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FUSION OF DEFORMED NUCLEI

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Potentials between two axial-symmetric deformed (prolate-prolate, prolate-oblate, oblate-oblate) nuclei are discussed in detail. The fusion reaction cross-sections are evaluated in the approach, which takes into account deformations of both nuclei and various orientations of colliding nuclei. The evaluated fusion cross-sections agree well with available experimental data.

1. Introduction

The ground state of nuclei is characterized by shape, which can be spherical or deformed. Nuclei of various shapes are used in collision experiments. The shapes of colliding nuclei are very important for the entrance-barrier height and other properties of a nuclear reaction [1]. It has been shown both experimentally and theoretically that the sub-barrier fusion of spherical and well-deformed nuclei in the ground state is strongly enhanced by deformation [1, 2].

The nucleus-nucleus potential between two deformed nuclei is evaluated in linear approximations on deformations values as a rule [3]. Simple expression for the potential between two axial-symmetric deformed arbitrary-oriented nuclei has been proposed recently [1], which takes into account linear and quadratic terms on quadrupole deformations of both nuclei. Therefore it is interesting to discuss the role of quadratic terms on quadrupole deformations on the fusion cross section around barrier induced by deformed nuclei. We discuss our approximation to the nucleus-nucleus potential in Sec. 2 and present results for the fusion reaction cross sections in Sec. 3.

2. Nucleus-nucleus potential

The interaction potential between nuclei consists of the Coulomb $V_C(R,\Theta_1,\Theta_2,\Phi)$, nuclear $V_N(R,\Theta_1,\Theta_2,\Phi)$ and centrifugal parts. When centrifugal part is absent, then the interaction potential is

$$V(R,\Theta_1,\Theta_2,\Phi) = V_C(R,\Theta_1,\Theta_2,\Phi) +$$

$$+ \eta V_N(R,\Theta_1,\Theta_2,\Phi),$$

where R is the distance between of their mass-centers and Θ_1,Θ_2 and Φ the angles, which define the relative orientation of two axial-symmetric, deformed nuclei, see Fig. 1. We introduce factor η, which determines the strength of nuclear interaction part. This factor will be used for fitting cross section data in the next section. Note that $\eta = 1$ usually. Shape of deformed nuclei in the intrinsic coordinate system is described as $R_i(\vartheta_i') = R_{i0}\left[1 + \sum_{l \geq 2} \beta_{il} Y_{l0}(\vartheta_i')\right]$, where β_{il} is the parameter of l-pole surface deformation of nuclei i ($i = 1, 2$) and R_{i0} is the radius parameter.
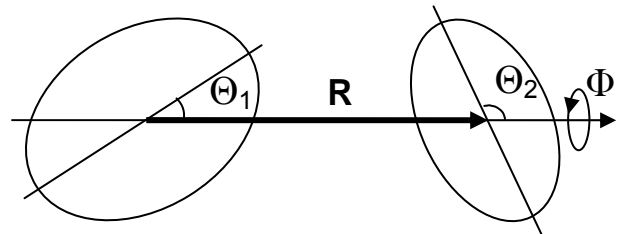

Fig. 1. The angles Θ_1,Θ_2 and Φ, and distance between of mass-centers R describing the arbitrary orientation of colliding nuclei.

The Coulomb interaction energy is related to a six-dimensional integral. We evaluate this integral and obtain [1]

$$V_C(R,\Theta_1,\Theta_2,\Phi) = \frac{Z_1 Z_2 e^2}{R}\{1 + \sum_{l \geq 2}\left[f_{1l}(R,\Theta_1,R_{10})\beta_{1l} + f_{1l}(R,\Theta_2,R_{20})\beta_{2l}\right] +$$

$$+ f_2(R,\Theta_1,R_{10})\beta_{12}^2 + f_2(R,\Theta_2,R_{20})\beta_{22}^2 + f_3(R,\Theta_1,\Theta_2,R_{10},R_{20})\beta_{12}\beta_{22} +$$

$$+ f_4(R,\Theta_1,\Theta_2,\Phi,R_{10},R_{20})\beta_{12}\beta_{22}\},$$

where f are the function of both the distance R and corresponding angles. As a rule, the values of the ground state deformation parameters in stable nuclei satisfy the condition $\beta_2^2 \approx \beta_{l>2}$, therefore linear β_{i2} and quadratic $\beta_{12}^2, \beta_{22}^2, \beta_{12}\beta_{22}$ terms on the

quadrupole deformation parameters as well as linear $\beta_{l>2}$ terms on high-multipolarity deformation parameters are taken into account.

