ЯДЕРНА ФІЗИКА ТА ЕНЕРГЕТИКА 2010, т. 11, № 3, с. 239 - 245

УДК 539.14

THE EXCITATION OF AN INDEPENDENT-PARTICLE GAS BY A TIME DEPENDENT POTENTIAL WELL

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The order-to-chaos transition in the dynamics of independent classical particles gas was studied by means of the numerical simulations. The excitation of the gas for containers whose surfaces are rippled according to Legendre polynomials P_2, P_3, P_4, P_5, P_6 was followed for ten periods of oscillations. Spheroidal deformations were also considered. Poincare sections and Lyapunov exponents have been calculated showing different degrees of chaoticity depending on the shape and amplitude of oscillations. For P_2 polynomial the reaction of a gas to the periodic container deformation is mostly elastic as P_2 deformation especially for not very big deformations is almost like an integrable spheroid. For other polynomials the situation is more or less chaotic with a chaoticity increasing with the increasing order of the polynomial.

Keywords: one-body dissipation, order-to-chaos transitions, nuclear collective dynamics.

Introduction

In recent years it became apparent that the collective nuclear dynamics is very much related to the nature of the nucleonic motion. If the nucleonic motion is ordered the nucleus behaves like an elastic solid whereas when it is chaotic the nucleus behaves like a viscous fluid. The behaviour of the nucleus is important in physical processes like fission or heavy ion collisions where a great amount of the collective energy is dissipated into a chaotic nucleonic motion.

In this paper we study the excitation of a classical gas of independent particles in a container changing its shape periodically. These studies are continuation of studies where the excitation of the classical and quantal gas were reported earlier [1 - 4]. In paper [3] a systematic comparison was carried out between quantal and classical computer simulations of the excitation of independent particles in a time-dependent potential well undergoing one cycle of the oscillation. The results for five Legendre polynomial deformations P_2, P_3, P_4, P_5, P_6 were compared with the one body dissipation model in the form of the wall formula [5]. These comparisons show that the wall formula often reproduces the overall trend of the numerical simulations but drastic deviations especially at low deformation speeds are present. We introduce, as before [2] the adiabaticity parameter η being the ratio of the biggest wall speed to the biggest speed of particles:

$$\eta = \alpha\omega R_0 / v_F , \qquad (1)$$

where α is the relative amplitude and ω is the frequency for surface vibrations, R_0 is the equilibrium radius parameter, v_F is the Fermi velocity of particles (all notations are specified in more details below). If one considers in classical calculations low deformation speed corresponding to the adiabaticity parameter [3] equal for instance to $\eta = 0.02$ then for P_2 type of vibrations around sphere with an amplitude $\alpha = 0.2$ one gets within one period an average number of particle collisions with the wall equal to about 36. The same number for $\eta = 0.6$ is equal to 1.3. For $\eta = 0.6$ and $\alpha = 0.2$ the angular velocity (in units where Fermi velocity $v_F = 1$ and radius of the nucleus $R_0 = 1$) is equal to 3. So the time period is about 2 which means that it corresponds to the time of the fastest particles to cross the nucleus. In such a case within one period particles do not have enough time to recognize what is the shape of the container and therefore independently on the shape the excitation of the gas will be close to that given by the wall formula.

The situation is different when one goes to longer time evolution like ten periods of the oscillation. Then the average number of particle collisions with the wall in the same situation (P_2 deformation, $\alpha = 0.2$, $\eta = 0.6$) is equal to almost 18 and that means that the gas is feeling the shape of the nucleus it is moving in. Therefore for P_2 deformation which is almost integrable the excitation of the gas after 10 oscillations is more than three times smaller than the one obtained from the wall formula.

In all the cases where there is enough number of particle collisions with the wall (more than about ten) the ratio between the wall formula excitation and the one obtained in numerical simulations equals to $3-4$. The same ratio for the P_3 deformation is equal to $1.2-1.6$ and for P_6 deformation which is almost completely chaotic it is $0.94-1.14$ and this

means that in the wall formula there should be a degree of chaoticity of a gas for a given shape taken into account [6]. We emphasize that the friction coefficient is determined through the average of the energy rate $< dE / dt >$ proportional to the change of the energy ΔE for a large enough time Δt. Therefore, the energy change ΔE for a larger interval Δt of the time averaging measures more precisely the friction coefficient.

