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THE NUMERICAL SOLUTION OF FADDEEV'S EQUATIONS AND THE CALCULATION OF *ND* - SCATTERING CROSS SECTION IN FUNDAMENTAL K-HARMONIC APPROXIMATION

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We have considered the problem of neuteron-deuteron scattering at low energies. Representing solution of corresponding Faddeev's equations as sum of asymptotic wave function and rapidly convergencing series of hyperspherical harmonics, we have calculated basic term of this series with value of full moment K = 0. The angular distribution of cross section for *nd*-scattering at 3,28 MeV neutron energy has been computed and compared with the experiment.

At present in the number of papers [1 - 8] the method of hyperspherical functions is widely used for solution of three-body Faddeev's equations (also known as the K-harmonic method [9 - 11]). Each of the component of full wave function $\Psi = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)}$, which satisfies system of three bounded Faddeev's equations [11, 12], expands in series of full set of hyperspherical functions [1-7]. Such series has rapidly convergence for bound states of three-body system, therefore in practice one can take into account a few number of lower K-harmonics. However, K-harmonics series has weak convergence (or divergence) for continuum, particularly for scattering problem when impact particle interacts with two bounded ones. In that case, we have proposed to expand difference $\Psi - \Phi$ in K-harmonics series, where Φ is the asymptotic part of Ψ [7, 8, 13]. Such expansion has the rapid convergence due to short-range pair potentials.

In this paper we consider *nd*-scattering at low neutron energy in non-spin approximation and without taking into account of deuteron D-wave. All our further calculations are carried out for c.m. system and $\hbar = c = 1$, table of symbols like paper [7]. Let the index number 1 corresponds to the incident neutron and numbers 2, 3 – to the nucleons of deuteron. As *NN* potential we choose the Hülten one [11]

$$V(r) = -V_0 \frac{e^{(\alpha-\beta)r}}{1-e^{(\alpha-\beta)r}}, \quad V_0 \cong 35\,\text{MeV}, \quad \beta = 7\alpha \cong 1,623\,\text{fm}^{-1}. \tag{1}$$

The first approximation for expansions for $\Psi^{(1)} - \Phi_{\vec{p}}$, $\Psi^{(2)}$ and $\Psi^{(3)}$ is [7]

$$\Psi^{(1)} = \Phi_{\vec{p}} - \frac{1}{\sqrt{\pi^3}} B^{(1)}(\rho), \quad \Psi^{(2,3)} = \frac{1}{\sqrt{\pi^3}} B^{(2,3)}(\rho), \tag{2}$$

where $\Phi_{\vec{p}}$ is the asymptotic wave function (plane wave) of three-nucleon system [14], ρ is the collective variable. The functions $B^{(j)}(\rho)$ (j = 1, 2, 3) combine into the system of bounded integral equations which can be solved by nmerical computation. As an example, we calculated $B^{(j)}(\rho)$ for neutron energy $E_n = 0,5; 1,75; 5,5$ MeV in lab system (see Fig. 1).

Note, that any component of $\text{Im}\, B^{(j)}(\rho)$ is absent when value of relative E_n less than binding energy of deuteron $\varepsilon \cong 2,23$ MeV. All dependencies $B^{(j)}(\rho)$ on Fig. 1 are characterized by maxima at $\rho \to 0$ and have monotone decreasing when ρ increases.

