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SHELL OSCILLATIONS IN SYMMETRY ENERGY

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The procedure of derivation of the symmetry energy from the shift of neutron-proton chemical potentials $\Delta\lambda = \lambda_n - \lambda_p$ is suggested. We observe the nonmonotonic (sawtooth) shape of the β-stability line given by the asymmetry parameter as a function of mass number. The behavior of the symmetry energy coefficient $b_{\rm sym}(A)$ at fixed neutron excess $D = N - Z$ is analyzed. We show the relation of local maxima of the β-stability line to mass numbers of the double-closed shells.

Keywords: symmetry energy, isotopic shift, β-stability line, shell oscillations.

Many static and dynamic features of nuclei are sensitive to isospin degree of freedom. Thenuclear β-stability line is derived by the balance of both the symmetry, $E_{\rm sym}$, and the Coulomb, E_C, energies. However the extraction of $E_{\rm sym}$ and E_C from the nuclear binding energy is not a simple problem because of its complicated dependency on the mass number A in finite nuclei [1]. The standard procedure of extraction of the symmetry energy from a fit of mass formula to the experimental binding energies [2] is not free from ambiguities and does not allow one to separate the symmetry energy into the volume, surface and curvature contributions directly.

In the present work, we use a non-standard procedure of extraction of the symmetry and Coulomb energies from the experimental data using the dependence of the isospin shift of neutron-proton chemical potentials $\Delta\lambda(X) = \lambda_n - \lambda_p$ on the asymmetry parameter $X = (N-Z)/(N+Z)$ for nuclei beyond the beta-stability line. This procedure allows one to represent the results for the A-dependence of energies $E_{\rm sym}$ and E_C in a transparent way, which can be easily used for the extraction of the smooth volume and surface contributions as well as the shell structure.

Considering the asymmetric nuclei with a small asymmetry parameter $X = (N-Z)/A \ll 1$ and assuming the leptodermous property, the total energy per nucleon E/A can be represented in the following form of A, X-expansion

$$\frac{E}{A} \equiv e_A = e_0(A) + b_{\rm sym}(A)\, X^2 + \frac{E_C(X)}{A}, \qquad (1)$$

where $e_0(A)$ includes both the bulk and the surface energies, $b_{\rm sym}(A)$ is the symmetry energy coefficient, $E_C(X)$ is the total Coulomb energy

$$E_C = \frac{3}{20}\frac{A\, e^2}{R_C}(1-X)^2 \qquad (2)$$

and R_C is the Coulomb radius of the nucleus. The beta-stability line $X = X^*(A)$ can be directly derived from Eq. (1) using the condition

$$\left(\frac{\partial \frac{E}{A}}{\partial X}\right)_A = 0 \Rightarrow X^*(A) = \frac{e_C^*(A)}{b_{\rm sym}^*(A) + e_C^*(A)}, \qquad (3)$$

where

$$e_C(A) = \frac{3}{20}\frac{Ae^2}{R_C}.$$

Along the beta-stability line, the binding energy per particle is then given by

$$\frac{E^*}{A} = e_0^*(A) + b_{\rm sym}^*(A)\, X^{*2} + \frac{E_C(X^*)}{A}, \qquad (4)$$

the upper index "*" indicates that the corresponding quantity is determined by the variational conditions (3) taken for fixed A and $X = X^*$ on the beta-stability line. For any given value of A, the binding energy can be extended beyond the beta-stability line as

$$\frac{E}{A} = \frac{E^*}{A} + b_{\rm sym}^*(A)\,(X - X^*)^2 + \frac{\Delta E_C(X)}{A}, \qquad (5)$$

where $\Delta E_C(X) = E_C(X) - E_C(X^*)$. The symmetry energy coefficient $b_{\rm sym}^*(A)$ contains the A-independent bulk term, $b_{\rm sym,vol}^*$, and the A-dependent surface contribution $b_{\rm sym,surf}^* A^{-1/3}$,

$$b_{\rm sym}^*(A) = b_{\rm sym,vol}^* + b_{\rm sym,surf}^* A^{-\frac{1}{3}}. \qquad (6)$$

In general, the surface symmetry energy $b_{\rm sym,surf}^* A^{-1/3}$ includes also the high order curvature correction $\sim A^{-2/3}$ [3].

