\partial^2}{\partial z^2}D(z,t) \tag{4.1}$$

as a matrix vector multiplication: collecting all the terms we get

$$L\psi(n\Delta t, n\Delta z) = L^N\psi_n^n \quad \text{n } [1, ..., N], \tag{4.2}$$

where L^N is the tri-diagonal matrix given by

$$L^N = \begin{pmatrix} d_1 & p_1 & 0 & 0 \\ r_2 & d_2 & p_2 & 0 \\ 0 & r_i & d_i & p_i \\ 0 & 0 & r_N & d_N \end{pmatrix}, \tag{4.3}$$

where

$$r_i = -\frac{F_{i-1}}{2\Delta z} + \frac{D_{i-1}}{2\Delta z^2}, .. i \in [2, N],$$
$$d_i = -\frac{D_i}{\Delta z^2}, .. i \in [1, N], \tag{4.4}$$
$$p_i = \frac{F_{i+1}}{2\Delta z} + \frac{D_{i+1}}{2\Delta z^2}, .. i \in [1, N-1].$$

The approximations introduced above suggest the following approximation to the PDE:

$$\frac{\psi^{n+1}-\psi^{n}}{\Delta t}=L^{N}\left(\theta\psi^{n+1}+(1-\theta)\psi^{n}\right), \qquad (4.5)$$

where $\theta \in [0, 1]$ is a parameter of the method. For $\theta = 0$ the derivative on the right hand side is evaluated only for the current approximation ψ^n. For all values $\theta > 0$ the derivative is evaluated for a mixture of ψ^n and ψ^{n+1}. In these cases one has to solve a system of linear equations to compute ψ^{n+1} from ψ^n: by rearranging the terms in (4.5) one gets

$$\left(I-\Delta t\theta L^{N}\right)\psi_i^{n+1}=\left(I-\Delta t(1-\theta)L^{N}\right)\psi_i^{n}, \qquad (4.6)$$

where I is the $N \times N$ identity matrix. The choice of the parameter θ affects the stability of the method. Common choices are $\theta = 0$ (Euler scheme), $\theta = 1/2$ (Crank Nicolson scheme) and $\theta = 1$ (implicit Euler scheme). In our own implementation we use the Crank Nicolson method. For $\theta < 1/2$ the PDE method is stable, independently of the choice of Δt. For $\theta < 1/2$ one gets a bound on Δt, depending on the smallest of the eigenvalue λ_i^N. A more detailed analysis shows that this eigenvalue is approximately equal to $-4D_i/\Delta z^2$ (with exact equality for $F_i = 0$) and thus for $\theta < 1/2$ the method is stable only if

$$\Delta t<\frac{\Delta z^{2}}{2D_{\max}(1-2\theta)}. \qquad (4.7)$$

And for $\theta > ½$ the method is stable if

$$\Delta t<\frac{\Delta z^{2}}{2D_{\max}}. \qquad (4.7a)$$

By using the equations discussed previously the numerical calculation procedure is as follows. One has to construct the set of the linear algebraic equations following the Eq. (4.6). Taking into account the matrix of operators (4.3) the left side of the Eq. (4.6) is written as multiplication of the tri-diagonal matrix (4.3) on the vector ψ_i^{n+1}. The right side of Eq. (4.6) can be written as a vector f_i, which is obtained by multiplying matrix $(I - \Delta t(1 - \theta)L^N)$ on the known ψ_i^n. In general a tri-diagonal system for N unknowns may be written as

$$\begin{pmatrix} b_1 & c_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_i & b_i & c_i \\ 0 & 0 & a_N & b_N \end{pmatrix}\begin{pmatrix}\psi_1^{n+1}\\ \psi_2^{n+1}\\ \psi_i^{n+1}\\ \psi_N^{n+1}\end{pmatrix}=\begin{pmatrix} f_1\\ f_2\\ f_i\\ f_n\end{pmatrix}, \qquad (4.8)$$

where

$$a_i=-\frac{T}{2}\left(\frac{D_{i-1}}{\Delta z^{2}}+\frac{F_{i-1}}{\Delta z}\right);. \qquad b_i=1+T\frac{D_i}{\Delta z^{2}};$$

$$c_i=-\frac{T}{2}\left(\frac{D_{i+1}}{\Delta z^{2}}-\frac{F_{i+1}}{\Delta z}\right); \qquad (4.9)$$

and

$$f_i=\psi_i^{n}+\frac{W}{2}\left(-\frac{F_{i+1}\psi_{i+1}^{n}-F_{i-1}\psi_{i-1}^{n}}{\Delta z}+\right.$$

$$\left.+\frac{D_{i+1}\psi_{i+1}^{n}-2D_i\psi_i^{n}+D_{i-1}\psi_{i-1}^{n}}{\Delta z^{2}}\right). \qquad (4.10)$$