The numerical simulations

In the present paper we report calculations concerning the excitation of a gas of classical particles undergoing ten cycles of the oscillations around a deformed shape. The container is a sharp-walled cavity of the infinite depth and the shape of the cavity is defined by a time-dependent radius $R(\theta,t)$ given by:

$$R(\theta,t) = R_0\left[1+\alpha_n(t)P_n\left(cos\theta\right)+\alpha_1(t)P_1\left(cos\theta\right)\right]/\lambda(t), \tag{2}$$

where $\lambda(t)$ is a normalization factor ensuring volume conservation and $\alpha_1(t)$ ensures a fixed position of the center of mass for odd multipolarities n. $P_n\left(cos\theta\right)$ are Legendre polynomials and $\alpha_n(t)$ is a periodic function of time:

$$\alpha_n(t) = \alpha_{st} + \alpha_n^0 cos\left(\omega t\right), \tag{3}$$

where α_{st} is a static deformation around which the system vibrates and α_n^0 is an amplitude of vibrations. In addition to five Legendre polynomials: P_2, P_3, P_4, P_5, P_6 we considered also a cavity whose surface is given by:

$$\left(x^2+y^2\right)/a^2(t)+z^2/c^2(t)=1, \tag{4}$$

where $a(t) = R_0\left[1+\alpha_{st}+\alpha cos\left(\omega t\right)\right]^{-1/2}$ and $c(t) = R_0\left[1+\alpha_{st}+\alpha cos\left(\omega t\right)\right]$.

We start our oscillations from a maximum displacement of the P_n deformation equal to $\alpha_{st}+\alpha_n^0$. The amplitude of the oscillations α_n^0 is taken as $\alpha\sqrt{(2n+1)/5}$ and this ensures that for small α the r.m.s. deviation of the surface from the sphere is the same for all multipolarities n. We introduce, as before [2] the adiabaticity parameter η being the ratio of the biggest wall speed to the biggest speed of particles:

$$\eta = \alpha\omega R_0 / v_F . \tag{5}$$

The equation to the improved wall formula for the relative-energy increase of the gas was derived in [7]:

$$\Delta E / E_0 = C\left(I + 3I^2/8\right), \tag{6}$$

where C is given in terms of the first two moments of the initial velocity distribution: $C = 2\overline{v_0}^2 / \overline{v_0^2}$. The bar means averaging in the phase space. For the initial Fermi-gas velocity distribution one has $\overline{v_0} = 3v_F/4$ and $\overline{v_0^2} = 3{v_F}^2/5$. In Eq. (6), I is an integral over time and over the surface of the squares of normal velocities of the surface elements dS:

$$I(t) = \left[1/\left(\overline{v_0}V\right)\right]\int_0^t dt \oint \dot{n}^2 dS, \tag{7}$$

where V is the volume of the container. The integral, Eq. (7), has to be in general evaluated numerically.

For small amplitude vibrations around the sphere one can use an approximate expression [1]:

$$\Delta E / E_0 = \tau + \tau^2/5, \tag{8}$$

where $\tau = (3/4)\alpha\eta\left[\omega t - (1/2)sin(2\omega t)\right]$.

In Fig. 1 the relative increase of the energy of the gas of particles during ten periods of oscillations at $\alpha_{st}=0$ and an amplitude $\alpha=0.1$ is presented. The upper two rows correspond to an adiabaticity parameter $\eta=0.1$ and lower two rows to $\eta=0.6$. The corresponding vibrating shapes are indicated in each picture. For the adiabaticity parameter $\eta=0.1$ only the P_5 and P_6 vibrations behave in a dissipative way following very nicely the wall formula (dashed curves). For other shapes the excitation energy follows the wall formula for no more than one period of oscillations. This can be understood on the basis of the number of particles collisions with the wall. For these conditions the average number of collisions during ten periods of oscillations is equal to 35 which means that for one period it is 3.5. In such a situation the gas during one period is not able to recognize the shape it is moving in and to build correlations between consecutive collisions.

