GIANT NEUTRON HALO IN NUCLEI BEYOND BETA-STABILITY LINE

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The radii of nucleon distribution and neutron skin in nuclei beyond the β -stability line are studied within the extended Thomas - Fermi approximation. We show that the growth of neutron skin in unstable nuclei does not obey the saturation condition because of the neutron coat. The neutron coat indicates the possibility of giant neutron halo which is growing with moving away from the beta-stability line. We demonstrate the presence of strong shell oscillations in the charge radius R_C and the relation of R_C to the isospin shift of neutron-proton chemical potentials $\Delta \lambda = \lambda_n - \lambda_p$ for nuclei beyond the beta-stability line at fixed value of mass number A.

Keywords: extended Thomas - Fermi approximation, Skyrme force, beta-stability line, giant neutron halo.

Introduction

In the vicinity of the beta-stability line, the average changes in binding energy E and nuclear radius R with nucleon content obey the saturation properties. The volume part E_{vol} of binding energy and the nuclear volume itself are proportional to the particle number A with $E_{vol} = -b_V A$ and $R = r_0 A^{1/3}$, where $b_{\nu} > 0$ and r_0 are constants. Both values of b_{ν} and r_0 depend, however, on the isotopic asymmetry parameter X = (N-Z)/(N+Z). This dependence comes from the difference in saturation bulk density, $\rho_0 \sim r_0^{-3}$, for nuclei with different values of X. The saturation density ρ_0 becomes smaller beyond the beta-stability line for neutron-rich nuclei where more neutrons are pushed off to form the "neutron coat". One can expect that the growth of neutron skin in neutron-rich nuclei violates the saturation property $R \sim A^{1/3}$ for the nuclear radius providing an existence of neutron halo (giant neutron halo) effect [1].

In this paper we study the deviation of neutron distribution from the saturation behavior in neutron-rich nuclei. We study the influence of spin-orbit and Coulomb forces on the neutron, $\sqrt{\langle r_n^2 \rangle}$, and proton, $\sqrt{\langle r_p^2 \rangle}$, root mean square radii as well as the relation of the shift $\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$ to the surface symmetry energy. We study the problems related to the nucleon redistribution within the surface region of the nucleus and, in particular, the neutron coat and the neutron excess for the nuclei beyond the beta-stability line.

We combine the extended Thomas-Fermi approximation (ETFA) and the direct variational method assuming that the proton and neutron distributions are sharp enough, i.e., that the corresponding densities $\rho_n(\mathbf{r})$ and $\rho_n(\mathbf{r})$ fall from

their bulk values to zero within a thin surface region. In our consideration, the thin-skinned densities $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ are generated by the profile functions which are eliminated by the requirement that the energy of the nucleus should be stationary with respect to variations of these profiles.

Direct variational approach

We will use the extended Thomas-Fermi approximation (ETFA) which is one of practical realization of general Hohenberg - Kohn theorem [2] on the unique functional relation between the ground state energy and the local density of particles for any fermion system. The total kinetic energy of the many-body fermion system is given by the semiclassical expression [3, 4] as follows

$$E_{kin}\{\rho_n, \rho_p\} \equiv E_{kin}\{\rho_q, \nabla \rho_q\} = \int d\mathbf{r} \ \varepsilon_{kin}[\rho_n(\mathbf{r}), \rho_p(\mathbf{r})],$$
(1)

where $\varepsilon_{kin}[\rho_n, \rho_p] = \varepsilon_{kin,n}[\rho_n] + \varepsilon_{kin,p}[\rho_p]$, and

$$\varepsilon_{kin,q}[\rho_q,\nabla\rho_q] = \frac{\hbar^2}{2m} \left[\frac{3}{5} (3\pi^2)^{2/3} \rho_q^{5/3} + \frac{1}{36} \frac{(\nabla\rho_q)^2}{\rho_q} + \frac{1}{3} \nabla^2 \rho_q \right].$$

Here ρ_q is the nucleon density with q = n for neutron and q = p for proton.

