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A FIELD-THEORETICAL TREATMENT OF TWO-NUCLEON SYSTEMS: NUCLEON-NUCLEON SCATTERING AND DEUTERON PROPERTIES

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The “clothing” procedure in quantum field theory is applied for the description of nucleon-nucleon ($N-N$) scattering and deuteron properties. We consider the system of interacting fermion and meson fields with the Yukawa-type couplings to introduce trial interactions between "bare" particles. Special unitary transformations are used to express the primary total Hamiltonian through new creation/annihilation operators for the so-called clothed particles (these quasiparticles of our approach). We are focused upon the Hermitian and energy-independent interactions (quasipotentials) between the clothed nucleons, being built up in the second order in the coupling constants. The interactions are the kernels of integral equations for the T-matrix of $N-N$ scattering and the deuteron wave function in momentum space. We discuss distinctions between our quasipotentials and the Bonn potential. Numerical solutions of these equations are compared with those by the Bonn group.

Keywords: quantum field theory, nucleon-nucleon scattering, deuteron, Bonn potential.

Introductory remarks

The main purpose of this work is to develop a consistent field-theoretic approach to the description of two nucleon systems using the method of unitary “clothing” transformations (UCTs) (see the survey [1] and refs. therein). We start with the total Hamiltonian for meson-nucleon system

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = \\ = H_F(\alpha) + V(\alpha) + \text{mass and vertex counterterms}, \tag{1}$$

where the free part $H_F(\alpha)$ and the interaction $V(\alpha)$ depend on creation (destruction) operators $\alpha^\dagger(\alpha)$ in the so-called bare particle representation (BPR). To be more definite, let us consider fermions (nucleons and antinucleons) and bosons ($\pi-$, $\eta-$, $\rho-$, $\omega-$mesons, etc.) interacting via the Yukawa-type couplings for scalar (s), pseudoscalar (ps) and vector (v) mesons (see, e.g., [2]). Then, using a trick prompted by the derivation of Eq. (7.5.22) in [3] to eliminate in a proper way the vector-field component $\varphi_{\rm v}^0$, we have $V(\alpha) = V_s + V_{ps} + V_{\rm v}$ with

$$V_s = g_s \int d\vec{x}\, \bar{\psi}(\vec{x})\psi(\vec{x})\varphi_s(\vec{x}), \tag{2}$$

$$V_{ps} = ig_{ps} \int d\vec{x}\, \bar{\psi}(\vec{x})\gamma_5\psi(\vec{x})\varphi_{ps}(\vec{x}), \tag{3}$$

$$V_{\rm v} = \int d\vec{x} \left\{ g_{\rm v}\bar{\psi}(\vec{x})\gamma_\mu\psi(\vec{x})\varphi_{\rm v}^\mu(\vec{x}) + \frac{f_{\rm v}}{4m}\bar{\psi}(\vec{x})\sigma_{\mu\nu}\psi(\vec{x})\varphi_{\rm v}^{\mu\nu}(\vec{x}) \right\} + \\ + \int d\vec{x} \left\{ \frac{g_{\rm v}^2}{2m_{\rm v}^2}\bar{\psi}(\vec{x})\gamma_0\psi(\vec{x})\bar{\psi}(\vec{x})\gamma_0\psi(\vec{x}) + \frac{f_{\rm v}^2}{4m^2}\bar{\psi}(\vec{x})\sigma_{0i}\psi(\vec{x})\bar{\psi}(\vec{x})\sigma_{0i}\psi(\vec{x}) \right\} \tag{4}$$

with the boson fields φ_b and the fermion field ψ, where $\varphi_{\rm v}^{\mu\nu}(\vec{x}) = \partial^\mu\varphi_{\rm v}^\nu(\vec{x}) - \partial^\nu\varphi_{\rm v}^\mu(\vec{x})$ is the tensor of the vector field included. The mass (vertex) counterterms are given by Eqs. (32) - (33) of Ref. [4] (the difference $V_0(\alpha) - V(\alpha)$ where a primary interaction $V_0(\alpha)$ is derived from $V(\alpha)$ replacing the "physical" coupling constants by "bare" ones). It should be noted that a second term in Eq. (4) is not a Lorentz scalar. The appearance of such terms is typical of theories with derivative couplings or/and spin $j \geq 1$. A more detailed discussion of this issue can be found in [5, 6].