Using Eq. (5), one can establish an important relation for the chemical potential λ_q ($q = n$) for a neutron and $q = p$ for a proton) beyond the beta-stability line. Namely, for the fixed A we obtain the following result from Eqs. (1) and (4)

$$\Delta\lambda(X) = \lambda_n - \lambda_p = \left(\frac{\partial E}{\partial N}\right)_Z - \left(\frac{\partial E}{\partial N}\right)_N =$$

$$= 2\left(\frac{\partial\left(\frac{E}{A}\right)}{\partial X}\right)_A = 4\left[b^*_{\rm sym}(A) + e^*_C(A)\right](X - X^*), \quad (7)$$

where

$$\lambda_n = \left(\frac{\partial E}{\partial N}\right)_Z, \quad \lambda_p = \left(\frac{\partial E}{\partial Z}\right)_N. \quad (8)$$

On the beta-stability line, it follows from Eq. (7) that $\Delta\lambda(X)_{X=X^*} = 0$, as it should be from the definition of the beta-stability line. We point out that for finite nuclei, the condition $\Delta\lambda = 0$ on the beta-stability line is not necessary fulfilled explicitly, because of the discrete spectrum of the single particle levels for both the neutrons and the protons near the Fermi surface. Based on the definition of the beta-stability line (3) one can rewrite Eq. (7) as

$$\frac{\Delta\lambda(X)}{4} = \left[b_{\rm sym}(A) + e_C(A)\right] X - e_C(A). \quad (9)$$

The quantity $\partial(E/A)/\partial X$ in Eq. (7) can be evaluated within the accuracy of $\sim 1/A^2$ using the finite differences which are based on the experimental values of the binding energy per nucleon $\mathcal{B}(N,Z) = -E(N,Z)/A$. Namely

$$\left(\frac{\partial\left(\frac{E}{A}\right)}{\partial X}\right)_A = \frac{A}{4}\left[\mathcal{B}(N-1, Z+1) - \mathcal{B}(N+1, Z-1)\right]. \quad (10)$$

Since the difference (10) is taken for $\Delta Z = -\Delta N = 2$ the pairing effects do not affect the resulting accuracy. It was shown [4] that linear dependence $\Delta\lambda$ on X by Eq. (7) is reproduced quite well experimentally. This allows extraction of the values of quantities $b^*_{\rm sym}(A)$, $e^*_C(A)$ and X^* for a given mass number A.

The dependency of the symmetry energy coefficient $b^*_{\rm sym}$ on the mass number A obtained from the experimental nuclear masses using Eqs. (7) and (10) shows the strong shell oscillations with the amplitude of about 15 % [4]. For the purpose of comparison, one could recall that shell effects contribute about 1 % to the nuclear mass. We have performed the fit of experimental data for $b^*_{\rm sym}$ to the leptodermous-like functional form of Eq. (6). The change of the intervals in mass numbers A for the fitting procedure leads to a significant difference in the surface contribution $b^*_{\rm sym,surf}$ to the symmetry energy. By fitting all of the available information for $A \geq 12$, we have obtained $b^*_{\rm sym,vol} = 26.5$ MeV and the surface contribution $b^*_{\rm sym,surf} = -25.6$ MeV with the surface-to-volume ratio $r_{S/V} = |b^*_{\rm sym,surf}|/b^*_{\rm sym,vol} \approx 1$. From the fit for $A \geq 50$ where the leptodermous expansion is more justified it has been obtained $b^*_{\rm sym,vol} = 32.5$ MeV, $b^*_{\rm sym,surf} = -56.3$ MeV and $r_{S/V} \approx 1.7$.

In Fig. 1, we have plotted the value of $X^*(A)$ (solid dots). The dashed and dotted lines in Fig. 1 were derived from the extended Thomas-Fermi approximation for SkM and SLy230b Skyrme forces, respectively, see Ref. [3]. The solid (thick) line in Fig. 1 was obtained using the empirical formula [5]