Simple parameterization of the nuclear part of interaction potential between two spherical nuclei $V_N^0(d^0(R_{sph}, R_{10}, R_{20})$ is proposed in Ref. [4]. This parameterization is obtained in the framework of the semi-microscopic approximation for the Skyrme energy-density functional. Applying the proximity theorem [5] we can estimate the nuclear part of the potential between deformed nuclei at fixed distance and orientation [1]

$$V_N(R, \Theta_1, \Theta_2, \Phi) \approx \frac{C_{10} + C_{20}}{[(C_1^{II} + C_2^{II})(C_1^{\perp} + C_2^{\perp})]^{1/2}} \times$$

$$\times V_N^0(d(R, \Theta_1, \Theta_2, \Phi)).$$

Here $d(R, \Theta_1, \Theta_2, \Phi)$ is the distance between the closest points, belonging to different nuclei at orientation Θ_1, Θ_2, Φ; $C_1^{II}, C_2^{II}, C_1^{\perp}, C_2^{\perp}$ are, correspondingly, the curvatures of deformed nuclei (1 and 2) at these closest points and $C_{10} = 1/R_{10}$ and $C_{20} = 1/R_{20}$ the curvatures of spherical nuclei (1 and 2). Useful expressions for curvatures $C_1^{II}, C_2^{II}, C_1^{\perp}, C_2^{\perp}$ are given in Ref. [1]. The distance between the closest points of deformed nuclei is evaluated numerically.