We will follow the concept of effective nucleonnucleon interaction using the Skyrme-type force. The total energy functional for charged nucleus is given by

$$E_{tot}\{\rho_q, \nabla \rho_q\} = E_{kin}\{\rho_q, \nabla \rho_q\} + E_{SK}\{\rho_q, \nabla \rho_q\} + E_C\{\rho_p\},$$
(2)

where $E_{SK}\{\rho_q, \nabla \rho_q\}$ is the potential energy of NN-interaction

$$E_{SK}\{\rho_q, \nabla \rho_q\} = \int d\mathbf{r} \ \varepsilon_{pot}[\rho_n(\mathbf{r}), \rho_p(\mathbf{r})], \qquad (3)$$

 $\varepsilon_{pot}[\rho_n(\mathbf{r}), \rho_p(\mathbf{r})]$ is the potential energy density and $E_C\{\rho_p\}$ is the Coulomb energy. The potential energy (3) also includes the energy of spin-orbit interaction. Considering the asymmetric nuclei with $X = (N-Z)/A \ll 1$, we will introduce the isotopic particle densities, namely the total density $\rho_+ = \rho_n + \rho_p$ and the neutron excess density $\rho_- = \rho_n - \rho_p$ with $\rho_- \ll \rho_+$. We apply the direct variational method [5] and assume the density profile functions $\rho_+(r)$ and $\rho_-(r)$ to be a power of the Fermi function as

$$\rho_{+}(r) = \rho_{0}f(r), \quad \rho_{-}(r) = \rho_{1}f(r) - \frac{1}{2}\rho_{0}\frac{df(r)}{dr}\Delta.$$
 (4)

Here, $f(r) = \left[1 + \exp\left[(r - R)/a\right]\right]^{-\eta}$, the values ρ_0 and ρ_1 are related to the bulk density, R is the nuclear radius, a is the diffuseness parameter and Δ is the parameter of neutron skin (see below). The profile functions $\rho_+(r)$ and $\rho_-(r)$ have to obey the condition of the neutron and proton number conservation. For the ground state of nucleus, the unknown parameters $\rho_{0,1}, R, a, \Delta, \eta$ and the total energy E_{tot} itself can be derived from the variational principle

$$\delta(E - \lambda_n N - \lambda_n Z) = 0, \qquad (5)$$

where the variation with respect to all possible small changes of $\rho_{0,1}$, R, a, Δ and η is assumed. The Lagrange multipliers λ_n and λ_p are the chemical potentials for neutrons and protons respectively, and both of them are fixed by the condition of particle number conservation.

As mentioned above, the parameter Δ in profile functions of Eq. (4) is related to the neutron skin. It can be easily seen from the derivation of the rms radii of the neutron and proton density distributions

$$\sqrt{\langle r_q^2 \rangle} = \sqrt{\int d\mathbf{r} \, r^2 \, \rho_q(r) / \int d\mathbf{r} \, \rho_q(r)} \ . \tag{6}$$

Using Eqs. (6) and (4) one obtains the size of the neutron skin

$$\sqrt{\left\langle r_{n}^{2}\right\rangle }-\sqrt{\left\langle r_{p}^{2}\right\rangle }pprox$$

$$\approx \sqrt{\frac{3}{5}} \frac{\Delta}{1 - X^2} \left(1 + \frac{\Delta}{2R} \frac{X}{1 - X^2} \right) + O\left(\left(\frac{a}{R} \right)^3 \right). \tag{7}$$

Note that the evaluation of the variational conditions leads to an additional dependence of the variational

parameters $\rho_{0,1}$, R, a, Δ and η on the external parameters A and X. The value of Δ disappears in symmetric nuclei at X=0 and depends slightly on the Skyrme force parametrization. In case of the SkM forces we have numerically calculated the dependence of Δ on X for A=120 and fitted it by the following formula

$$\Delta(X) \approx 0.90X + 1.47X^2$$
. (8)