$$X^*(A) = 0.4\frac{A}{A+200}. \quad (11)$$

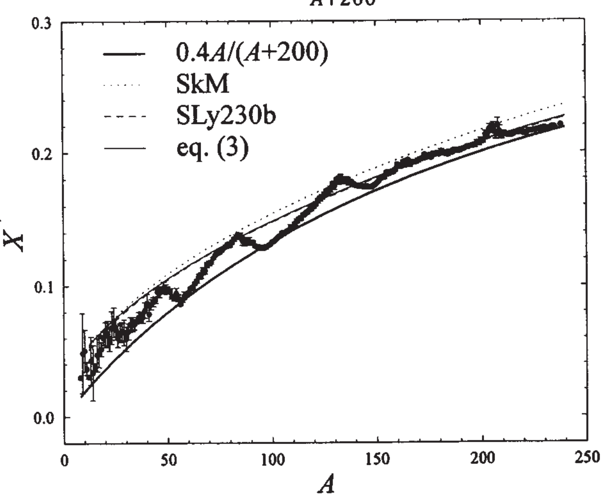

Fig. 1. Asymmetry parameter X^* versus the mass number A. Filled circles are the experimental data, dashed lines show calculations using Skyrme forces SkM and SLy230b, see Ref. [3]. Solid lines present the functions $X^*(A) = 0.4\,A/(A+200)$ [5] (thick) and $X^*(A) = e^*_C(A)/[b^*_{\rm sym}(A) + e^*_C(A)]$ (see Eq. (3)) with $e^*_C(A) = 0.17A^{2/3}$ and $b^*_{\rm sym}(A) = 26.5 - 25.6A^{-1/3}$ (thin).

The "experimental" curve $X^*(A)$ in Fig. 1 shows the non-monotonic (sawtooth) shape as a function of the mass number A. This behavior is the consequence of shell structure of single particle levels near Fermi surface for both the neutrons and the protons. Because of this shell structure, the Fermi levels for protons and neutrons can coincide by chance only. In agreement with Eq. (3), the smooth behavior of $X^*(A)$ is achieved by a fit of the symmetry and the Coulomb energy coefficients. Thin solid line in Fig. 1 is obtained from Eq. (3) with $e^*_C(A) = 0.17A^{2/3}$ and $b^*_{\rm sym}(A) = 26.5 - 25.6A^{-1/3}$.

To make shell oscillations in $X^*(A)$ more transparent with respect to well-known magic nucleon numbers of the closed shells [6], let consider the value of the chemical potential shift $\Delta\lambda$

at fixed neutron excess, $D = N - Z = AX$. As seen from Eq. (9), for the zero neutron excess the value of $\Delta\lambda$ is not affected by the symmetry energy and completely determined by $e_C(A)$. In Fig. 2 we show the A-dependence of the Coulomb energy coefficient obtained from Eq. (9) using all available data with $D = 0$. Solid line represents the smooth function

$$e_C(A) = 0.207A^{\frac{2}{3}} - 0.174A^{\frac{1}{3}} \qquad (12)$$

obtained using the fit to the experimental data shown in Fig. 2 as filled circles. Approximating the contribution of the Coulomb interaction to $\Delta\lambda$ by function (12), we can now extract $b_{sym}(A)$ at fixed $D \neq 0$ from the experimental values of $\Delta\lambda$ by use of Eq. (9). We note that estimation of e_C given by Eq. (12) was made inside quite narrow interval of nuclear masses $8 \leq A \leq 58$, and the use of Eq. (12) does not make much sense for masses $A > 60$ due to the extrapolation error. Nevertheless, some qualitative conclusions based on estimation (12) are still possible for heavy nuclei.

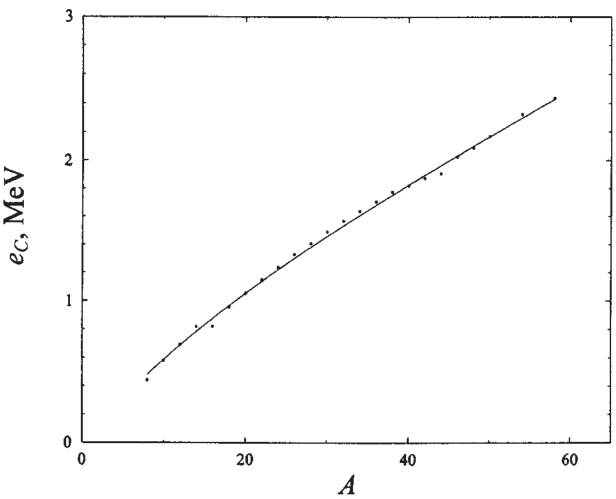

Fig. 2. Coulomb energy coefficient e_C as a function of mass number A. Symbols show experimental data for nuclei with $N = Z$. Solid line represent the smooth function $e_C(A) = 0.207A^{2/3} - 0.174A^{1/3}$.