The total nucleus-nucleus potential depends strongly on angles Θ_1, Θ_2, Φ. We present potentials for various systems in Ref. [1] and in Figs. 2 - 4.
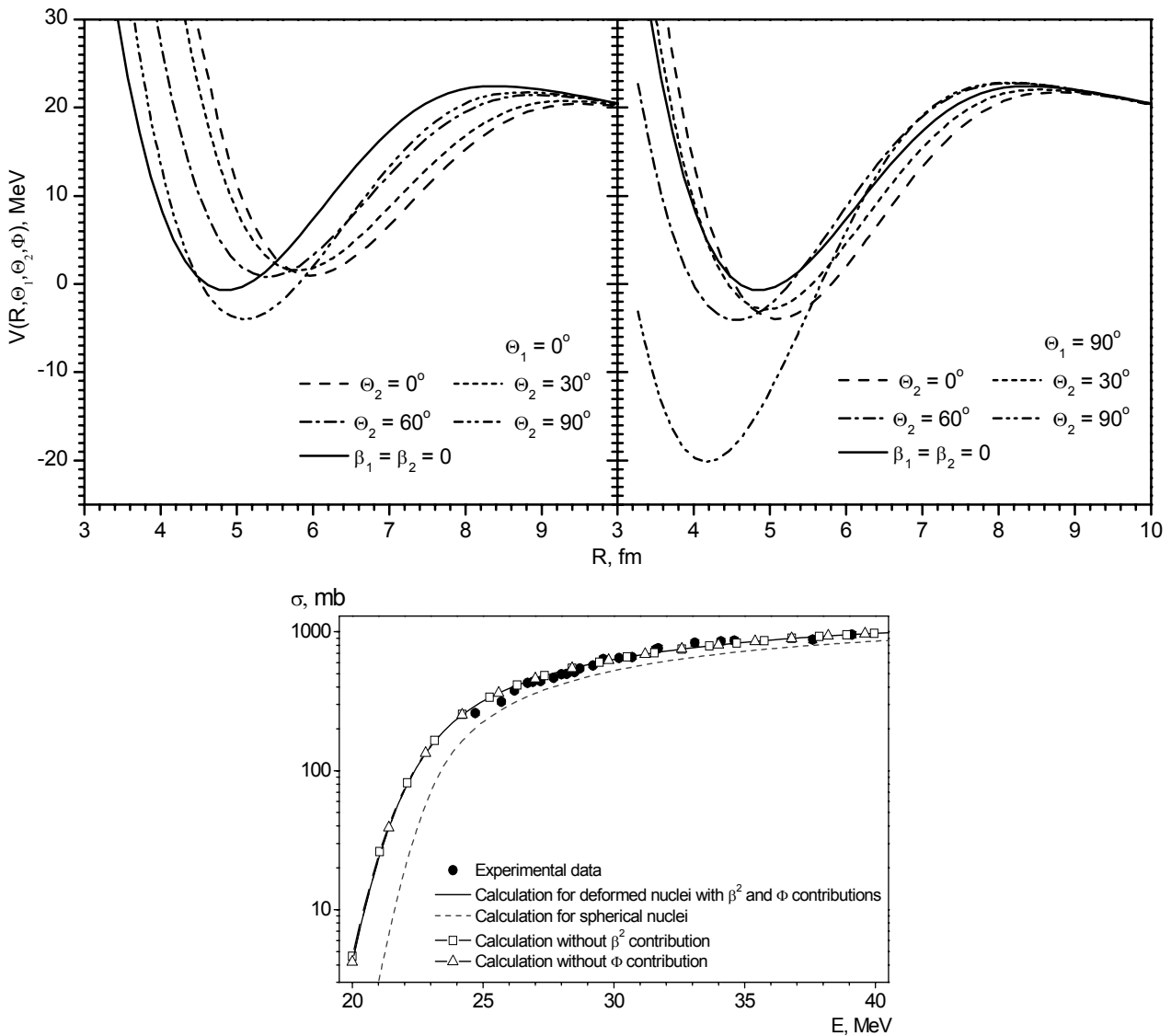