Analytic expressions for the quasipotentials in momentum space

The transition to the clothed particle representation (CPR) is doing with help of a unitary transformation $\alpha_c = W\alpha W^\dagger$, where $W = \exp R$ with $R^\dagger = -R$. Its generator R is chosen in such a way to remove all “bad” terms from $V(\alpha)$ (by definition [1], the latter prevent the bare vacuum and the bare one-particle states to be H eigenstates). After this, the primary Hamiltonian $H(\alpha)$ can be represented in the form $H(\alpha) = K_F(\alpha_c) + K_I(\alpha_c) \equiv K(\alpha_c)$.

The free part of the new decomposition is determined by $K_F(\alpha_c) = H_F(\alpha_c)$ while K_I contains only interactions responsible for physical processes in the system. In particular, the $2 \leftrightarrow 2$ interactions of interest stem from the commutator $[R, V]$, namely:

$$K_I^{(2)}(\alpha_c) = K(NN \to NN) + K(\bar{N}\bar{N} \to \bar{N}\bar{N}) + K(N\bar{N} \to N\bar{N}) + K(bN \to bN) \tag{5}$$

It is important to realize that operator $K(\alpha_c)$ is the same Hamiltonian $H(\alpha)$ but written in other representation. Accordingly [1, 4] the operator responsible for $N-N$ interaction in the CPR has the following structure:

$$K(NN \to NN) = \sum_b K_b(NN \to NN) \equiv K_{NN},$$

$$K_b(NN \to NN) = \int \sum_\mu d\vec{p}'_1 d\vec{p}'_2 d\vec{p}_1 d\vec{p}_2 V_b(1',2';1,2) b_c^\dagger(1') b_c^\dagger(2') b_c(1) b_c(2), \tag{6}$$

where the symbol $\sum_\mu$ denotes the summation over nucleon spin projections, $1 = \{\vec{p}_1, \mu_1\}$, etc.

For our evaluations of the c-number matrices V_b we have employed some experience from Refs. [1, 4] to get in the second order in the coupling constants

$$V_b(1',2';1,2) = \frac{1}{(2\pi)^3} \frac{m^2}{\sqrt{E_{\vec{p}'_1} E_{\vec{p}'_2} E_{\vec{p}_1} E_{\vec{p}_2}}} \delta\left(\vec{p}'_1 + \vec{p}'_2 - \vec{p}_1 - \vec{p}_2\right) v_b(1',2';1,2), \tag{7}$$

$$v_s(1',2';1,2) = -\frac{g_s^2}{2} \bar{u}(\vec{p}'_1) u(\vec{p}_1) \frac{1}{(p_1 - p'_1)^2 - m_s^2} \bar{u}(\vec{p}'_2) u(\vec{p}_2), \tag{8}$$

$$v_{ps}(1',2';1,2) = \frac{g_{ps}^2}{2} \bar{u}(\vec{p}'_1) \gamma_5 u(\vec{p}_1) \frac{1}{(p_1 - p'_1)^2 - m_{ps}^2} \bar{u}(\vec{p}'_2) \gamma_5 u(\vec{p}_2), \tag{9}$$

$$v_{\mathrm{v}}(1',2';1,2) = \frac{1}{2} \frac{1}{(p'_1 - p_1)^2 - m_{\mathrm{v}}^2} \times$$

