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OPTICAL POTENTIAL BASED ON SKYRME FORCES FOR DESCRIBING THE ELASTIC NUCLEON-NUCLEUS SCATTERING

© 2010 V. V. Pilipenko[1], V. I. Kuprikov[1], A. P. Soznik[2]

[1] *National Scientific Centre "Kharkiv Institute of Physics and Technology", Kharkiv,*
[2] *Civil Defense University, Kharkiv*

The model of microscopic optical potential based on calculations of the one-particle Green function mass operator using the effective Skyrme nucleon-nucleon forces has been applied for describing cross sections and analyzing powers of the elastic nucleon-nucleus scattering. The Skyrme force parameters have been optimized by fitting a chosen angular distribution of the elastic neutron-nucleus scattering with simultaneous control of main characteristics of the nuclear matter and the binding energy and rms charge radius of the target nucleus. The found Skyrme forces have been used to analyze differential cross sections and analyzing powers of elastic neutron scattering by nuclei in a wide range of mass numbers. The calculations have given a satisfactory description of experimental data on elastic neutron-nucleus scattering and reasonable values of the main parameters for the symmetric nuclear matter and even-even nuclei. Analogous calculations have been performed for describing experimental data on the elastic proton-nucleus scattering, which have yielded encouraging results.

Keywords: Green function, Skyrme nucleon-nucleon forces, elastic neutron scattering, analyzing power, elastic proton scattering.

Introduction

The optical model is one of the basic theoretical approaches to the analysis of nucleon-nucleus (*NA*) collisions. There still remains an actual problem of constructing microscopic *NA* optical potentials (MOP) which would be applicable to description of experimental data on the *NA*-scattering proceeding from a given nucleon-nucleon (*NN*) interaction. It is of significant interest to study possibilities of developing a unified theory of shell-model and MOP on the basis of given *NN*- forces. Here, for this purpose we use the effective Skyrme forces. The Hartree-Fock method with the effective Skyrme interaction (SHF) [1, 2] is one of the main approaches to calculating the structure of atomic nuclei that allows describing satisfactorily many properties of nuclei. The use of the SHF approach for calculations of the MOP for the *NA*-scattering was first considered in [3]. In [4 - 7] the real and imaginary parts of MOP for the *NA*-interaction were constructed on the basis of effective Skyrme forces and calculations of the mass operator of the one-particle Green function in the nuclear matter and local density approximations. In [5, 6] in the real part of MOP allowance was made for the rearrangement (saturation) potential that is related with the density dependence of the effective forces and it was shown that its influence is rather important. In [7] the MOP with the density-dependent Skyrme forces was calculated without taking account of the rearrangement potential and the nucleon densities were used in a phenomenological form whereas we employ SHF nucleon densities calculated in a self-consistent way. The model was somewhat refined in [8, 9] by means of using the real part of the MOP in the form of the Hartree-Fock potential for finite nuclei with allowance for the terms depending on the nuclear density gradients and the spin-orbit potential.

Because of substantial ambiguity in determining the Skyrme force parameters and strong distinctions in the scattering cross sections calculated by the model under consideration with different forces, we may hope to improve the description of *NA*-scattering by modifying these parameters. In the paper, we use new variants of the Skyrme forces found from optimizing the description of angular distributions of the elastic neutron-nucleus scattering and nuclear structure in order to analyze them in calculations of cross sections and analyzing powers of the neutron- and proton-nucleus scattering, the nuclear matter characteristics and the structure of finite nuclei.