Fig. 2. Upper panels - Nucleus-nucleus potentials for the system ^{24}Mg + ^{24}Mg for various orientations. The potential between spherical nuclei for the system ^{24}Mg + ^{24}Mg is also presented on each panel. Down panel - The ^{24}Mg + ^{24}Mg fusion cross section. See text for notations.
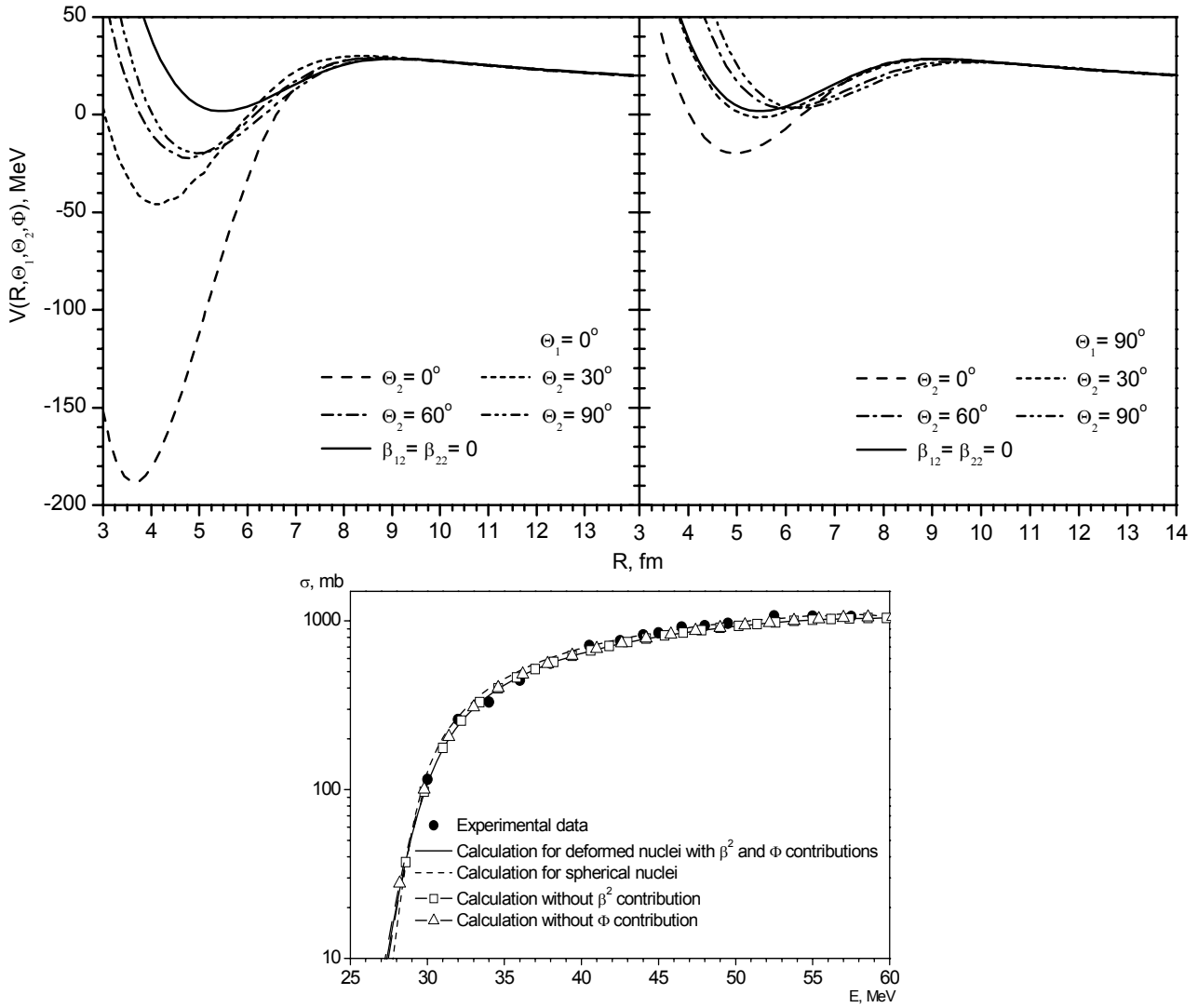
Fig. 3. Upper panels - Nucleus-nucleus potentials for the system ^{28}Si + ^{28}Si for various orientations.
Down panel - The ^{28}Si + ^{28}Si fusion cross section. The notations are the same as in Fig. 2.

3. Fusion cross sections

Various orientations of deformed nuclei occurs during collisions; therefore the fusion reaction cross-section induced by two deformed nuclei is given as

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E} \sum_l (2l+1) < T_l(E,\Theta_1,\Theta_2,\Phi) > .$$

Here μ is the reduced mass of colliding nuclei and $<T_l(E,\Theta_1,\Theta_2,\Phi)>$ is the transmission coefficient averaged over all possible orientations of colliding nuclei.

We use the WKB approximation for evaluation of the transmission coefficient for sub-barrier energies at orientation Θ_1,Θ_2,Φ

$$T_l(E,\Theta_1,\Theta_2,\Phi) = ,$$

$$= \{1 + \exp[\frac{2}{\hbar}\int_a^b \sqrt{2\mu(V(R,\Theta_1,\Theta_2,\Phi)\text{-}E)}dR]\}^{-1},$$

where a and b are, respectively, the inner and outer turning points. The transmission coefficient is obtained using the Hill-Wheeler approach [6] for over-barrier collision energies.