$$\times \Bigg[\bar{u}(\vec{p}'_1) \left\{ (g_{\mathrm{v}} + f_{\mathrm{v}}) \gamma_\nu - \frac{f_{\mathrm{v}}}{2m} (p'_1 + p_1)_\nu \right\} u(\vec{p}_1) \bar{u}(\vec{p}'_2) \left\{ (g_{\mathrm{v}} + f_{\mathrm{v}}) \gamma^\nu - \frac{f_{\mathrm{v}}}{2m} (p'_2 + p_2)^\nu \right\} u(\vec{p}_2) -$$

$$- \bar{u}(\vec{p}'_1) \left\{ (g_{\mathrm{v}} + f_{\mathrm{v}}) \gamma_\nu - \frac{f_{\mathrm{v}}}{2m} (p'_1 + p_1)_\nu \right\} u(\vec{p}_1) \times$$

$$\times \bar{u}(\vec{p}'_2) \frac{f_{\mathrm{v}}}{2m} \left\{ \left\{ (\hat{p}'_1 + \hat{p}'_2 - \hat{p}_1 - \hat{p}_2) \gamma^\nu - (p'_1 + p'_2 - p_1 - p_2)^\nu \right\} u(\vec{p}_2) \right\} \Bigg], \tag{10}$$

where m_b the mass of the clothed boson (its physical value) and $\hat{q} = q_\mu \gamma^\mu$. In the framework of the isospin formalism one needs to add the factor $\vec{\tau}(1)\vec{\tau}(2)$ in the corresponding expressions.

The derivation of the vector-boson contribution (10) is discussed more detail in [5, 6]. One should stress, that the first UCT enables us to remove the non-invariant terms (second term in Eq. (4)) directly in the Hamiltonian. In our opinion, such a cancellation, is a pleasant feature of the CPR.

The corresponding relativistic and properly symmetrized $N-N$ interaction, the kernel of integral equations for the $N-N$ bound and scattering states, is determined by

$$\left\langle \Omega \right| b_c(\vec{p}'_1) b_c(\vec{p}'_2) K_b(NN \to NN) b_c^\dagger(\vec{p}_1) b_c^\dagger(\vec{p}_2) \left| \Omega \right\rangle =$$

$$= -V_b(1',2';1,2) - V_b(2',1';2,1) + V_b(1',2';2,1) +$$

$$+ V_b(2',1';1,2). \tag{11}$$

Application to the elastic $N-N$ scattering

In order to evaluate the $N-N$ scattering amplitude for the collision energy E we will regard a field operator T in the CPR, that meets the equation

$$T_{cloth}(E+i0)=K_I+K_I(E+i0-K_F)^{-1}T_{cloth}(E+i0)\,, \tag{12}$$

and whose matrix elements $\langle NN|T_{cloth}(E+i0)|NN\rangle$ on the energy shell $E=E_1+E_2=E_1^{'}+E_2^{'}$ can be expressed through the phase shifts and mixing parameters.

If in Eq. (12) we approximate K_I by $K_I^{(2)}$, then initial task of evaluating the CPR matrix elements can be reduced to solving the equation

$$\langle 1',2'|T_{NN}(E)|1,2\rangle=\langle 1',2'|K_{NN}|1,2\rangle+$$

$$+\langle 1',2'|K_{NN}(E+i0-K_F)^{-1}T_{NN}(E)|1,2\rangle. \tag{13}$$

For practical applications one prefers to work with the corresponding R–matrix that meets the equation

$$\langle 1'2'|\bar{R}(E)|12\rangle=\langle 1'2'|\bar{K}_{NN}|12\rangle+$$

$$+\int\limits_{34}\sum\langle 1'2'|\bar{K}_{NN}|34\rangle\frac{\langle 34|\bar{R}(E)|12\rangle}{E-E_3-E_4} \tag{14}$$

with $\bar{R}(E)=R(E)/2$ and $\bar{K}_{NN}=K_{NN}/2$, where the operation $\int\limits_{34}\sum$ means the summation over nucleon polarizations and the p.v. integration over nucleon momenta.