Description of the model

When calculating the MOP and the nuclear structure we use the effective *NN* Skyrme interaction in the form:

$$
\begin{aligned}
V_{ij} = V(\mathbf{r},\rho) &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}) + \\
&+\tfrac{1}{2}t_1(1+x_1P_\sigma)[\mathbf{k}'^2\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{k}^2] + \\
&+t_2(1+x_2P_\sigma)\mathbf{k}'\delta(\mathbf{r})\mathbf{k} + \tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\gamma(\mathbf{R})\delta(\mathbf{r}) + \\
&+iW_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)[\mathbf{k}'\times\delta(\mathbf{r})\mathbf{k}]. \qquad (1)
\end{aligned}
$$

Here, $\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j$ and $\mathbf{R}=(\mathbf{r}_i+\mathbf{r}_j)/2$ are the relative and center-of-mass radius-vectors of two nucleons *i, j*; ρ_n, ρ_p and $\rho=\rho_n+\rho_p$ are the neutron, proton and the total densities of the target nucleus; $\mathbf{k} = -i\partial/\partial\mathbf{r}$ and $\mathbf{k}' = i\partial/\partial\mathbf{r}'$ are the momentum operators of the relative nucleon motion in the initial and final states; P_σ is the spin permutation operator. The quantities t_n, x_n ($n = 0-3$), γ and W_0 are phenomenological Skyrme force parameters.

The MOP is found from calculations of the mass operator $M_{\alpha\alpha}$ of the one-particle Green function ($M_{\alpha\beta} = <\alpha|M|\beta>$, α, β being the final and initial states of the incident nucleon) up to the Goldstone diagrams of the second order. We choose the zero-order approximation for the mass operator $M_{\alpha\alpha}^{(0)}$ in the form of the mean self-consistent Hartree-Fock potential. This choice cancels a certain class of diagrams in all orders of the perturbation theory, in particular, the first-order diagrams for $M_{\alpha\alpha}^{(1)}$. In the SHF theory the variation of the Hartree-Fock energy functional with the density-dependent *NN*-forces results in the appearance of the rearrangement potential. According to this, the zero-order approximation of $M_{\alpha\alpha}^{(0)}$ is

$$M_{\alpha\alpha}^{(0)} = U_{\alpha\alpha}^{(HF)} = U_{\alpha\alpha}^{(0)} + U_{\alpha\alpha}^{(R)}, \tag{2}$$

where the standard Hartree-Fock potential $U_{\alpha\alpha}^{(0)}$ and the rearrangement potential $U_{\alpha\alpha}^{(R)}$ have the form

$$U_{\alpha\alpha}^{(0)} = \sum_{\lambda} <\alpha\lambda|V(1-P_{12})|\alpha\lambda> n_\lambda, \tag{3}$$

$$U_{\alpha\alpha}^{(R)} = \sum_{\lambda\mu} <\alpha|<\lambda\mu|\frac{1}{2}\delta(\mathbf{r}-\mathbf{R})\frac{\partial V}{\partial\rho}(1-P_{12})|\lambda\mu> n_\lambda n_\mu|\alpha>. \tag{4}$$

Here, P_{12} is the nucleon permutation operator, n_λ are the occupation numbers of the one-particle Hartree-Fock states with energies ε_λ.

The imaginary part of the sought MOP arises in the second order of perturbation theory:

$$W_{\alpha\alpha} \equiv \operatorname{Im} M_{\alpha\alpha}^{(2)} = \frac{1}{2}\operatorname{Im}\sum_{\lambda\mu\nu}\langle\alpha\mu|V(1-P_{12})|\lambda\nu\rangle\langle\lambda\nu|V(1-P_{12})|\alpha\mu\rangle\,\frac{n_\mu(1-n_\lambda)(1-n_\nu)}{\varepsilon_\alpha+\varepsilon_\mu-\varepsilon_\lambda-\varepsilon_\nu+i\delta}. \tag{5}$$

As in our papers [8, 9], in the real part of MOP we take account of the terms that are related to the non-uniformity of the nucleon densities in finite nuclei and the spin-orbit *NA*-potential. Therefore, the MOP has the following form

$$U(r,E) = V(r,E) + \frac{1}{r}V_{SO}(r)(\mathbf{l}\cdot\boldsymbol{\sigma}) + iW(r,E) + V_C(r), \tag{6}$$

where the real central part of the MOP is determined by the expression

$$V(r,E) = \frac{m_q^*}{m_q}\left[V^{(0)}(r) + V^{(R)}(r) + V^{(\Delta)}(r) + V^{(m)}(r)\right] + \left(1 - \frac{m_q^*(r)}{m_q}\right)\left[E - V_C(r)\right], \tag{7}$$