The potential for various orientation angles and the fusion cross-sections for prolate-prolate (^{24}Mg + ^{24}Mg), oblate-oblate (^{28}Si + ^{28}Si) and prolate-oblate (^{154}Sm + ^{28}Si) systems are presented in Figs. 2 - 4. The experimental fusion cross-section values for these reactions are taken from Refs. [7 - 9]. Experimental data for fusion cross sections are shown by dots in these Figures. The experimental values of the quadrupole deformation parameters ($\beta_2(^{24}\text{Mg}) = 0.438$, $\beta_2(^{28}\text{Si}) = -0.407$, $\beta_2(^{154}\text{Sm}) = 0.341$) are picked up from Moscow State University center for nuclear data [10]. These values of quadrupole deformation are extracted from experimental values of the reduced probabilities B(E2) [10]. We reevaluate the values of quadrupole deformation according to the volume conservation condition and B(E2) experimental value, because we
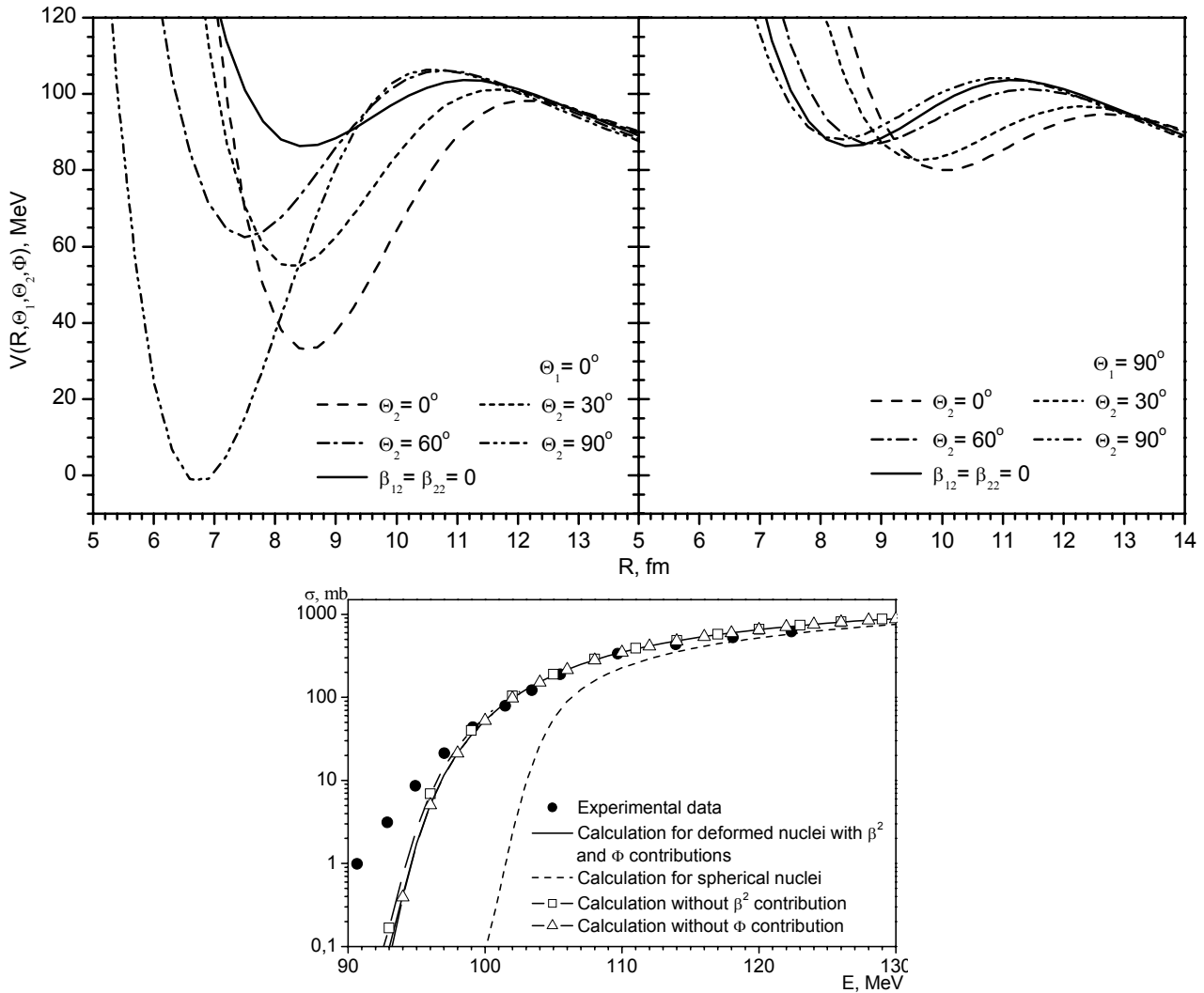
Fig. 4. (Upper panels) Nucleus-nucleus potentials for the system ^{28}Si + ^{154}Sm for various orientations. (Down panel) The ^{28}Si + ^{154}Sm fusion cross section. The notations are the same as in Fig. 2.