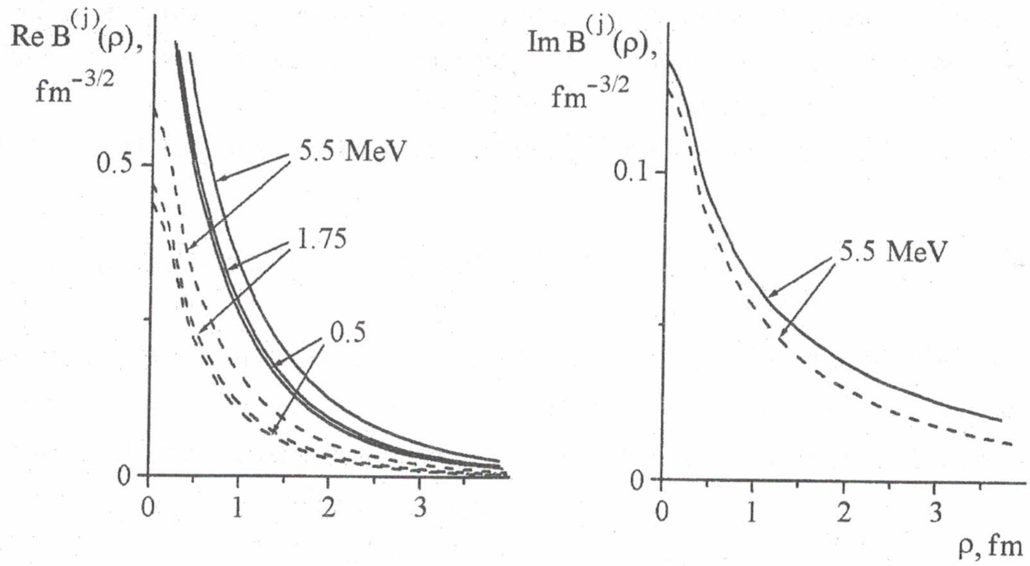

Fig. 1. Radial dependencies of functions $B^{(j)}$. Solid (dash) curves correspond to j = 2, 3 (j =1).

Besides $\Phi_{\bar{p}}$, the full wave function Ψ contains divergent spherical wave $B(\rho) = B^{(1)}(\rho) + B^{(2)}(\rho) + B^{(3)}(\rho)$

$$B(\rho) = \frac{\pi m}{\rho^2}\int_0^\infty d\rho' \rho'^3 B(\rho')\left(\bar{V}_{12} + \bar{V}_{31} + \bar{V}_{23}\right)P_+(k_0,\rho,\rho') +$$

$$+\frac{\pi^{5/2} m}{\rho^2}\int_0^\infty d\rho' \rho'^3 B(\rho')\left(\overline{V_{12}\Phi_{\bar{p}}} + \overline{V_{31}\Phi_{\bar{p}}}\right)P_+(k_0,\rho,\rho'), \quad k_0 = \sqrt{2m(E_n-\varepsilon)}, \tag{3}$$

$$P_+(k_0,\rho,\rho') = -i\left[J_2(k_0\rho)H_2^{(1)}(k_0\rho')\Theta(\rho'-\rho) + J_2(k_0\rho')H_2^{(1)}(k_0\rho)\Theta(\rho-\rho')\right], \tag{4}$$

where m is the neutron mass, $J_2(k_0\rho)$ is the Bessel function of second kind and $H_2^{(1)}(k_0\rho)$ is the first Hankel one, $\Theta(\rho'-\rho)$ is the Heavyside function, values $\bar{V}_{12}, \bar{V}_{31}, \bar{V}_{23}, \overline{V_{12}\Phi_{\bar{p}}}$ and $\overline{V_{31}\Phi_{\bar{p}}}$ were taken from [7].

The functions $\mathrm{Re}\,B(\rho)$ and $\mathrm{Im}\,B(\rho)$ for set of neutron energies are shown on Fig. 2.

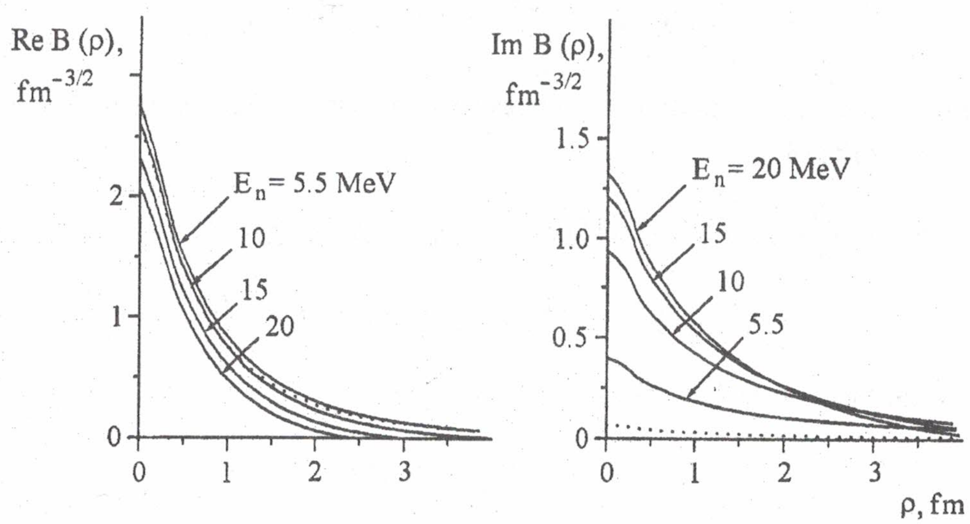

Fig. 2. Radial dependencies of computed functions B. Dot curves correspond to $E_n = 3,28$ MeV.

Here the values of $\mathrm{Im}\,B(\rho)$ are monotone increasing at $\rho \to 0$ when E_n increases, while energy dependence of $\mathrm{Re}\,B(\rho)$ has more complicated behavior. The dot curves corresponding to $E_n = 3{,}28$ MeV [15] will be necessary for our calculations below.