that coincides with the local Hartree-Fock potential for finite nuclei [3]. Here, $V^{(0)} + V^{(R)}$ is the Hartree-Fock potential in the nuclear matter approximation including the rearrangement potential $V^{(R)}$, the term $V^{(\Delta)}(r)$ depends on the radial derivatives of the nuclear densities according to the SHF theory of finite nuclei, m_q and m_q^* are the mass and the effective mass of the incident nucleon of sort $q = n, p$ in the nuclear medium, the term $V^{(m)}(r)$ depends on the radial derivatives of m_q^* and arises in the transformation from the non-local Hartree-Fock equation to the Schrödinger equation with energy-dependent local potential (E is the incident nucleon energy in the c.m. frame) [3]. The explicit form for these quantities and the SHF spin-orbit potential $V_{SO}(r)$ is presented in [8, 9]. The Coulomb potential $V_C(r)$ must be taken into account in the case of proton scattering. The imaginary part of MOP is calculated in the nuclear matter and local-density approximations and depends on the nucleon densities and the incident-nucleon wave vector magnitude k_α (see [8, 9]), the latter being determined from the following dispersion law:

$$k_\alpha^2 = \frac{2m_q^*}{\hbar^2}\left[E - V^{(0)} - V^{(R)} - V^{(\Delta)} - V_C\right]. \tag{8}$$

The nucleon densities, spin densities and kinetic energy densities entering into expressions for the MOP are found from the self-consistent calculations of the target nucleus structure by solving the Hartree-Fock equations with the effective Skyrme interaction and the constant pairing forces in the BCS approximation and with allowance for the direct and exchange parts of the Coulomb potential.

Results of calculations

We have carried out many calculations of differential cross sections of the neutron-nucleus scattering using this model of MOP with different known sets of the Skyrme interaction parameters.
In Fig. 1 we present typical examples of such calculations for the case of the differential cross section $\sigma(\theta) \equiv d\sigma(\theta)/d\Omega$ for the elastic scattering of 13.9 MeV neutrons by ^{116}Sn nuclei with popular variants of the Skyrme forces: Sly4 [10], SkM* [11], Ska [12]. It can be seen that some of the sets of the Skyrme force parameters provide an encouraging description of the experimental data (all experimental data on cross sections and analyzing powers considered in this paper were taken from the electronic nuclear data tables http://www-nds.iaea.org/exfor/exfor.htm). At the same time, there is a considerable spread of the calculation results for different Skyrme forces.

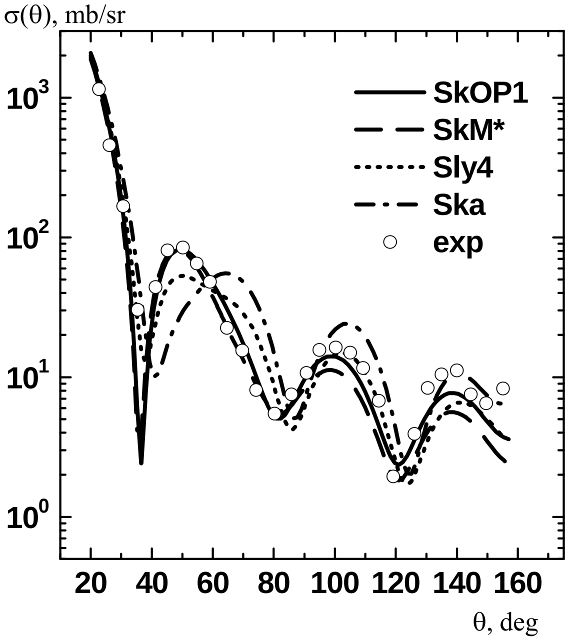


Fig. 1. Differential cross section of the elastic n + ^{116}Sn scattering at the neutron energy 13.9 MeV calculated with different Skyrme forces.