$W = \Delta t(1 - \theta)$, $T = \Delta t\theta$. To solve a tri-diagonal system of Eq. (4.8) one can use so called "the tri-diagonal matrix algorithm" also known as the "Thomas algorithm" [10], which is a simplified form of Gauss elimination. By Thomas algorithm the solution can be obtained in N operations instead of N^3 required by Gaussian elimination.

5. The constrained interpolation profile (CIP) method

The CIP method is well developed and known as a general numerical solver for solid, liquid, gas, and plasmas. In Refs. [11 - 12] one can find the detailed mathematical treatments of CIP method. This method is a kind of semi-Lagrangian scheme and has been extended to treat incompressible flow in the framework of compressible fluid. Since it uses primitive Euler representation, it is suitable for multiphase analysis by the FPE.

A numerical solver for the FPE in a stochastic momentum cooling process by using a CIP method is described in details in Refs. [13 - 14]. The main idea of the CIP method is that the Eq. (2.1) is separated to a coherent phase and an incoherent phase for a numerical calculation procedure. The incoherent term is written as the diffusion equation, while the coherent term is described by the advection equation. For solving the advection term of the equation the CIP method is used.

The cooling term is described as the advection term, and the diffusion term is occurred by the beam signal and the amplifier noises. The FPE can be numerically solved as the advection-diffusion equation. The CIP method is a useful scheme in the numerical calculation for the advection equation. Using the CIP method one can numerically solve nonlinear equations including the advection term with less discredited grid numbers.

6. Coefficients F and D with the option of considering feedback through the beam

Prior to the calculation of the precision and stability of the FPE numerical solutions given in previous Sections the coefficients F and D FPE with the option considering feedback through the beam must be defined. These coefficients characterise an electronic system of stochastic cooling. Their optimization is a subject of many works.

The coherent coefficient F

The coherent coefficient F depending on the momentum $\delta = dp/p$ is written by [16]

$$F(\delta) = 2ef_0^2\sqrt{n_p n_k Z_K Z_P}\sum_{n=n1}^{n2}\mathrm{Re}\left\{\frac{G(\delta,n)}{1-S(\delta,n)}\right\}, \qquad (6.1)$$

e – electron charge; Z_k, Z_p – electrical impedances coupled to the beam in the circuit convention, n_p, n_k – number of the pick-up and kicker units; f_0 – revolution frequency; $G(\delta, n)$ – the total gain from the beam current at the pick-up to the kicker voltage. It is complex voltage gain including the filter and phase shifts in the pick-ups and kickers. The gain is reduced by factor (1 - S) due to the beam-feedback effect. The function $S(\delta, n)$ is so-called open-loop gain. Summing is done over all harmonics from n1 to n2.

The total gain of the system $G(\delta, n)$ is defined by

$$G(\delta,n) = G_{gain}(\delta,n)M_{mix}(\delta,n)P(n)^2, \qquad (6.2)$$

where G_{gain} is complex voltage gain:

$$G_{gain}(\delta,n) = \frac{i}{2}G_{const}\left(1-\exp\left(-i2\pi n\eta\delta\right)\right), \qquad (6.3)$$

G_{const} – electrical gain; η - slip factor calculated by formula: $\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$. $M_{mix}(\delta, n)$ is mixing between pick-up and kicker as a function of the transit time ΔT_{pk} between them.

$$M_{mix}(\delta,n) = \exp\left(-i2\pi n f_0\eta_{pk}\delta\Delta T_{pk}\right). \qquad (6.4)$$

$P(n)$ is sensitivity factor of pick-up or kicker electrodes inputted in the program.