The parameter Δ is also related to the number, N_S , of neutrons in surface region of the nucleus ("neutron coat"). Substituting Eqs. (4) into condition of the particle conservation and using the leptodermous expansion, we obtain for the neutron excess N-Z the following expression

$$N - Z \approx N_{\nu} + N_{\rm s} \,, \tag{9}$$

where

$$N_V = \frac{4\pi}{3} R^3 \left(1 + 3\kappa_0(\eta) \frac{a}{R} + 6\kappa_1(\eta) \frac{a^2}{R^2} \right) \rho_1,$$

$$N_S = 4\pi R^2 \left(1 + 2\kappa_0(\eta) \frac{a}{R} + 2\kappa_1(\eta) \frac{a^2}{R^2} \right) \frac{\rho_0}{2} \Delta,$$

and $\kappa_i(\eta)$ are the generalized Fermi integrals derived in Ref. [5]. The first term $N_{\nu} \sim R^3$ on the right hand side of Eq. (9) is due to redistribution of the neutron excess within the nuclear volume while the second one $N_S \sim R^2$ is the number of neutrons within neutron coat. In Fig. 1 we have plotted N_S for neutron-rich nuclei in the vicinity of Sn nucleus (Z=50) (solid line). Two additional lines show the influence of the spin-orbit and the Coulomb forces on the neutron coat N_S .

In general, the change of the radius R of nucleon distribution with the nucleon number A is caused by two factors. There is a simple geometrical change $R \propto A^{1/3}$. An additional change can occur due to the polarization effect (the bulk density distortion) with moving away from the beta-stability line. In particular, the size of neutron skin is sensitive to the symmetry and the Coulomb energies. We expand the total energy $E_{tot}(\rho_0, X)/A$ around the saturation density $\rho_{0,eq}$ and the isotopic asymmetry parameter X on the beta-stability line as

$$E_{tot}(\rho_0, X) / A = E_{tot}(\rho_{0,eq}, X^*) / A + \frac{K_A}{18\rho_{0,eq}^2} (\rho_0 - \rho_{0,eq})^2 + \frac{P_A}{\rho_{0,eq}^2} (X - X^*)^2 (\rho_0 - \rho_{0,eq}), \qquad (10)$$

where K_A is the incompressibility of finite nucleus

$$K_{A} = 9 \rho_{0,eq}^{2} \frac{\partial^{2} E_{tot}(\rho_{0}, X^{*}) / A}{\partial \rho_{0}^{2}} \bigg|_{A, \alpha = 0}$$
(11)

and P_A is the partial pressure related to the symmetry and the Coulomb energies

$$P_{A} = \rho_{0,eq}^{2} \frac{\partial}{\partial \rho_{0}} \left[b_{V,sym}(\rho_{0}) + b_{S,sym}(\rho_{0}) A^{-1/3} - \alpha_{C}(\rho_{0}) A^{2/3} \right]_{\rho_{0} = \rho_{0,eq}}, \tag{12}$$

where $b_{V,sym}(\rho_0)$ and $b_{S,sym}(\rho_0)$ are the volume and the surface symmetry coefficients and $\alpha_C(\rho_0) = 3e^2/20(4\pi\rho_0/3)^{V_3}$. As seen from Eq. (10), the deviation from the beta-stability line $(X \neq X^*)$ implies the change in the bulk density ρ_0 . The corresponding change is dependent on the incompressibility K_A and the partial pressure P_A . For an arbitrary fixed value of X, the equilibrium density $\rho_{0,X}$ is derived by the condition

$$\frac{\partial}{\partial \rho_0} E_{tot}(\rho_0, X) / A \Big|_{A, \rho_0 = \rho_{0, X}} = 0. \tag{13}$$

$$\begin{array}{c} 25 \\ --- \text{ No Coulomb} \\ \text{No SO interaction} \end{array}$$

Fig. 1. N_S for neutron-rich nuclei in the vicinity of Sn (Z = 50) (solid line). The dotted and dashed lines show the influence of the spin-orbit and the Coulomb forces on the neutron coat N_S . The arrow shows the position of Sn nucleus on the β -stability line.