After the angular-momentum decomposition Eq. (14) splits into to the set of integral equations

$$\bar{R}^{JST}_{l'l}(p',p)=\bar{V}^{JST}_{l'l}(p',p)+$$

$$+\sum_{l''}\text{p.v.}\int\limits_0^\infty\frac{q^2\,dq}{2(E_p-E_q)}\bar{V}^{JST}_{l'l''}(p',q)\bar{R}^{JST}_{l''l}(q,p) \tag{15}$$

to be solved for each submatrix $\bar{R}^{JST}$ composed of the elements $\bar{R}^{JST}_{l'l}(p',p)\equiv\bar{R}^{JST}_{l'l}(p',p;2E_p)$, where $E_p=\sqrt{\vec{p}^{\,2}+m^2}$ the collision energy in the center of mass system (c.m.s.), m the nucleon mass.

In our case such a decomposition means the transition to the matrix elements between the states $|p(lS)JM_J\rangle$, which have been constructed as common eigenstates of the operator K_F and the field linear- and angular-momentum operators

$$|p(lS)JM_J\rangle=\int d\,\hat{\vec{p}}\,Y_{lm_l}(\hat{\vec{p}})\big(lm_lSM_S\,|\,JM_J\big)\left(\frac{1}{2}\mu_1\frac{1}{2}\mu_2\,|\,SM_S\right)b^\dagger(\vec{p}\mu_1)b^\dagger(-\vec{p}\mu_1)|\Omega\rangle\,. \tag{16}$$

with the unit vector $\hat{\vec{p}}=\vec{p}/p$.

The deuteron equation

Now, we consider a $K(\alpha_c)$ eigenstate from the NN sector, to be represented as

$$|\psi_{NN}\rangle=\sum_{\mu_1\mu_2}\int d\vec{p}_1d\vec{p}_2\psi_{NN}(\vec{p}_1\mu_1,\vec{p}_2\mu_2)b^\dagger(\vec{p}_1\mu_1)b^\dagger(\vec{p}_2\mu_2)|\Omega\rangle\,. \tag{17}$$

In the approximation $K_I=K_I^{(2)}$, the eigenvalue equation $K(\alpha_c)|\psi_{NN}\rangle=E|\psi_{NN}\rangle$ has the form

$$[K_F+K_{NN}]|\psi_{NN}\rangle=E|\psi_{NN}\rangle\,. \tag{18}$$

In turn the deuteron state at rest can be written as the superposition

$$|\psi_d^M\rangle=\sum_{l=0,2}\int\limits_0^\infty dq\,q^2\,|q(l1)1M\rangle\psi_l^d(q)\,, \tag{19}$$

with coefficients $\psi_l^d(q)=\langle q(l1)1M|\psi_{NN}\rangle$ that satisfy the equations

$$\psi_l^d(p)=\frac{1}{M_d-2E_{\vec{p}}}\sum_{l'}\int\limits_0^\infty dq\,q^2\bar{V}_{l\,l'}^{J=1,S=1,T=0}(p,q)\psi_{l'}^d(q)\,, \tag{20}$$

where $M_d=2m-\varepsilon_d$ the deuteron mass and ε_d the deuteron binding energy.