In two lower lines of Fig. 1, one can see a strong non-adiabatic effect in comparison with upper plots. For the adiabaticity parameter $\eta=0.6$, the average number of the particles collisions with the wall during ten periods of oscillations is equal to about 7. It means that the same number of collisions during one period at $\eta=0.1$ will be reached now in five periods. This is the case for spheroidal (SPH) and

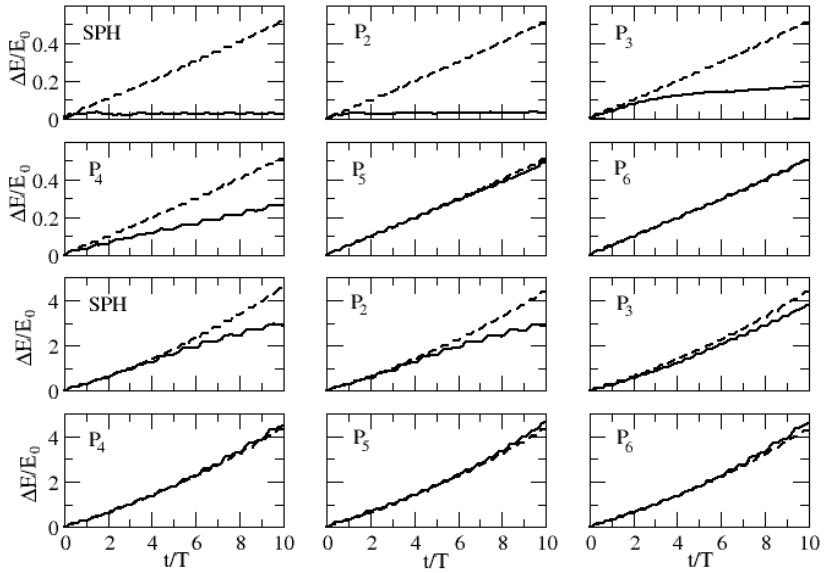

Fig. 1. Relative excitation of a gas of particles ΔE with respect to the initial energy of the gas E_0 during ten periods of oscillations around a spherical shape for six different shape deformations. The amplitude of the vibrations is $\alpha = 0.1$ and the adiabaticity parameter $\eta = 0.1$ (upper two rows) and $\eta = 0.6$ (lower two rows). Solid lines indicate results of the computer simulations and dashed ones are the wall formula predictions.

P_2 vibrations where calculations up to five periods follow pretty well the wall formula. For shapes from P_3 up to P_6 the gas behaves in a dissipative way.

In Fig. 2 the same is plotted but for $\alpha_{st} = 0.3$ which means that now we are looking for oscillations around the deformed shape. The picture is pretty much the same as presented in Fig. 1. Within this time, P_3 and P_4 at $\eta = 0.1$ behave however in a more dissipative way and the situation with spheroid and P_2 at $\eta = 0.6$ is somewhat different.

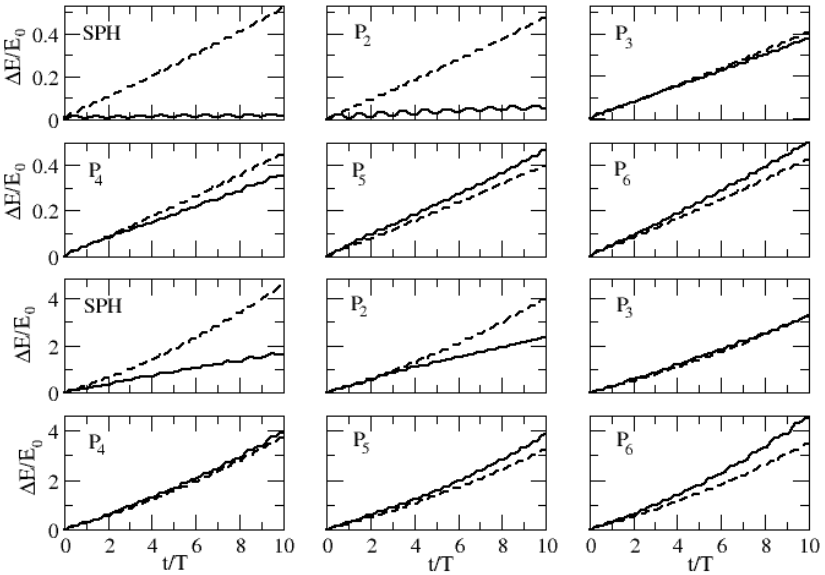

Fig. 2. The same as in Fig. 1 but for the vibrations around a deformed shape $\alpha_{st} = 0.3$.