Using Eqs. (10) and (13), we obtain the expression for the shift of bulk density (polarization effect) in the neutron rich nuclei

$$\rho_{0,X} = \rho_{0,eq} - 9 \frac{P_A}{K_A} (X - X^*)^2. \tag{14}$$

The equilibrium partial pressure P_A is positive and thereby $\rho_{0,X} < \rho_{0,eq}$, see also Refs. [6, 7]. We point out that in general the sign of the equilibrium partial pressure P_A depends on the Skyrme force parametrization and this fact can be used for the Skyrme force selection [8].

Radii of nucleon distributions and neutron skin

As noted above, the bulk density $\rho_{0,X}$ is smaller for neutron-rich nuclei, more neutrons should be pushed off to enrich the skin providing the polarization effect. Thus, the rms radius $\sqrt{\langle r_n^2 \rangle}$ of neutron distribution does not necessarily obey the saturation condition $\sqrt{\langle r_n^2 \rangle} \sim A^{1/3}$. As a consequence, the nuclei with significant excess of neutrons exhibit neutron skin, i.e., they are characterized by larger radii for the neutron than for proton distributions.

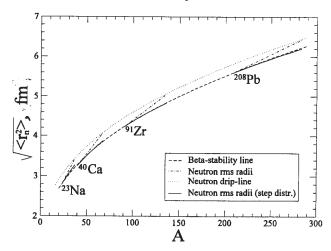


Fig. 2. The rms neutron radii beyond the beta-stability line for spherically symmetric nuclei ²³Na, ⁴⁰Ca, ⁹¹Zr and ²⁰⁸Pb.

The neutron coat N_s (see also Fig. 1) indicates the possibility of giant neutron halo which is growing with moving away from the beta-stability line [1]. In Fig. 2 we have plotted the rms radii of neutron distribution from Eq. (6) as a function of A. The deviation of $\sqrt{\langle r_n^2 \rangle}$ from the saturation behavior $\sim A^{1/3}$, obtained for the spherically symmetric nuclei, demonstrates the appearance of giant neutron halo when approaching the drip line. To extract a simple geometrical change of the radii we have made calculations with a step neutron distribution $\rho_n(r) = \rho_{0,n}\Theta(r-R_n)$, where the radius of the neutron distribution has saturation behavior $R_n = r_{0,n}A^{1/3}$. The results of the calculations are

shown in Fig. 2 by the solid lines. As one can see from Fig. 2 the solid lines are very close to the beta-stability line. The difference between the dash-dotted and solid lines gives the value of the polarization effect.

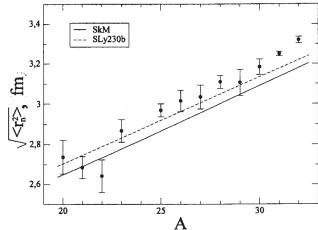


Fig. 3. The rms radius of neutron distribution in Na isotopes.

An occurrence of the giant halo in Na isotopes is shown in Fig. 3. We can see from this figure that the ETFA results agree quite well with the experimental data from [9]. The sensitivity of $\sqrt{\langle r_n^2 \rangle}$ calculation of to the choice of the Skyrme force can be also seen.

The results for the charge radius $\sqrt{\langle r_p^2 \rangle}$ are shown in Fig. 4. The rms radius $\sqrt{\langle r_p^2 \rangle}$ of proton distribution indicates the non-monotonic behavior due to the shell effects. Such behavior of $\sqrt{\langle r_p^2 \rangle}$ correlates with A-dependence of the Coulomb radius R_C . To derive R_C we use $E_{tot}(\rho_0, X)$ on the beta-stability line and establish an important relation for the chemical potentials λ_q (q=n for neutron and q=p for proton) beyond the beta-stability line. Namely, for fixed A, we obtain

$$\Delta \lambda(X) = \lambda_n - \lambda_p = \frac{\partial E}{\partial N} \Big|_{Z} - \frac{\partial E}{\partial Z} \Big|_{N} = 2 \frac{\partial (E/A)}{\partial X} \Big|_{A} =$$

$$= 4 \Big[b_{sym}^*(A) + e_C^*(A) \Big] (X - X^*), \qquad (15)$$

where $b_{sym}^*(A) = b_{V,sym}^*(\rho_{0,eq}) + b_{S,sym}^*(\rho_{0,eq})A^{-1/3}$,

$$e_C^*(A) = 0.15Ae^2 / R_C$$
, (16)

and

$$\lambda_n = \left(\frac{\partial E}{\partial N}\right)_Z, \quad \lambda_p = \left(\frac{\partial E}{\partial Z}\right)_N.$$
 (17)