Results of numerical calculations and their discussion

In order to compare our calculations and those obtained in [7] with the potential B, first of all, one needs to regularize the expressions (8) - (10) by introducing the phenomenological cutoffs

$$F_b(p',p)=\left[\frac{\Lambda_b^2-m_b^2}{\Lambda_b^2-(p'-p)^2}\right]^{n_b}\equiv F_b[(p'-p)^2] \tag{21}$$

in the c.m.s taken from [2, 6]. It means the substitution

$$\langle\vec{p}'\mu'_1\mu'_2|v_b^{UCT}|\vec{p}\mu_1\mu_2\rangle\equiv$$

$$\equiv -F_b^2(p',p)\left[v_b(1',2';1,2)+v_b(2',1';2,1)\right]. \qquad (22)$$

However, such a regularization does not remove all distinguish between our model and the Bonn potential. Replacing in equations (22)

$$F_b^2[(p'-p)^2]\left[(p'-p)^2-m_b^2\right]^{-1}$$

by

$$-F_b^2[-(\vec{p}'-\vec{p})^2]\left[(\vec{p}'-\vec{p})^2+m_b^2\right]^{-1}$$

and neglecting the tensor-tensor term

$$\frac{f_v^{\,2}}{4m^2}(E_{p'}-E_p)^2\bar{u}(\vec{p}^{\,'})[\gamma_0\gamma_\nu-g_{0\nu}]u(\vec{p})\bar{u}(-\vec{p}^{\,'})[\gamma^0\gamma^\nu-g^{0\nu}]u(-\vec{p}), \qquad (23)$$

we obtain approximate expressions that with the common factor $(2\pi)^{-3}m^2/E_{p'}E_p$ instead of $(2\pi)^{-3}m/\sqrt{E_{p'}E_p}$ are equivalent to Eqs. (E. 21) - (E. 23) from [2] (for details see [5]).

The best-fit parameters for the two models. The row *Potential B (UCT)* taken from Table A.1 [7] (obtained by solving Eqs. (15, 20) with a least squares fitting to Bonn values). All masses and cutoff parameters are in MeV, and $n_b = 1$ except for $n_\rho = n_\omega = 2$. The values in brackets for ρ – meson denote the ratio f_ρ/g_ρ

Model	Meson	π	η	ρ	ω	δ	$\sigma, T=0\ (T=1)$
Potential B	$g^2/4\pi\ [f/g]$	14.4	3	0.9 [6.1]	24.5	2.488	18.3773(8.9437)
	Λ	1700	1500	1850	1850	2000	2000(1900)
	m	138.03	548.8	769	782.6	938	720(550)
UCT	$g^2/4\pi\ [f/g]$	14.574	2.1	1.3 [5.935]	25.325	2.923	16.081(10.089)
	Λ	2200	1200	1450	2144	2092	2012(2200)
	m	138.03	548.8	769	782.6	938	693.66(562.07)

Our calculations of the R matrices and deuteron wave functions that meet the equations (15) and (20) are twofold. On the one hand, we will check reliability of our numerical procedure. On the other hand, we would like to show similarities and discrepancies between our results and those by the Bonn group. These results are depicted in Figs. 1 and 2. In case of the UCT potential after parameters fitting (Table) we have for the deuteron binding energy $\varepsilon_d = 2.224$ MeV and for the *D*-state probability $P_D = 5.494$ % (in case of Bonn potential $\varepsilon_d = 2.223$ MeV and $P_D = 4.986$ %). More detailed discussion can be found in [5, 6].