In Fig. 3 the excitation of the gas of particles at very slow motion of walls ($\eta = 0.02$) is plotted. In upper two rows a vibration around a sphere ($\alpha_{st} = 0$) and in lower two rows a vibration around a deformed shape ($\alpha_{st} = 0.3$) are shown. The picture again is pretty much the same as for bigger adiabaticity parameters $\eta = 0.1$ (see Figs. 1 and 2). With decreasing α at $\alpha_{st} = 0$ one obtains more ordering for each given multipolarity n.

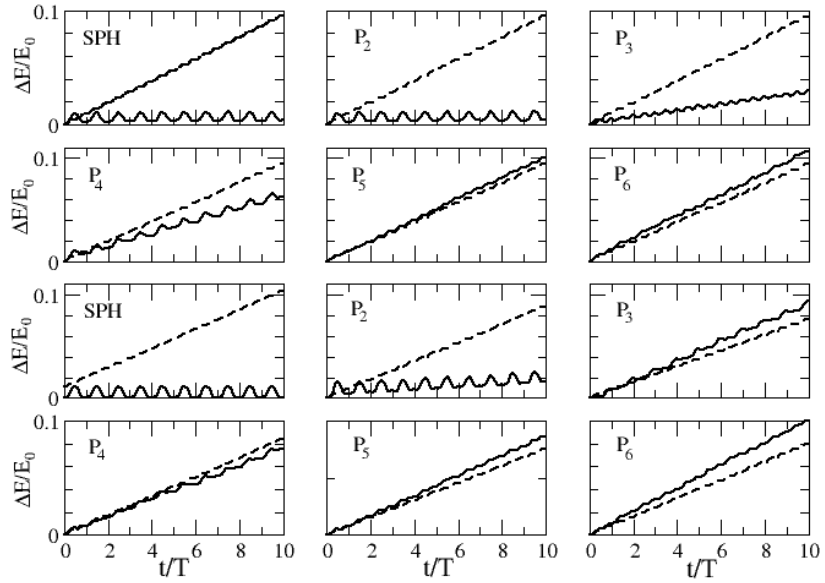

Fig. 3. Relative excitation of a gas of particles ΔE with respect to the initial energy of the gas E_0 during ten periods of oscillations around a spherical shape (two upper rows) and around a deformed shape $\alpha_{st} = 0.3$ (two lower rows) for six different shape deformations. The amplitude of the vibrations is $\alpha = 0.1$ and the adiabaticity parameter $\eta = 0.02$. Solid lines indicate results of the computer simulations and dashed ones are the wall formula predictions.

Poincare sections

In order to study the degree of chaoticity of particles in different containers we first calculate Poincare sections [2]. The Poincare sections are generated in the following way: we split our shape in the middle by an equatorial plane and every time the trajectory crosses this plane we notice the distance ρ from the symmetry axis and the corresponding component of the velocity v_ρ.