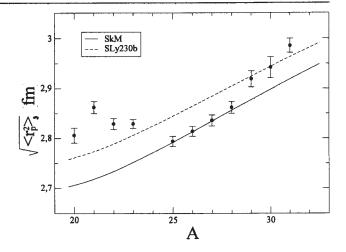


Fig. 4. The rms radius of proton distribution in Na isotopes.

On the beta-stability line, one has from Eq. (15) that $\Delta\lambda(X)_{X=X^*}=0$, as it has to be from the definition of the beta-stability line. We point out that for finite nuclei, the condition $\Delta\lambda=0$ on the beta-stability line is not necessarily fulfilled exactly because of the discrete spectrum of single particle levels for both the neutrons and the protons near Fermi surface. In agreement with Eq. (15), the slopes of straight lines $\Delta\lambda(X)$ allow us to derive the quantity $b_{sym}^*(A) + e_C^*(A)$. From the beta-stability condition $\Delta\lambda(X) = 0$ one can also derive the asymmetry parameter $X^*(A)$. Using the definition of the beta-stability line as

$$\frac{\partial E_{tot}(\rho_0, X) / A}{\partial X}\bigg|_{A X = X^*} = 0,$$

we obtain the symmetry energy coefficient $b_{sym}^*(A)$ and Coulomb energy parameter $e_C^*(A)$. Finally, using Eq. (16), we obtain the A-dependence of the Coulomb radius $R_C(A)$.

The quantity $\partial(E/A)/\partial X$ in Eq. (15) can be evaluated within the accuracy $\sim 1/A^2$ using the finite differences which are based on the experimental values of the binding energy per nucleon B(N,Z) = -E(N,Z)/A. Namely,

$$\left. \frac{\partial (E/A)}{\partial X} \right|_{A} = \frac{A}{4} \left[B(N-1, Z+1) - B(N+1, Z-1) \right].$$

(18)

Since the difference (18) is taken for $\Delta Z = -\Delta N = 2$, the pairing effects do not affect the resulting accuracy. Because of Eq. (16), this procedure allows us to derive the "experimental" value of the Coulomb radius $R_C(A)$. The A-dependence of the

"experimental" Coulomb radius $R_C^*(A)$, extracted from the experimental data [10], along the beta-stability line is presented in Fig. 5. The deviation of the Coulomb radius $R_C^*(A)$ from the smooth A-dependence is mainly due to the shell oscillations. We point out that the shell oscillations in $R_C(A)$ are correlated with the ones in A-dependence of the symmetry energy [11].

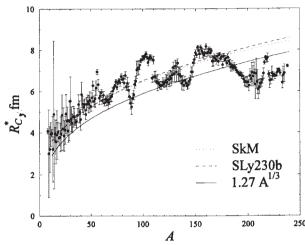


Fig. 5. The A-dependence of "experimental" Coulomb radius extracted from the experimental data [10].

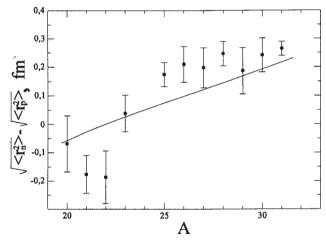


Fig. 6 Isovector shift of nuclear radius for Na isotopes.

The size of the neutron skin $\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$ is illustrated in Fig. 6. The line has been obtained from Eq. (7), and the experimental data were taken from Ref. [9]. As seen from Fig. 6, the skin size

 $\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$ is primarily linear with the asymmetry parameter X.