To obtain the expression for scattering amplitude one need to proceed from Schrödinger equation

$$\left\{E_n - \left[-\frac{1}{m}\frac{\partial^2}{\partial \vec{r}^2} + V(r)\right] + \frac{3}{4m}\frac{\partial^2}{\partial \vec{\rho}_1^2}\right\}\Psi(\vec{\rho}_1,\vec{r}) = \left[V(r_{12}) + V(r_{31})\right]\Psi(\vec{\rho}_1,\vec{r}), \tag{5}$$

where $\vec{r} \equiv \vec{r}_{23} = \vec{r}_2 - \vec{r}_3$, $\vec{\rho}_1 = \vec{r}_1 - (\vec{r}_2 + \vec{r}_3)/2$ is the relative vector between incident neutron and mass center of deuteron, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = \vec{\rho}_1 - \vec{r}/2$, $\vec{r}_{31} = \vec{r}_3 - \vec{r}_1 = -\vec{\rho}_1 - \vec{r}/2$. Assuming that $\vec{r}$ is finite and $\vec{\rho}_1 \to \infty$, we have $V(r_{12}) \to 0$, $V(r_{31}) \to 0$, $\Psi(\vec{\rho}_1,\vec{r}) \to \Phi_{\vec{p}} = \psi(\vec{\rho}_1)\varphi_d(\vec{r})$ and the equation (5) reduces to

$$\left[\frac{1}{m}\frac{1}{\varphi_d(\vec{r})}\frac{\partial^2 \varphi_d(\vec{r})}{\partial \vec{r}^2} - V(r)\right] + \frac{3}{4m}\frac{1}{\psi(\vec{\rho}_1)}\frac{\partial^2 \psi(\vec{\rho}_1)}{\partial \vec{\rho}_1^2} = -E_n\,. \tag{6}$$

From (6) it follows that

$$\psi(\vec{\rho}_1) = e^{i\vec{p}\vec{\rho}_1}, \quad E_n = -\varepsilon + \frac{3\vec{p}^2}{4m},$$

$$\frac{1}{m}\frac{\partial^2 \varphi_d(\vec{r})}{\partial \vec{r}^2} - V(r)\varphi_d(\vec{r}) = -\varepsilon\,\varphi_d(\vec{r}), \quad \varepsilon > 0. \tag{7}$$

The solution of (7), (1) is [11]

$$\varphi_d(\vec{r}) \equiv \varphi_d(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}}\,\frac{e^{-\alpha r} - e^{-\beta r}}{r}.$$

The amplitude of *nd*-scattering A can be found by using (5) and asymptotic of $\Psi(\vec{\rho}_1,\vec{r})$

$$\Psi(\vec{\rho}_1,\vec{r}) \equiv \Psi^{(+)}(\vec{\rho}_1,\vec{r}) \to \Phi_{\vec{p}} + A\frac{e^{ip\rho_1}}{\rho_1}, \quad |\vec{p}| = \frac{2}{3}\sqrt{2mE_n}, \quad \rho_1 \to \infty,$$

so we have [16]

$$A = -\frac{m}{3\pi}\int d\vec{\rho}_1 \int d\vec{r}\, \Phi^*_{\vec{p}}\left[V(r_{12}) + V(r_{31})\right]\Psi^{(+)}(\vec{\rho}_1,\vec{r}), \tag{8}$$

where $\Psi^{(+)}(\vec{\rho}_1,\vec{r})$ is exact wave function for three-nucleon system. According to K-harmonics method [11], we can rewrite (8) as

$$A = -\frac{\sqrt{3}m}{\pi}\int_0^\infty d\rho\,\rho^5 \int_0^{\pi/2} d\theta \cos^2\theta \sin^2\theta \int d\Omega_{\vec{\rho}_1} \int d\Omega_{\vec{r}}\, \varphi_d(r) e^{-i\vec{p}\vec{\rho}_1} \times$$

$$\times\left[V(|\vec{\rho}_1 - \vec{r}/2|) + V(|\vec{\rho}_1 + \vec{r}/2|)\right]\Psi^{(+)}(\vec{\rho}_1,\vec{r}), \tag{9}$$

where six-dimensional function $\Psi^{(+)}(\vec{\rho}_1,\vec{r})$ is

$$\Psi^{(\cdot)}(\vec{\rho}_1,\vec{r}) \approx \varphi_d(r) e^{i\vec{p}\vec{\rho}_1} + \frac{1}{\sqrt{\pi^3}}B(\rho).$$

The differential cross section can be found after partial integrations in (9), so we obtain