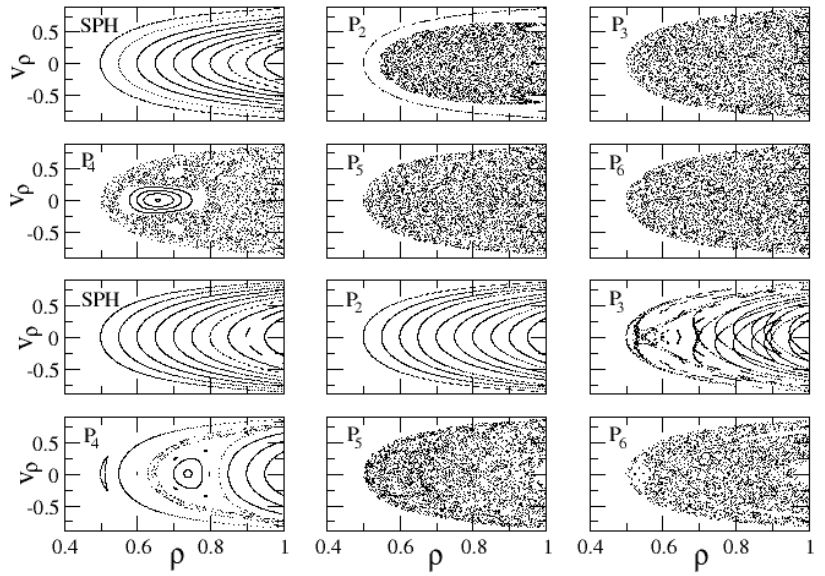

Fig. 4. Poincare sections for six shapes under the consideration at the deformation $\alpha = 0.05$ (two lower rows) and deformation $\alpha = 0.4$ (two upper rows). The Poincare sections are for projections of the angular momentum on the symmetry axis K with respect to the maximal ones equal to 0.5.

In this way we get points in two-dimensional phase space: (velocity v_ρ, position ρ). If the motion is integrable the particle in the phase space moves on a torus which intersecting with the equatorial plane will give a regular curve in (ρ, v_ρ) space. On the other hand if the motion is chaotic, the points will fill the whole space (ρ, v_ρ) in an irregular way.

In Fig. 4 the Poincare sections for six shapes under the consideration are presented. These sections are

presented for a mean projection of the angular momentum on the symmetry axis equal to 0.5 of the maximal projection, which in units we use ($R_0 = 1, v_F = 1$) is equal to ρ_{max}. In each picture there are ten initial conditions considered corresponding to the initial ρ value spread equally between $\rho_{min} = 0.5$ and ρ_{max} corresponding to the shape boundary at the equatorial plane. The sections are presented for the small deformation $\alpha = 0.05$ which corresponds to the mean value for the oscillations around sphere with the vibration amplitude equal 0.1 (two lower rows). The two upper rows correspond to the deformation $\alpha = 0.4$. For lower value of the deformation $\alpha = 0.05$ the spheroid and P_2 shapes are fully ordered and P_5 and P_6 shapes are fully chaotic. For the P_3 and P_4 shapes the situation is somewhere in between. It confirms the fact that for $\eta = 0.1$ (see Fig. 1) the excitation for P_3 and P_4 vibrations is much smaller than the one obtained with the wall formula whereas for P_5 and P_6 vibrations the calculated excitations follow very nicely the wall formula. With decreasing the deformation α, the Poincare sections for all multipolarities n tend to those of the spherical shape.

For bigger deformation the situation is ordered only for spheroid and all other shapes are chaotic. However, surprisingly the P_4 shape shows a sign of order. For increasing K one finds more ordering for each Legendre polynomial degree n because of diminishing of the phase space volume accessible for classical motion.

Lyapunov exponents

For more quantitative studies of the chaotic behaviour the method of Lyapunov exponents Λ have been used [8]. In this method one looks for an exponential divergence in time for two trajectories with nearly the same initial conditions. For N-dimensional phase space there are N maximal Lyapunov exponents Λ out of which some of them can be positive or negative. In the case of regular motion all of the Lyapunov exponents Λ are equal to zero.