To improve the description of angular distributions for the elastic *NA*-scattering, we have tried to search for some new variants of the Skyrme forces using an optimization procedure that requires a satisfactory description of both the scattering processes and the main characteristics of the nuclear structure simultaneously [8. 9]. This procedure includes fitting of one selected differential cross section of the neutron scattering with allowance for conditions ensuring an acceptable description of the uniform symmetric nuclear matter characteristics and of the binding energy and rms radius of the target nucleus. The controlled characteristics of the nuclear matter were the average binding energy per nucleon *E/A*, the equilibrium density ρ_{eq}, the symmetry energy ε_τ and the effective nucleon mass in nuclear matter m^*. Therefore, the set of independently varied parameters was chosen as follows: ρ_{eq} , *E/A*, ε_τ , m^*, t_2 , x_1 , x_2 , x_3, and W_0 . Other Skyrme parameters, t_0 , t_3 , t_1, and x_0 , were determined through the varied parameters at each step of the variation. The binding energy and rms radius of the target nucleus were controlled by using the modified $\tilde{\chi}^2$ value [8, 9]. Using this optimization procedure, we have succeeded to find the most promising variants of the Skyrme forces when fitting the differential cross section of the 13.9 MeV neutron scattering by ^{116}Sn nuclei. For the optimization, the starting Skyrme forces were Ska for the density dependence index γ = 1/3 and Sly4 and SkM* for γ = 1/6. We have obtained two variants of the Skyrme parameter sets [9], SkOP1 (γ = 1/6) and SkOP2 (γ = 1/3), that are presented in [9].

The result of the cross section fitting for the forces SkOP1 is presented in Fig. 1, which shows improving the description of these experimental data. The forces SkOP2 yield practically the same result, as SkOP1 [9]. The main characteristics of the symmetric nuclear matter calculated with the found parameter sets have values [9] close to those given by the known Skyrme forces. The binding energies and charge radii of nuclei calculated by the SHF theory with the found forces in a wide range of mass numbers agree with their experimental data within 1.5 % accuracy.

We have used the found Skyrme force variants to calculate differential cross sections and polarizations for the neutron and proton scattering on other target nuclei in a wide mass number range and at other nucleon energy values. In Fig. 2 we present the differential cross sections and analyzing powers of the elastic neutron scattering on different nuclei at neutron energy of 14 MeV calculated with the SkOP1 forces together with the experimental data. The figure shows that the calculations provide a satisfactory agreement with the data. The results obtained with the SkOP2 forces are practically the same and we do not show them. The results of analogous calculations for the elastic proton-nucleus scattering at 16 MeV are shown in Fig. 3. Although we have not involved *pA*-scattering data in the fitting procedure when determining the Skyrme force parameters, nevertheless the calculations with these forces yield quite encouraging description of such data.