change values of the nuclear radii R_{10}, R_{20} and take into account quadratic term $(\beta_2)^2$, see for details Eqs. (22) - (24) in [1].

The values of η and radii R_{10}, R_{20} are found by fitting the cross-section values at high (over-barrier) collision energies. Such approach for determination of nuclear potential parameters is common for theoretical description of fusion reactions around barrier [11, 12], because the fusion cross section at over-barrier energies is determined by the Coulomb barrier height and the curvature of potential barrier. In contrast to this the fusion cross-section values at sub-barrier energies are defined by the width of potential barrier, the shape of capture well and the reaction mechanism [12, 13].

We present the fusion cross-sections in Figs. 2 - 4 obtained in the framework of various approaches for the sake of clarifying a role of various orientations and a accuracy of nucleus-nucleus potential evaluation on the cross-section values.

The results obtained in the most accurate our

approximation are shown by solid lines in Figs. 2 - 4. We take into account both linear and quadratic terms on the quadrupole deformation parameters in the potential and averaging on all possible orientation angles Θ_1, Θ_2 and Φ in this case.

Dot lines are cross-sections obtained in the approach, when spherical shapes of both nuclei are proposed. So we neglect deformation of colliding nuclei in this case. We clearly see strong effect of deformation on the fusion cross section at sub-barrier energies by comparing solid and dots lines in Figs. 2 - 4. This effect is especially strong in the case of systems with prolate nuclei, see Figs. 2 and 4. The oblate deformation gives weak influence on the fusion cross section, see Fig. 3.

Lines with squares are results, that has been calculated without taking into account of quadratic terms on the quadrupole deformation parameters in the potential and taking into account averaging on orientation angles Θ_1, Θ_2 and Φ. We can see the influence of the second-order terms on the quadru-

pole deformation parameter on the fusion cross section at sub-barrier energies by comparing the results presented by solid lines and lines with squares. We see in Figs. 2 - 4 that the cross-sections for light systems depend on $(\beta_2)^2$ terms weakly. This effect can see only for systems with prolate nuclei. The enhancement of sub-barrier cross section induced by deformation and the influence of $(\beta_2)^2$ terms rise with mass number of nuclei involved in reaction.

Lines with triangles show fusion cross section values obtained in the model, that takes into account linear and quadratic terms on the quadrupole deformation parameters in the potential and averaging on orientation angles Θ_1 and Θ_2. As a result, we can see the effect of averaging on orientation angle Φ on fusion cross section only for system ^{154}Sm + ^{28}Si at very low collision energy. The nucleus-nucleus potential depends weakly on Φ [1, 14]. As the result, the influence of rotation on orientation angle Φ on the fusion cross-section is negligible for systems light nuclei.

Note that averaging on space angles Θ_1, Θ_2 and Φ are related to the integral weights $\sin(\Theta_1)$ and $\sin(\Theta_2)$, therefore averaging on space angles gives no-trivial effect. Some orientations are rare occurred during collision (as, for example, orientation with $\Theta_1 = \Theta_2 = 0^\circ$ with the lowest barrier height), while other orientations are taken place more often (as, for example, orientation with $\Theta_1 = \Theta_2 = 90^\circ$ with the highest barrier height).

