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UNIFIED SEMICLASSICAL APPROACH TO ISOSCALAR COLLECTIVE EXCITATIONS IN NUCLEI

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A semiclassical model based on the solution of the Vlasov kinetic equation for finite systems with a moving surface has been used to study the isoscalar collective modes in heavy spherical nuclei. Within this model, a unified description of both low-energy surface modes and higher-energy giant resonances has been achieved by introducing a coupling between surface vibrations and the motion of single nucleons. An analytical expression for the isoscalar multipole response function can be derived by using a separable approximation for the residual interaction between nucleons. The response function obtained in this way gives a good qualitative description of the isoscalar (monopole, quadrupole and octupole) response in heavy nuclei. Although shell effects are not explicitly included in the theory, our semiclassical response functions are very similar to the quantum ones. This happens because of the well known close relation between classical trajectories and shell structure.

1. Introduction

It is well known that the isoscalar multipole response of nuclei displays both low- and high-energy collective modes [1]. Also known is that semiclassical models have difficulties in describing both these systematic features of the isoscalar response, in particular, models based on fluid dynamics, see e.g. [2], can explain the giant resonances, but fail to describe the low-energy collective modes. On the other hand it is known from quantum studies that the coupling between the motion of individual nucleons and surface vibrations plays an essential role in low-energy nuclear collective motions, see e.g. [3, 4]. Semiclassical models of the fluid-dynamical type, where the Vlasov kinetic equation is reduced to the equations of motion for the local quantities like particle density, current density etc., do not contain explicitly the single-particle degrees of freedom.

In the present work we study the isoscalar collective modes in nuclei by using a semiclassical approach that includes the single-particle degrees of freedom explicitly and thus allows for an account the coupling between individual nucleons and surface motion. Our approach is based on the direct solution of the linearized Vlasov kinetic equation for finite systems with moving surface [5, 6]. The coupling between the motion of individual nucleons and the surface vibrations is obtained by treating the nuclear surface as a collective dynamical variable, like in the liquid drop model. In regard to the conception of the free moving surface, our approach can be called the Fermi liquid drop model of nuclear collective excitations. In Sect.2 the isoscalar multipole response function based on the semiclassical approach of [5, 6] is discussed. Several applications of the theory to the study of isoscalar vibrations of different multipolarity are given in Sect. 3. We concentrate our attention on the isoscalar monopole, quadrupole and octupole collective modes in heavy spherical nuclei.

2. Isoscalar multipole response function

We treat the nucleus like a system of interacting nucleons confined to a spherical cavity with perfectly reflecting walls that are allowed to move. We assume that the fluctuations of the phase-space density induced by a weak external force can be described by the linearized Vlasov equation, which is usually a differential equation in seven variables. For spherical systems this equation can be reduced to a system of two (coupled) differential equations in the radial coordinate alone [5]. This is achieved by means of a change of variables and a partial-wave expansion:

$$\delta f(\mathbf{r},\mathbf{p},\omega)=\sum_{LMN}[\delta f^{L+}_{MN}(\varepsilon,\lambda,r,\omega)+\delta f^{L-}_{MN}(\varepsilon,\lambda,r,\omega)](D^L_{MN}(\alpha,\beta,\gamma))^* Y_{LN}(\tfrac{\pi}{2}\tfrac{\pi}{2}). \quad (1)$$

The functions $\delta f_{MN}^{L\pm}(\varepsilon, \lambda, r, \omega)$ are partial-wave components of the (Fourier transformed in time) density fluctuations for particles with energy ε, magnitude of angular momentum λ and radial position r, the $\pm$ sign distinguishes between particles having positive or negative components of the radial momentum p_r. The other terms in the expansion are Wigner matrices and spherical harmonics.

In order to solve the one-dimensional linearized Vlasov equation for the $\delta f_{MN}^{L\pm}$ functions we must specify the boundary conditions satisfied by these functions. Different boundary conditions allow us to study different physical properties of the system, so the fixed-surface boundary conditions employed in [5] were adequate to study giant resonances, but different (moving-surface) boundary conditions [6] must be introduced in order to study also surface and compression modes. We assume that the external force can also induce oscillations of the system the usual liquid-drop model expression

$$R(\vartheta,\varphi,t) = R + \sum_{LM} \delta R_{LM}(t) Y_{LM}(\vartheta,\varphi) \tag{2}$$

and the boundary condition satisfied by the functions $\delta f_{MN}^{L\pm}$ at the nuclear surface is taken as [6]

$$\delta f_{MN}^{L+}(R) - \delta f_{MN}^{L-}(R) = 2F'(\varepsilon) i\omega p_r \delta R_{LM}(\omega). \tag{3}$$