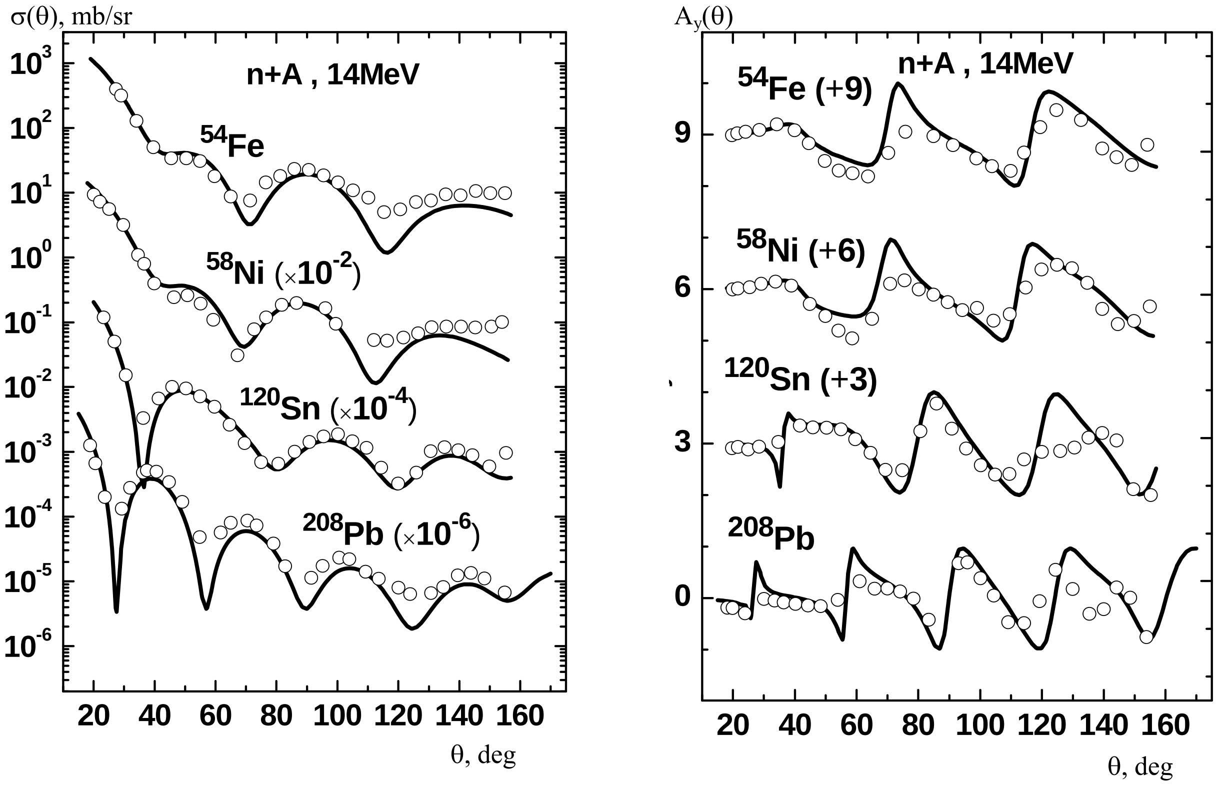


Fig. 2. Differential cross sections and analyzing powers of the elastic *nA*-scattering at 14 MeV for the SkOP1 forces. The numbers in parentheses are the offset values.