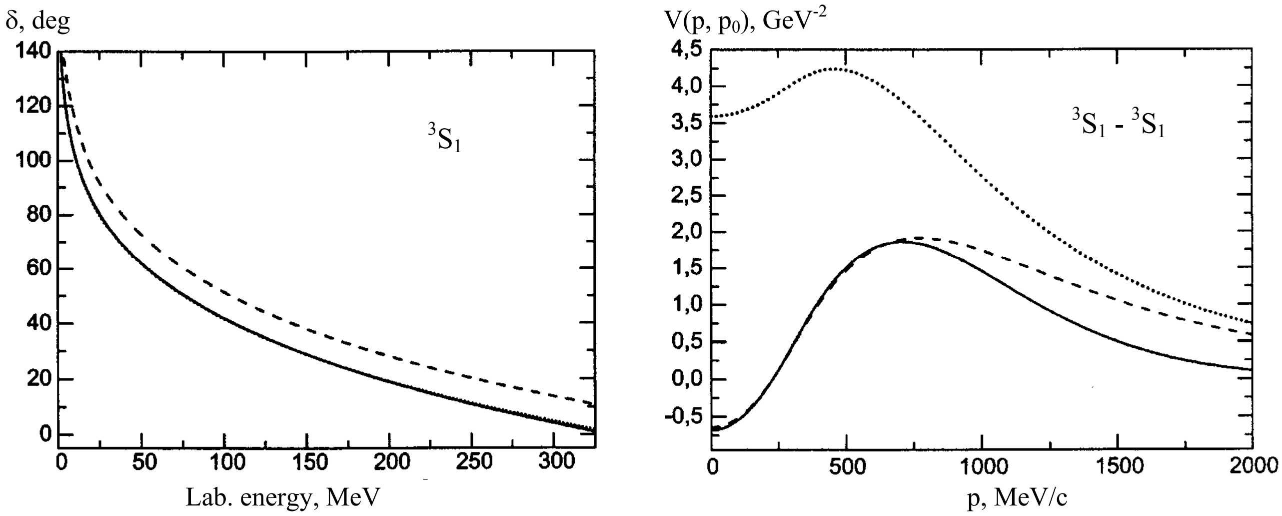

Fig. 1. The $^3S_1$ neutron-proton phase shift and the corresponding off-shell potential. Solid curves calculated for Potential B. Dashed (dotted) – for UCT potential with Potential B (UCT) parameters from the Table . On-shell momentum p_0 is fixed to 265 MeV.

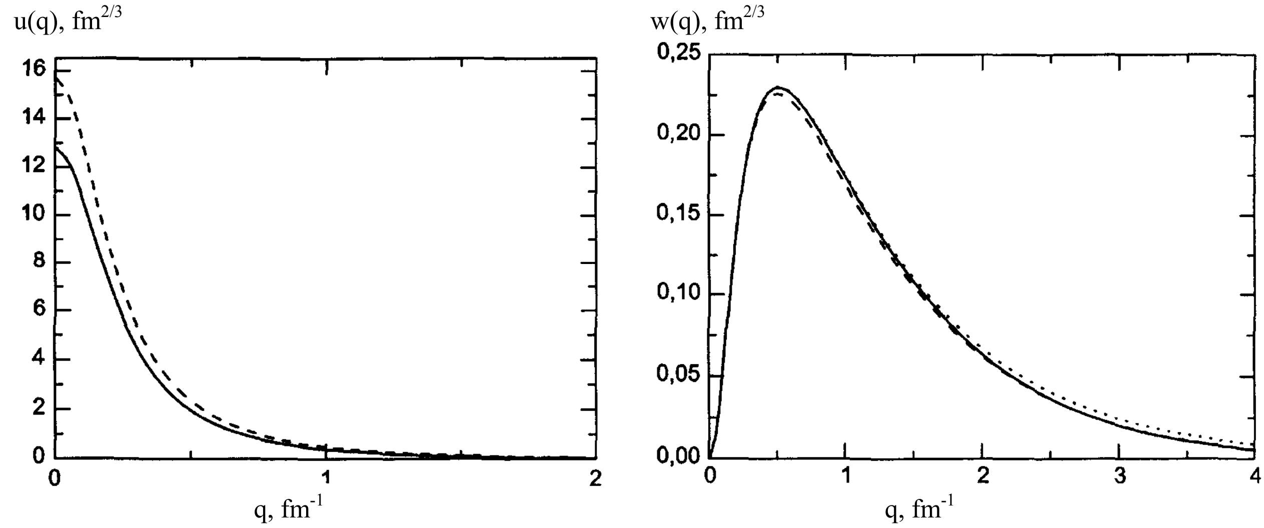

Fig. 2. Deuteron wave functions $u(q) = \psi_0^d(p)$ and $w(q) = \psi_2^d(p)$. Other notations as in Fig. 1.