Summary

We have applied the direct variational method within the extended Thomas-Fermi approximation with effective Skyrme-like forces to the description of the radii of nucleon distributions. In our consideration, the thin-skinned nucleon densities $\rho_{p}(\mathbf{r})$ and $\rho_{n}(\mathbf{r})$ are generated by the profile functions which are eliminated by the requirement for the energy of the nucleus to be stationary with respect to variations of these profiles. The advantage of the direct variational method is the possibility to derive the equation of state for finite nuclei: dependence of the binding energy per particle or the pressure on the bulk density ρ_0 . We have evaluated the partial pressure P_A which includes the contributions from the symmetry and the Coulomb energies. The pressure P_A is positive driving off the neutrons in neutron-rich nuclei to the skin.

Using the leptodermous properties of the nucleon densities $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$, we have established the possibility of giant neutron halo in neutron-rich nuclei. The effect of giant halo increases with moving away from the beta-stability line. In Fig. 2 this fact is demonstrated as deviation of the rms radii of neutron distribution from the saturation behavior $\sim A^{1/3}$ in the nuclei beyond the beta-stability line.

The average behavior of the rms radii of nucleon distributions $\sqrt{\langle r_q^2 \rangle}$ and the size of neutron skin are satisfactorily described within the extended Thomas-Fermi approximation. The sensitivity of the calculations of $\sqrt{\left\langle r_q^2 \right\rangle}$ to force parametrization can be used for Skyrme force selection. The charge radius of proton distribution shows the strong shell oscillations with mass number. We demonstrated the relation of the Coulomb radius to the isospin shift of the neutron-proton chemical potentials $\Delta \lambda = \lambda_n - \lambda_p$ for nuclei beyond the betastability line at fixed value of mass number A.

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ГІГАНТСЬКЕ НЕЙТРОННЕ ГАЛО В ЯДРАХ ПОЗА ЛІНІЄЮ БЕТА-СТАБІЛЬНОСТІ

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У рамках розширеного наближення Томаса - Фермі досліджено радіуси нуклонного розподілу та нейтронної шкіри в ядрах поза лінією бета-стабільності. Показано, що внаслідок наявності нейтронної шуби зростання нейтронної шкіри в нестабільних ядрах не задовольняє умові насичення. Нейтронна шуба вказує на можливість існування гігантського нейтронного гало в ядрах, яке зростає при віддалені від лінії бета-стабільності. Продемонстровано наявність сильних оболонкових осциляцій зарядового радіуса R_C та зв'язок R_C з ізоспіновим зсувом нейтронно-протонних хімічних потенціалів $\Delta \lambda = \lambda_n - \lambda_p$ для ядер поза лінією бета-стабільності з фіксованим масовим числом A.

Ключові слова: розширене наближення Томаса - Фермі, сили Скірма, лінія бета-стабільності, гігантське нейтронне гало.

ГИГАНТСКОЕ НЕЙТРОННОЕ ГАЛО В ЯДРАХ ВДАЛИ ОТ ЛИНИИ БЕТА-СТАБИЛЬНОСТИ

В. М. Коломиец, С. В. Лукьянов, А. И. Санжур

В рамках расширенного приближения Томаса - Ферми исследованы радиусы нуклонного распределения и нейтронной кожи в ядрах вдали от линии бета-стабильности. Показано, что вследствие существования нейтронной шубы возростание нейтронной кожи в нестабильных ядрах не удовлетворяет условию насыщения. Нейтронная шуба указывает на возможность существования гигантского гало в ядрах, которое увеличивается при удалении от линии бета-стабильности. Продемонстрировано существование сильных оболочечных осцилляций зарядового радиуса R_C и связь R_C с изоспиновым сдвигом нейтронно-протонных химических потенциалов $\Delta \lambda = \lambda_n - \lambda_p$ для ядер вдали от линии бета-стабильности с фиксированным масовым числом A.

Ключевые слова: расширенное приближение Томаса - Ферми, силы Скирма, линия бета-стабильности, гигантское нейтронное гало.

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