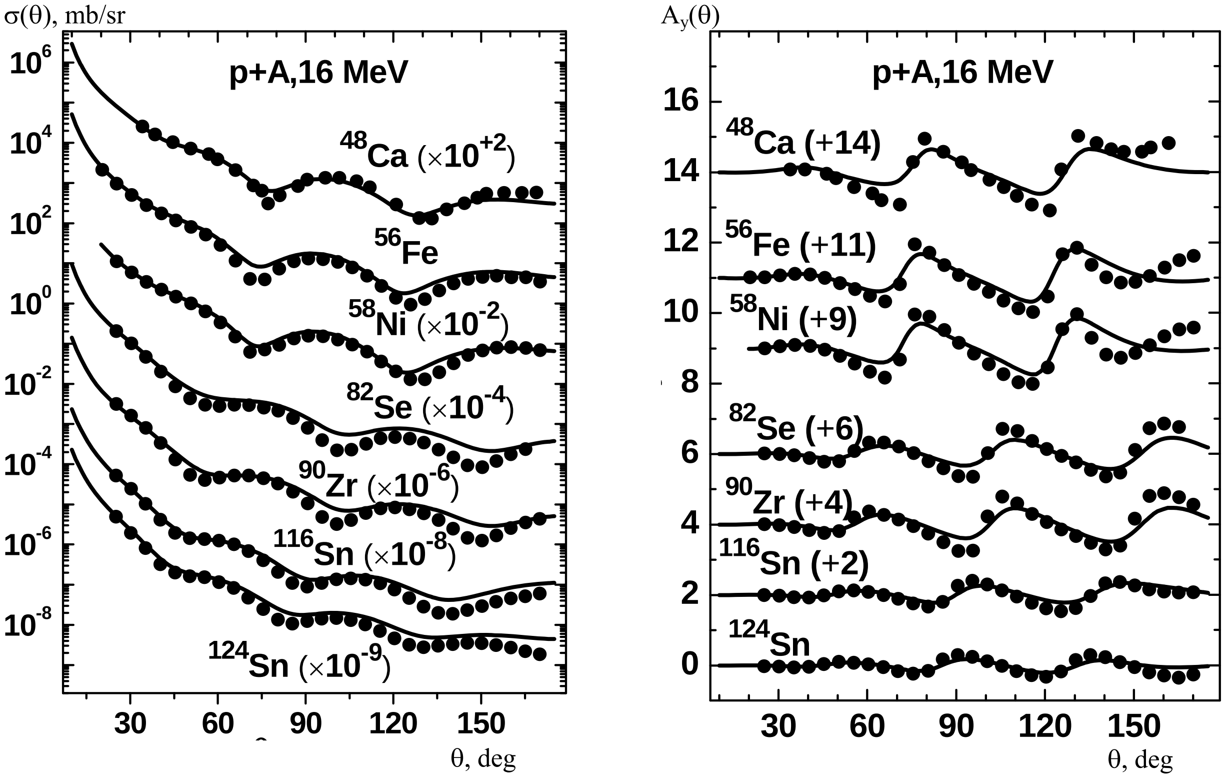
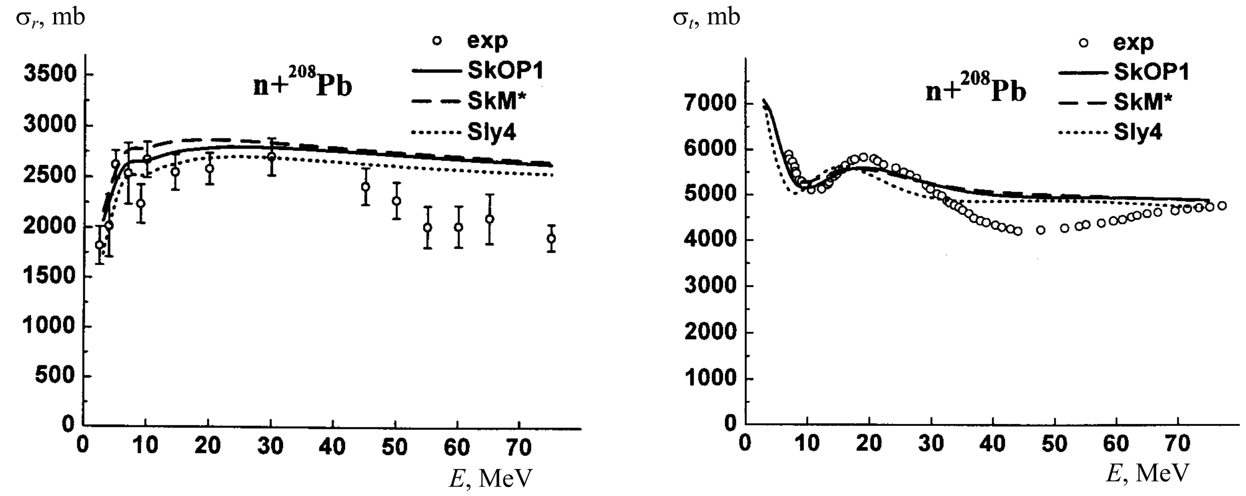


Fig. 3. Differential cross sections and analyzing powers of the elastic *pA*-scattering at 16 MeV for the SkOP1 forces. The numbers in parentheses are the offset values.

We have also applied the developed model of MOP to calculations of the reaction and total interaction cross sections that are important integrated characteristics of the *NA*-scattering. By way of example, the calculated energy dependences of these cross sections for the n+^{208}Pb scattering are displayed in Fig. 4 for the known forces Sly4 and SkM* and the new forces SkOP1. The model provides a reasonable description of the experimental reaction cross sections in the energy region



a *b*

Fig. 4. Reaction cross section (*a*) and total cross section (*b*) for the neutron scattering on ^{208}Pb nuclei as functions of the neutron energy for the SkOP1, Sly4, and SkM* forces.

extending up to 30 MeV both with standard forces and with the forces obtained here.

The model under consideration reproduces qualitatively the energy behavior of the total cross section, the modification of Skyrme force parameters leading to a shift of the cross section oscillations toward higher energies, so that their position agrees better with that observed in the experiment. The calculations performed for other target nuclei give close results.