$S(\delta, n)$ is the open-loop gain:

$$S(\delta,n) = ef_0^2\sqrt{n_p n_k Z_p Z_k}G_{gain}(\delta,n)M_{mix}(\delta,n)B_{btf}(\delta,n)P\left(n\right)^2. \qquad (6.5)$$

Here $B_{btf}(\delta, n)$ is beam transfer function, which is calculated summing over all momentum spread and harmonics by formula

$$B_{btf}(\delta,n) = -\frac{\pi}{n|K|}\frac{\partial\psi}{\partial E} + \\ +\Delta E\Delta n\sum_{\delta_{kk}=\delta_{\min}}^{\delta_{\max}}\frac{d\psi}{dE}\frac{i}{\left(\Delta\omega(\delta,n)-\Delta\omega(\delta_{kk},n)\right)}, \delta_{kk}\neq\delta, \qquad (6.6)$$

$\Delta\omega$ – is a frequency shifts due to momentum deviation and harmonic: $\Delta\omega\left(\delta,n\right) = 2\pi n f_0\eta\delta$. K is constant parameter: $|K| = 2\pi\eta f_0\frac{\gamma}{\gamma+1}\frac{1}{E_0}$.

The incoherent coefficient D

The incoherent coefficient D given in ref. [16, 17] consists of two noise power components. One is the amplifier noise, and the other the beam noise. The D is written by

$$D\left(\delta\right) = 2\cdot\left[kTf_0^2 n_k Z_K\sum_{n=n1}^{n2}\frac{|G(\delta,n)|^2}{|1-S(\delta,n)|^2\cdot P(n)^2} + \frac{2\pi e^2 f_0^4 n_p n_k Z_P Z_K\psi(\delta)}{|\mathrm{K}|}\sum_{n=n1}^{n2}\frac{|G(\delta,n)|^2}{n|1-S(\delta,n)|^2}\right], \qquad (6.7)$$

k is Boltzmann's constant; T is the temperature of the electronic devise (K); Z_P and Z_K are the impedances of a pick-up and a kicker; e – electron charge. One can see that the noise power components proportional to the gain G^2 of system and open-loop gain S. If one does not want to take the feedback by the beam into account, one should just set $S(\delta, n)$= 0 into the code.

For simulations the latest parameters of the stochastic cooling system, which is planned to be used in the CR [15], are used. The parameters necessary here for the coefficients are summarized in the Table. Fig. 1 shows calculated for these parameters incoherent coefficient.

It is necessary to notice that incoherent coefficient D oscillates because of oscillations ψ function at equilibrium state. But in case of PDE method such oscillates can be decreased.

Main parameters of the Cooling System and the CR machine for antiprotons

Number of particles, N	10^8
Kinetic energy, E_0, MeV/u	3000
Initial momentum spread, $\Delta p/p$ (1σ), %	0.35
Transition energy, γ_{tr}	3.69
Frequency slip factor of the ring, η	-0.017
Frequency slip factor: pick-up- kicker, η_{pk}	-0.041
Ring circumference, C, m	221.45
Path length from pick-up to kicker, P, m	81
Bandpass, (f_{min}-f_{max}), GHz	1-2
Minimum harmonic number, n_1	800
Maximum harmonic number, n_2	1600
Effective temperature for amplifier noise, K	73
Pick-up impedance, Z_p, Ω	720
Kicker-impedance, Z_c, Ω	2880
Number of pick-ups, n_p	2
Number of kickers, n_k	2

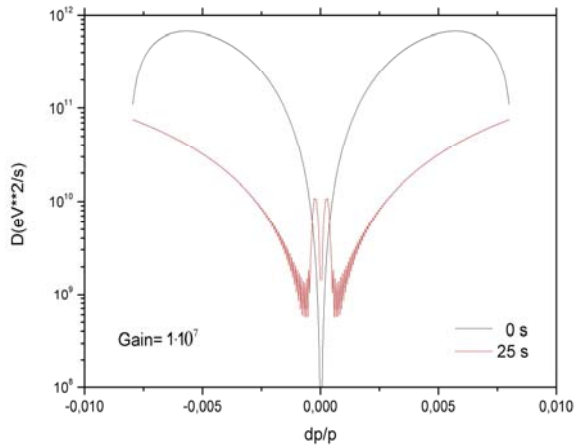

Fig. 1. Incoherent coefficient D calculated by formula (6.7) using antiproton beam parameters given in the Table.

7. Numerical simulation (CR, antiproton beam cooling)

This Section presents the results of the stochastic cooling process numerically calculated by FOPLEQ code, where the all methods presented in the previous Sections are implemented. The parameters of the stochastic cooling system given in the Table are used. This system is planned to be applied in the Collector Ring (CR). The coherent and incoherent coefficients F and D are calculated by formulae described in Section 6. The feed back effect is included in simulations.