Summary

We have seen how starting from the field Hamiltonian one can reduce a very complex field-theoretical problem to an approximate description typical of the relativistic quantum mechanics. The method of UCTs has turned out to be appropriate in achieving this aim.

In particular, since the two representations, CPR and BPR, are unitarily equivalent the description of the $N-N$ scattering can be reduced to the three-dimensional *LS*-type equation for the T-matrix in momentum space. Such a conversion becomes possible owing to the property of $K_I^{(2)}$ to leave the two-nucleon sector and its separate subsectors to be invariant.

Special attention has been paid to the elimination of auxiliary field components. We encounter such a necessity for interacting vector and fermion fields when in accordance with the canonical formalism the interaction Hamiltonian density embodies not only a scalar contribution but nonscalar terms too. It has proved (at least, for the primary ρN and ωN couplings) that the UCT method allows us to remove such noncovariant terms directly in the Hamiltonian. To what extent this result will take place in higher orders in coupling constants it will be a subject of further explorations.

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ТЕОРЕТИКО-ПОЛЬОВИЙ РОЗГЛЯД ДВОНУКЛОННИХ СИСТЕМ: НУКЛОН-НУКЛОННЕ РОЗСІЯННЯ ТА ВЛАСТИВОСТІ ДЕЙТРОНА

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“Clothing” процедура у квантовій теорії поля застосовується для опису нуклон-нуклонного розсіювання та властивостей дейтрона. Розглядається система ферміонних і мезонних полів, що взаємодіють за допомогою зв'язку типу Юкави, для введення потрійної взаємодії між “голими” частинками. Використовуючи спеціальне унітарне перетворення, повний гамільтоніан системи виражається через нові оператори народження/знищення для "одягнених" частинок (квазічастинок нашого підходу). Основна увага приділяється ермітівським, не залежним від енергії операторам взаємодії (квазіпотенціалам) між "одягненими" нуклонами, побудованим у другому порядку за константами взаємодії. Дані квазіпотенціали є ядрами інтегральних рівнянь для T-матриці нуклон-нуклонного розсіяння і дейтронної хвильової функції в імпульсному представленні. Обговорюється відмінність отриманих квазіпотенціалів і боннського потенціалу. Чисельний розв'язок рівнянь порівнюється з аналогічними результатами боннської групи.

Ключові слова: квантова теорія поля, нуклон-нуклонне розсіяння, дейтрон, боннський потенціал.

ТЕОРЕТИКО-ПОЛЕВОЕ РАССМОТРЕНИЕ ДВУХНУКЛОННЫХ СИСТЕМ: НУКЛОН-НУКЛОННОЕ РАССЕЯНИЕ И СВОЙСТВА ДЕЙТРОНА

Е. А. Дубовик

"Clothing" процедура в квантовой теории поля применяется для описания нуклон-нуклонного рассеяния и свойств дейтрона. Рассматривается система фермионных и мезонных полей, взаимодействующих посредством связи типа Юкавы, для введения тройного взаимодействия между "голыми" частицами. Используя специальное унитарное преобразование, полный гамильтониан системы выражается через новые операторы рождения/уничтожения для "одетых" частиц (квазичастиц нашего подхода). Основное внимание уделяется эрмитовским, не зависящим от энергии операторам взаимодействия (квазипотенциалам) между "одетыми" нуклонами, построенным во втором порядке по константам взаимодействия. Данные квазипотенциалы являются ядрами интегральных уравнений для T-матрицы нуклон-нуклонного рассеяния и дейтронной волновой функции в импульсном представлении. Обсуждаются различия полученных квазипотенциалов и боннского потенциала. Численное решение уравнений сравнивается с аналогичными результатами боннской группы.

Ключевые слова: квантовая теория поля, нуклон-нуклонное рассеяние, дейтрон, боннский потенциал.

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