Conclusions

A model of the nucleon-nucleus MOP whose real and imaginary parts are determined from calculations of the mass operator of the one-particle Green function with taking account of the rearrangement potential has been applied to study possibilities of a simultaneous usage of the effective Skyrme *NN*-forces for describing both the structure of even-even nuclei and the cross sections and analyzing powers of the elastic *NA*-scattering. An optimization of the Skyrme force parameters has been employed, basing on the fitting of a differential cross section of elastic *nA*-scattering with control of the basic characteristics of nuclear matter and the binding energy and charge radius of the target nucleus. This has allowed us to find two sets of modified Skyrme parameters, that improve description of the fitted cross section and also reasonably describe differential cross sections and analyzing powers for other cases of neutron- and proton-nucleus scattering up to projectile energies of 15 - 20 MeV, the energy dependences of reaction cross sections and total cross sections of the *NA*-interaction, as well as binding energies and charge radii of the considered even-even nuclei.

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ОПТИЧНИЙ ПОТЕНЦІАЛ НА ОСНОВІ СИЛ СКІРМА ДЛЯ ОПИСУ ПРУЖНОГО НУКЛОН-ЯДЕРНОГО РОЗСІЯННЯ

В. В. Пилипенко, В. І. Купріков, О. П. Сознік

Модель мікроскопічного оптичного потенціалу, що базується на розрахунках масового оператора одночастинкової функції Гріна з використанням ефективних нуклон-нуклонних сил Скірма, застосовано для опису перерізів та аналізуючих здатностей пружного нуклон-ядерного розсіяння. Параметри сил Скірма було оптимізовано шляхом фітування вибраного кутового розподілу пружного нейтрон-ядерного розсіяння з одночасним контролем основних характеристик ядерної матерії та енергії зв’язку й середньоквадратичного зарядового радіуса ядра-мішені. Знайдені сили Скірма застосовано для аналізу диференціальних перерізів та аналізуючих здатностей пружного розсіяння нейтронів ядрами в широкому діапазоні масових чисел. Розрахунки дали задовільний опис експериментальних даних з пружного нейтрон-ядерного розсіяння та розумні значення основних параметрів для симетричної ядерної матерії та парно-парних ядер. Було виконано аналогічні розрахунки для опису експериментальних даних з пружного протон-ядерного розсіяння, що дало обнадійливі результати.

Ключові слова: функція Гріна, нуклон-нуклонні сили Скірма, пружне розсіяння нейтронів, аналізуюча здатність, пружне розсіяння протонів.

ОПТИЧЕСКИЙ ПОТЕНЦИАЛ НА ОСНОВЕ СИЛ СКИРМА ДЛЯ ОПИСАНИЯ УПРУГОГО НУКЛОН-ЯДЕРНОГО РАССЕЯНИЯ

В. В. Пилипенко, В. И. Куприков, А. П. Созник

Модель микроскопического оптического потенциала, основанная на расчетах массового оператора одночастичной функции Грина с использованием эффективных нуклон-нуклонных сил Скирма, применена для описания сечений и анализирующих способностей упругого нуклон-ядерного рассеяния. Параметры сил Скирма были оптимизированы путем фитирования выбранного углового распределения упругого нейтрон-ядерного рассеяния с одновременным контролем основных характеристик ядерной материи и энергии связи и среднеквадратичного зарядового радиуса ядра-мишени. Найденные силы Скирма применены для анализа дифференциальных сечений и анализирующих способностей упругого рассеяния нейтронов ядрами в широком диапазоне массовых чисел. Расчеты дали удовлетворительное описание экспериментальных данных по упругому нейтрон-ядерному рассеянию и разумные значения основных параметров для симметричной ядерной материи и четно-четных ядер. Были проведены аналогичные расчеты для описания экспериментальных данных по упругому протон-ядерному рассеянию, которые дали обнадеживающие результаты.

Ключевые слова: функция Грина, нуклон-нуклонные силы Скирма, упругое рассеяние нейтронов, анализирующая способность, упругое рассеяние протонов.

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