As an example Fig. 2 shows the particle distributions as a function of energy during the stochastic cooling in the CR in every 2 s of cooling time. The anti-protons are collected into the central energy of the beam due to the stochastic cooling.

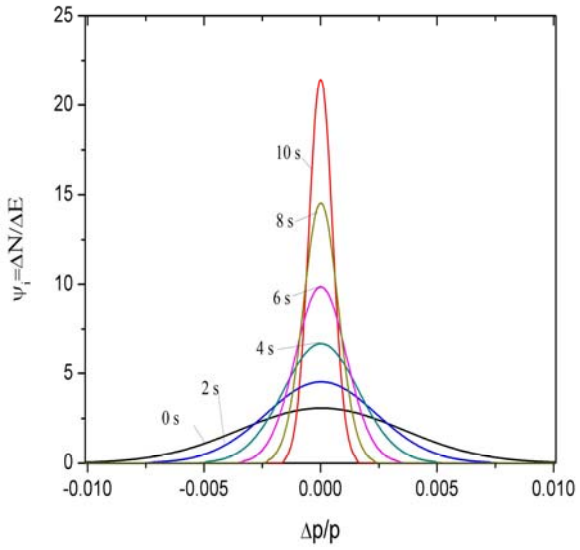

Fig. 2. Psi function evolution in every 2 s. Number of channels is 500. Number of particles is 10^8. Calculated by FOPLEQ code (CIP method) for antiproton beam. The time step is $dt = 1E$ - 4 s

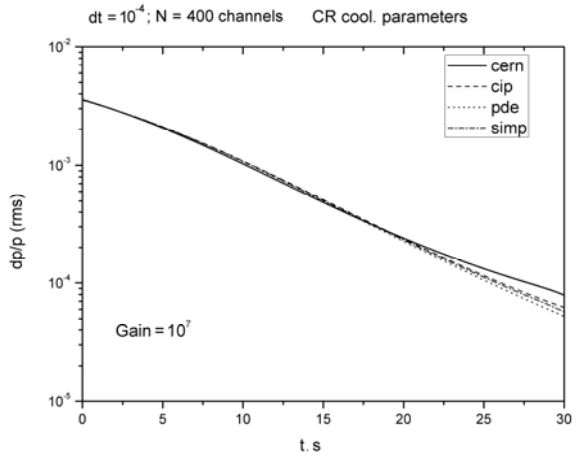

Fig. 3. Rms energy spread history during cooling in the CR. The label "cern" corresponds to the CERN code [17]. The labels "cip", "pde", "simp" indicate the methods implemented in the FOPLEQ code.

Fig. 3 demonstrates the rms momentum spread during the cooling process. It is confirmed that the numerical solvers (simple, CIP, PDE) used in FOPLEQ code quite well represent the result obtained by CERN code. Here the rms energy spread is calculated by difference methods, where N is the total particle number in the ring. Fig. 3a presents the comparison of RMS momentum spread results received by difference methods with difference number of channels. From these figure one can see that in case of PDE method it is needed less number of channels for the same accuracy.

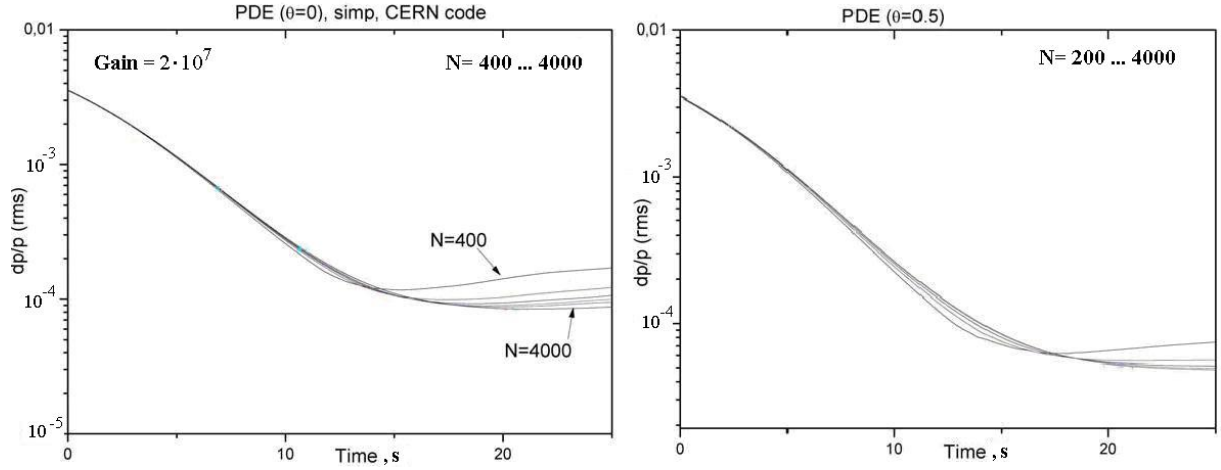

Fig. 3a. Rms energy spread calculates by simple method and CERN code – *left*; PDE method (FOPLEQ code) – *right* for different number of channels.