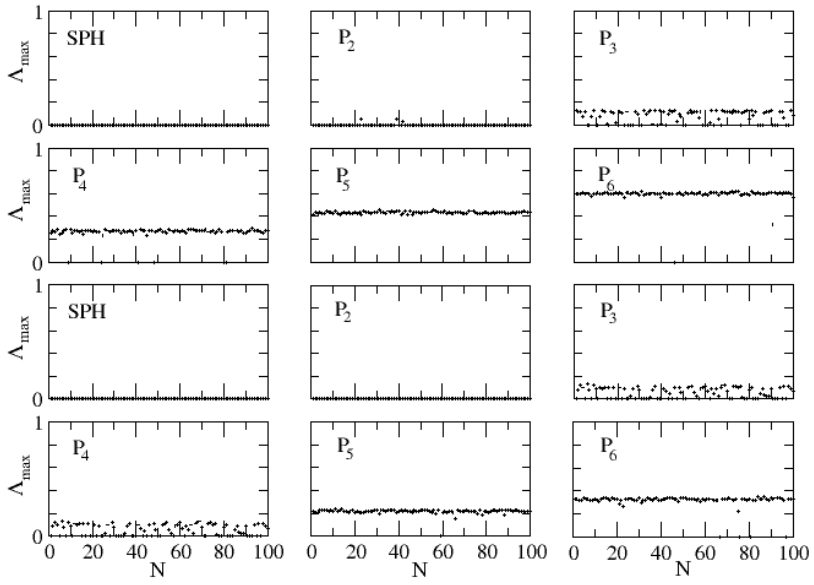

Fig. 5. Lyapunov exponents Λ_{max} for six shapes under the consideration at deformation $\alpha = 0.05$ (lower two rows) and deformation $\alpha = 0.4$ (upper two rows). The projection of the angular momentum on the symmetry axis $K = 0$.

Once the positive Lyapunov exponents Λ_{max} are presented in Figs. 5 and 6 as their largest values one controls the exponential instability leading to a chaos. In Fig. 5 these maximal Lyapunov exponents are plotted for the six shapes under the consideration at the deformation $\alpha = 0.05$ (lower two rows) and deformation $\alpha = 0.4$ (upper two rows). All the Lyapunov exponents are calculated for particles with the projection of the angular momentum $K = 0$ on the symmetry axis. For the deformation $\alpha = 0.05$ the situation is very much ordered (almost all the Lyapunov exponents are equal to zero) and this confirms what one can see in Poincare sections (see Fig. 4). In the P_3 and P_4 shapes at this deformation one has partially ordered and partially chaotic (some of the Lyapunov exponents are equal to zero and others have a finite positive value) and this is also confirmed by the Poincare sections (see Fig. 4). In the P_5 and P_6 shapes the dynamics seems to be fully chaotic (all the Lyapunov exponents have some finite positive values). This is also presented by the Poincare sections. For the deformation $\alpha = 0.4$ the

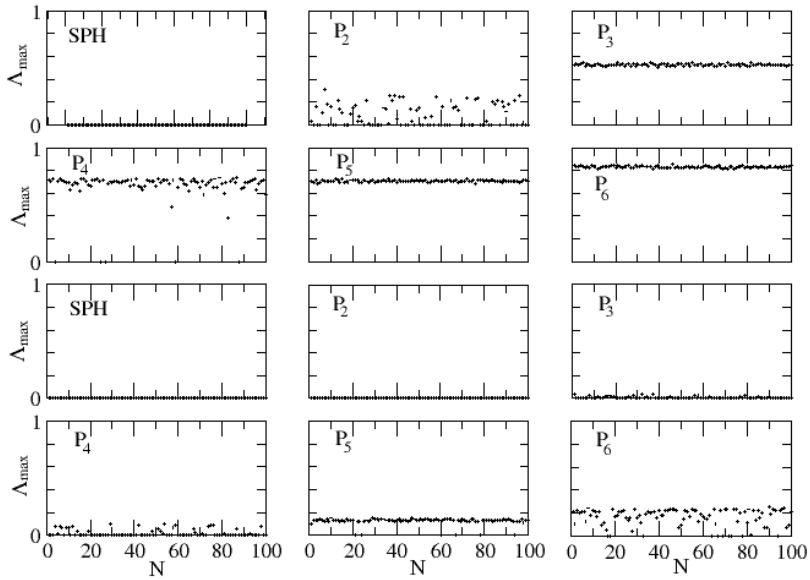

Fig. 6. The same as in Fig. 5 but for the projection of the angular momentum on the symmetry axis $K = 0.5$ of the maximal projection.

situation only in the spheroidal (SPH) shape is ordered, i.e. all the Lyapunov exponents are vanishing. For all other shapes, one finds a more chaotic behavior, and it is even chaotic for P_4 shape where Poincare sections show some trace of chaos. However situations presented in Figs. 4 and 5 are not strictly comparable as they refer to the two different projections of the angular momentum on the symmetry axis $K = 0$ and $K = 0.5$. The maximal Lyapunov exponents are the smaller the larger K for the given multipolarity n in line of the features of the Poincare sections.