If we consider very heavy collision system, as for example ^{150}Nd + ^{152}Sm, then we can see strong influence of terms with $(\beta_2)^2$ and β_4 deformations and rotation on orientation angle Φ on a di-nuclear capture cross-section, see Fig. 5. We define a cross-section of di-nuclear capture as the possibility to penetrate through barrier and form relatively long-live di-nuclear system in the capture well during nucleus-nucleus collision. If capture well absent, then nuclei touch to each other and reseparate quickly, i.e. the time of reaction is very short in this case. The shape of capture well is very important for the synthesis of super-heavy nuclei [1], the shape evolution from sticking di-nuclear to near-spherical compound nuclear system is taken place after relaxation of relative motion of two approaching nuclei in the capture well. The cross-section of di-nuclear capture for such heavy system depends on the accuracy on nucleus-nucleus potential evaluation strongly, see Fig. 5. Unfortunately there are no experimental data for such system.

The di-nuclear capture cross-section is small, because the capture well of nucleus-nucleus potential occurs at some orientation angles. The di-nuclear capture cross-section depends on the orientation angle Φ as well as on $(\beta_2)^2$ and β_4 terms in the potential strongly.
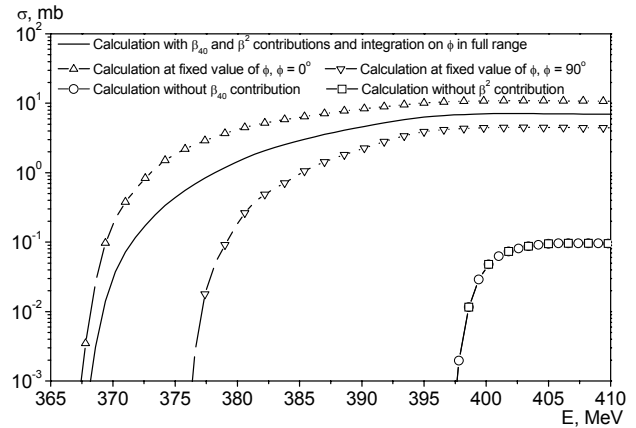

Fig. 5. The di-nuclear capture cross section for system ^{150}Nd + ^{152}Sm. The solid line is the most accurate our calculation. The lines with stars and squares are results evaluated without with β_4 and $(\beta_2)^2$ terms, respectively. The lines with up and down triangles are results obtained without averaging on orientation angle Φ at fixed values of $\Phi = 0^\circ$ and $\Phi = 90^\circ$ respectively.

In conclusions we note that:

- Interaction potential between two nuclei is strongly depended on both the surface deformations and the relative orientations of nuclei related to angles Θ_1, Θ_2 and Φ.

- Relative orientation of deformed nuclei essentially influences on height and position of potential barrier, as well as on the depth and width of capture well.

- The sub-barrier fusion cross section induced by deformed nuclei depends drastically on values of prolate deformation of surface.

- The contribution of the quadratic terms on deformation parameters into the nucleus-nucleus potential is very important for heavy systems only.

- Influence of orientation effects on sub-barrier fusion increase when charge and mass of interaction nuclei increase.

- Averaging on orientation angles Θ_1 and Θ_2 is very important for cross-section values at sub-barrier energies.

- Averaging on orientation angle Φ is important for cross-section values at sub-barrier energies for very heavy system only.

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ЗЛИТТЯ ДЕФОРМОВАНИХ ЯДЕР

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Детально розглянуто потенціал взаємодії двох деформованих аксіально-симетричних (витягнуто-витягнутих, витягнуто-сплюснутих, сплюснуто-сплюснутих) ядер. Розраховано переріз злиття двох ядер в наближенні, що враховує деформацію обох ядер та їх орієнтацію. Отриманий переріз злиття добре узгоджується з наявними експериментальними даними.

СЛИЯНИЕ ДЕФОРМИРОВАННЫХ ЯДЕР

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Подробно рассмотрен потенциал взаимодействия двух деформированных аксиально-симметричных (вытянуто-вытянутых, вытянуто-сплюснутых, сплюснуто-сплюснутых) ядер. Рассчитано сечение слияния двух ядер в приближении, которое учитывает деформацию обеих ядер и различные их ориентации. Полученное сечение слияния хорошо согласуется с доступными экспериментальными данными.