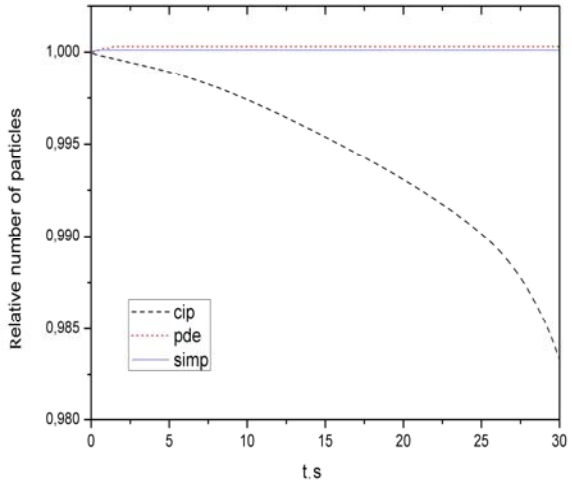

Fig. 4. Relative number of particles calculated by FOPLEQ code.

Fig. 4 shows the particle number during cooling process calculated by CIP, PDE and simple methods. The result of the CIP method gives an unphysical phenomenon after the long time beam cooling, because the distribution function has the quite sharp distribution at the centre due to the stochastic cooling.

8. Conclusions

A fast and accurate algorithm for numerical solution of FPE based on the solution of the parabolic PDE, where the Crank - Nicholson scheme is used, has been discussed in this paper. The special FOPLEQ code has been developed to study the beam dynamic in storage ring, where the stochastic cooling process must be used. In this code the PDE algorithm is applied. The stability, convergence and round-off errors of the algorithm are studied. The numerical results on FPE solution with PDE method are compared with other numerical methods. It was proved that the PDE is unconditionally stable and converged. This method and technique can be applied to solve any fractional (in space and time) PDE.

As an example in the paper the results of simulation with the FOPLEQ code are given for the stochastic cooling process at the CR storage ring.

In the near future this work will be extended to higher dimensions, and it will be applied to more realistic parameters for studying the dynamics of particles in storage rings.

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РОЗВ'ЯЗАННЯ РІВНЯННЯ ФОККЕРА - ПЛАНКА ДЛЯ ДОСЛІДЖЕННЯ СТОХАСТИЧНОГО ОХОЛОДЖЕННЯ В НАКОПИЧУВАЛЬНИХ КІЛЬЦЯХ

Пропонується так званий чисельний PDE-метод розв'язання рівняння Фоккера - Планка для дослідження динаміки пучків у накопичувальних кільцях, де застосовується стохастичне охолодження. Цей метод був використаний при розробці нової комп'ютерної програми FOPLEQ. Представлено результати чисельних розрахунків, отримані за допомогою цієї програми. Проведено порівняння цих результатів з результатами інших чисельних алгоритмів. Обговорюються питання застосування, стабільності, збіжності та точності запропонованого методу.

Ключові слова: рівняння Фоккера - Планка, стохастичне охолодження, еволюція розподілу частинок.

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РЕШЕНИЕ УРАВНЕНИЯ ФОККЕРА - ПЛАНКА ДЛЯ ИЗУЧЕНИЯ СТОХАСТИЧЕСКОГО ОХЛАЖДЕНИЯ В НАКОПИТЕЛЬНЫХ КОЛЬЦАХ

Предлагается так называемый численный PDE-метод решения уравнения Фоккера - Планка для изучения динамики пучков в накопительном кольце, где используется стохастическое охлаждение. Этот метод был использован при разработке новой компьютерной программы FOPLEQ. Представлены результаты численных расчетов, полученные с помощью этой программы. Проводится сравнение этих результатов с результатами других численных алгоритмов. Обсуждаются вопросы применения, стабильности, сходимости и точности предложенного метода.

Ключевые слова: уравнения Фоккера - Планка, стохастическое охлаждение, эволюция распределения частиц.

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