When one goes to the Lyapunov exponents calculated for particles with $K = 0.5$ (Fig. 6) then all the conclusions from above remain valid except that now in P_4 shape at the deformation $\alpha = 0.4$ the situation is partly chaotic and partly ordered which confirms what one can see in the Poincare sections (see Fig. 4).

Conclusions

We have extended the earlier research [3] to longer time evolutions (ten periods of oscillations) which allow to have enough number of the particle collisions with the walls and in this way to recognize the shape of the container for the gas it is moving in. When one goes to a longer time evolution, and oscillation beyond a small amplitude around the sphere one should be careful in applying a simplified wall formula [Eq. (11) in [1]] calculating the proper integral (7). Unfortunately, one is losing a general simplicity which under the proper rescaling of the amplitudes of vibrations gives the same energy dissipated for all shapes considered.

Looking at Figs. 1, 2 and 3 one could say that the wall formula is reproducing the computer simulations pretty well in the wide range of the wall speeds and especially for higher multipolarities. This agreement is the better the larger multipolarity for the same all other parameters. The correlation between the dissipative or elastic behaviour of the system and the degree of the chaoticity is well visible when one looks into the Poincare sections and Lyapunov exponents.

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ЗБУДЖЕННЯ ГАЗУ НЕЗАЛЕЖНИХ ЧАСТИНОК ЗАЛЕЖНОЮ ВІД ЧАСУ ПОТЕНЦІАЛЬНОЮ ЯМОЮ

Я. П. Блоцький, О. Г. Магнер, І. С. Яцишин

Перехід “порядок - хаос” у динаміці незалежних класичних частинок газу вивчався за допомогою чисельного моделювання. Збудження газу для комірок, поверхня яких деформувалась за поліноміальною залежністю (поліноми Лежандра P_2, P_3, P_4, P_5, P_6), спостерігались протягом десяти періодів коливань. Розглянуто також сфероїдальні деформації. Обчислені перерізи Пуанкаре та експоненти Ляпунова показали різні ступені хаотичності, що залежать від форми поверхні та амплітуди коливань. Для P_2 поліноміальної залежності реакція газу на періодичні деформації комірки є найбільш пружною, тому що P_2 деформація при дуже малих деформаціях майже така ж сама, як і для інтегрованого сфероїда. Для інших поліномів ситуація є хаотичною з хаотичністю, що зростає зі збільшенням порядку полінома.

Ключові слова: однотільна дисипація, переходи “порядок - хаос”, колективна ядерна динаміка.

ВОЗБУЖДЕНИЕ ГАЗА НЕЗАВИСИМЫХ ЧАСТИЦ ЗАВИСЯЩЕЙ ОТ ВРЕМЕНИ ПОТЕНЦИАЛЬНОЙ ЯМОЙ

Я. П. Блоцкий, А. Г. Магнер, И. С. Яцышин

Переход “порядок - хаос” в динамике независимых классических частиц газа изучался с помощью численного моделирования. Возбуждения газа для ячеек, поверхность которых деформировалась по полиномиальной зависимости (полиномы Лежандра P_2, P_3, P_4, P_5, P_6), наблюдались на протяжении десяти периодов колебаний. Рассмотрены также сфероидальные деформации. Вычисленные сечения Пуанкаре и экспоненты Ляпунова показали различные степени хаотичности, которые зависят от формы поверхности и амплитуды колебаний. Для P_2 полиномиальной зависимости реакция газа на периодические деформации ячейки является наиболее упругой, потому что P_2 деформация при очень малых деформациях такая же, как и для интегрированного сфероида. Для других полиномов ситуация является хаотической с хаотичностью, которая возрастает с увеличением порядка полинома.

Ключевые слова: однотельная диссипация, переходы “порядок - хаос”, коллективная ядерная динамика.

Received 07.06. 10,
revised - 28.10.10.