The symmetry energy coefficient as a function of mass number for the values of the neutron excess D =18, 22, 26 and 30 is shown in Fig. 3. It is seen from Fig. 3 that, qualitatively, $b_{sym}(A)$ has canyon-like behavior for given value of D. The width and the position of the bottom for such "canyon" depends on neutron excess. The left wall of the canyon corresponds to proton closed shell and the right wall corresponds to the neutron shell closure. In Fig. 3 the walls are located symmetrically with respect to A =132 which correspond to both neutron and proton closed shell (N = 82, Z = 50). From D = 18 to D = 30 the shape of $b_{sym}(A)$ changes to thinner and deeper canyon with the local minimum in the symmetry coefficient being located at mass number which corresponds to double (proton-neutron) magic number.

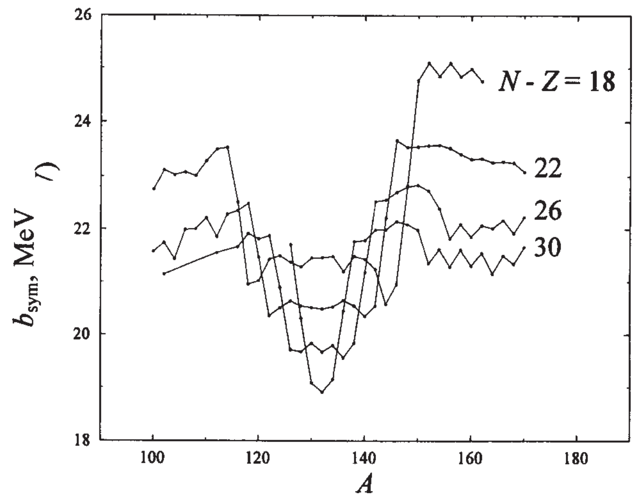

Fig. 3. The symmetry coefficient b_{sym} versus the mass number A at fixed neutron excess $D = N - Z$. The values of the neutron excess are specified by numbers near the curves.

Let now check the above reasoning with the shape of the beta-stability line presented in Fig. 1. We consider the sequence of the nucleon magic numbers [6]: 8, 20, 28, 50, 82, and 126. From this sequence one should expect special behavior of $X^*(A)$ nearby the following values of mass number $A = N + Z$: 28 (20 + 8), 48 (28 + 20), 78 (50 + 28), 132 (82 + 50), and 208 (126 + 82). The "experimental" line of beta-stability (see Fig. 1) has local maxima at mass numbers 24 (13 + 11), 48 (26 + 22), 84 (48 + 36), 133 (79 + 54), and 208 (126 + 82). We note that mass numbers of local maxima in Fig. 1 does not exactly follow double magic numbers. Moreover, they demonstrate only nearly closed shells for neutrons and protons, except A = 208. Nevertheless, one still can state that, at least approximately, there exist the correlation between the positions of maxima of the function $X^*(A)$ and double magic mass numbers.

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ОБОЛОНКОВІ ОСЦИЛЯЦІЇ В ЕНЕРГІЇ ІЗОТОПІЧНОЇ СИМЕТРІЇ

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Запропоновано метод отримання ізотопічної енергії симетрії виходячи з ізотопічного зсуву нейтронного та протонного хімічних потенціалів $\Delta\lambda = \lambda_n - \lambda_p$. Спостерігається немонотонна (пилкоподібна) форма залежності положення лінії β-стабільності від масового числа. Проаналізовано поведінку коефіцієнта енергії симетрії $b_{\text{sym}}(A)$ при сталому значенні нейтронного надлишку $D = N - Z$. Установлено кореляцію положення локальних максимумів лінії β-стабільності з магічними значеннями масового числа для подвійно заповнених оболонок.

Ключові слова: енергія симетрії, ізотопічний зсув, лінія β-стабільності, оболонкові осциляції.

ОБОЛОЧЕЧНЫЕ ОСЦИЛЛЯЦИИ В ЭНЕРГИИ ИЗОТОПИЧЕСКОЙ СИММЕТРИИ

В. М. Коломиец, А. И. Санжур

Предложен метод получения изотопической энергии симметрии исходя из изотопического сдвига нейтронного и протонного химических потенциалов $\Delta\lambda = \lambda_n - \lambda_p$. Наблюдается немонотонная (пилообразная) форма зависимости положения линии β-стабильности от массового числа. Проанализировано поведение коэффициента энергии симметрии $b_{\text{sym}}(A)$ при постоянном значении нейтронного избытка $D = N - Z$. Установлена корреляция положения локальних максимумов линии β-стабильности с магическими значениями массового числа для дважды заполненных оболочек.

Ключевые слова: энергия симметрии, изотопический сдвиг, линия β-стабильности, оболочечные осцилляции.

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