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{9\pi^2}\left|J^{(1)}(q) + J^{(2)}(p)\right|^2, \quad q = 2p\sin(\chi/2), \tag{10}$$

where χ is scattering angle,

$$J^{(1)}(q) = \frac{16\pi^2}{q}\int_0^\infty d\rho_1\, \rho_1 \sin(q\rho_1)\int_0^\infty dr\, r^2\varphi^2(r)\int_{-1}^{1} d\lambda V\left(\sqrt{\rho_1^2 + r^2/4 - \lambda\rho_1 r}\right),$$

$$J^{(2)}(p) = \frac{48\sqrt{2\pi}}{p}\int_0^\infty d\rho\, \rho^4 B(\rho)\int_0^{\pi/2} d\theta \cos^2\theta \sin\theta \sin\left(\sqrt{3/2}\, p\rho\sin\theta\right)\varphi_d\left(\sqrt{2}\,\rho\cos\theta\right)\times$$

$$\times\int_{-1}^{1} d\lambda V\left(\rho\sqrt{\frac{3}{2}\sin^2\theta + \frac{1}{2}\cos^2\theta - \lambda\frac{\sqrt{3}}{2}\sin 2\theta}\right).$$

Usually, the experimental distributions of $d\sigma/d\Omega$ for low-energy *nd*-scattering (e.g. [15]) are characterized by monotone decreasing from angles $\theta \sim 0^\circ$ and 180° to wide minimum at $\theta \approx 90-120^\circ$. Since we use zero-order approximation (K = 0) in our computations, it could expect a qualitative fitting. Therefore our calculations of $d\sigma/d\Omega$ values (10) and

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{9\pi^2}\left|J^{(1)}(q)\right|^2 \text{ (K-harmonic is absent)}, \tag{11}$$

were performed for range of scattering angles $80^\circ \le \theta \le 130^\circ$ (see Fig. 3).

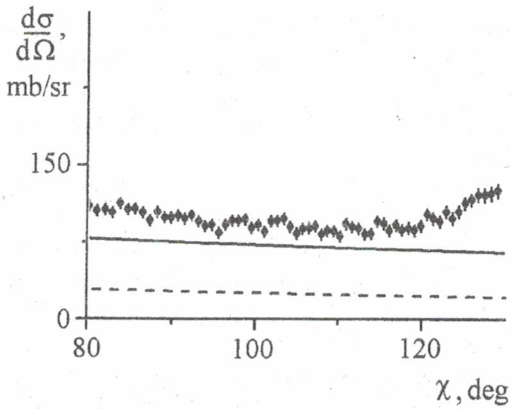

Fig. 3. Differential cross section of *nd*-scattering at $E_n = 3,28$ MeV. Experimental data were taken from [15].

There are dash curve corresponds to (11) and solid one to (10), thus one could see some improvement of experiment fitting just for the case when fundamental K-harmonic is taking into account. As qualitative fitting of experimental data [15] is reached, our future aim is to take into consideration both high K-harmonics with K > 0 in expansion for $\Psi^{(1,2,3)}$ and spins of interacting particles.

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ЧИСЕЛЬНИЙ РОЗВ'ЯЗОК РІВНЯНЬ ФАДДЄЄВА І РОЗРАХУНОК ПЕРЕРІЗУ *ND*-РОЗСІЯННЯ В НАБЛИЖЕННІ ОСНОВНОЇ К-ГАРМОНІКИ

В. К. Тартаковський, В. І. Ковальчук, І. В. Козловський

Після подання розв'язку рівнянь Фаддєєва для системи "нейтрон - дейтрон" у вигляді суми асимптотичної хвильової функції та швидкозбіжного ряду по К-гармонікам було розраховано основний член цього ряду з мінімальним значенням повного моменту K = 0. З використанням одержаного наближеного розв'язку обчислено кутовий розподіл нейтронів, розсіяних дейтронами при енергії 3,28 МеВ, яке порівнюється із відповідним експериментом.

ЧИСЛЕННОЕ РЕШЕНИЕ УРАВНЕНИЙ ФАДДЕЕВА И РАСЧЕТ СЕЧЕНИЯ *ND*-РАССЕЯНИЯ В ПРИБЛИЖЕНИИ ОСНОВНОЙ К-ГАРМОНИКИ

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Представив решение уравнений Фаддеева для системы "нейтрон - дейтрон" в виде суммы асимптотической волновой функции и быстро сходящегося ряда по К-гармоникам, был рассчитан основной член этого ряда с минимальным значением полного момента K = 0. С использованием полученного приближенного решения рассчитано угловое распределение нейтронов, рассеяных дейтронами при энергии 3,28 МэВ, которое сравнивается с соответствующим экспериментом.

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