This equation has been derived with the assumption that the equilibrium phase-space density is a function $F(\varepsilon)$ of the particle energy alone, $F'(\varepsilon)$ is its derivative. The boundary condition (3) corresponds to a mirror reflection of particles in the reference frame of the moving nuclear surface, it provides a coupling between the motion of nucleons and the surface vibrations. A self-consistency condition involving the nuclear surface tension is then used to determine the time (or frequency) dependence of the additional collective variables δR_{LM} (t) [6].

We are interest in the multipole response function that in the moving-surface case is defined as, see [7],

$$\tilde{R}_L(\omega) = \frac{1}{\beta} \int dr r^2 r^L \delta\bar{\varrho}_{LM}(r,\omega), \tag{4}$$

with

$$\delta\bar{\varrho}_{LM}(r,\omega) = \varrho_{LM}(r,\omega) + \varrho_0 \delta(r-R) \delta R_{LM}(\omega), \tag{5}$$

where

$$\delta\varrho(r,\omega) = \frac{8\pi^2}{2L+1} \frac{1}{r^2} \sum_{N=-L}^{L} |Y_{LN}(\tfrac{\pi}{2},\tfrac{\pi}{2})|^2 \int d\varepsilon \int d\lambda \frac{\lambda}{v_r(\varepsilon,\lambda,r)} [\delta f_{MN}^{L+}(\varepsilon,\lambda,r,\omega) + \delta f_{MN}^{L-}(\varepsilon,\lambda,r,\omega)].$$

The fluctuation $\delta\varrho_L(\mathbf{r},\omega)$ is the time Fourier transform of the density fluctuation $\delta\varrho_L(\mathbf{r},t)$ induced by an external isoscalar field $V_{ext}(\mathbf{r},t) = \beta\delta(t) r^L Y_{LM}(\hat{\mathbf{r}})$. The equilibrium density ϱ_0 appearing in Eq. (5) is $\varrho_0 = \frac{2}{3\pi^2} p_F^3$.

Now, assuming a simplified residual interaction of separable form,

$$v(r_1, r_2) = \kappa_L r_1^L r_2^L, \tag{6}$$

the moving-surface isoscalar collective response function of a spherical nucleus, described as a system of A interacting nucleons contained in a cavity of equilibrium radius $R = 1.2 A^{\frac{1}{3}}$ fm, can be found as [8]

$$\tilde{R}_L(s) = R_L(s) + S_L(s). \tag{7}$$

Instead of the frequency ω, as independent variable we have used the more convenient dimensionless quantity $s=\omega/(v_F/R)$ (v_F is the Fermi velocity). The response function $R_L(s)$, given by

$$R_L(s)=\frac{R_L^0(s)}{1-\kappa_L R_L^0(s)}, \tag{8}$$

describes the collective response in the fixed-surface limit. The response function $R_L^0(s)$ is analogous to the quantum single-particle response function and it is given explicitly by [7]

$$R_L^0(s)=\frac{9A}{8\pi}\frac{1}{\varepsilon_F}\sum_{n=-\infty}^{+\infty}\sum_{N=-L}^{N=L}(C_{LN})^2\int_0^1 dx x^2 s_{nN}(x)\frac{(Q_{nN}^{(L)}(x))^2}{s+i\varepsilon-s_{nN}(x)}, \tag{9}$$

where ε_F is the Fermi energy and the quantity ε is a vanishingly small parameter that determines the integration path at poles. The functions $s_{nN}(x)$ are defined as

$$s_{nN}(x)=\frac{n\pi+N\arcsin(x)}{x}. \tag{10}$$

The variable x is related to the classical nucleon angular momentum λ. The quantities C_{LN} in Eq. (9) are classical limits of the Clebsh-Gordan coefficients coming from the angular integration. In principle the integer N takes values between $-L$ and L, however only the coefficients C_{LN} where N has the same parity as L are nonvanishing. The coefficients $Q_{nN}^{(L)}(x)$ appearing in the numerator of Eq. (9) have been defined in Ref. [5], they are essentially the classical limit of the radial matrix elements of the multipole operator r^L and can be evaluted analytically for any L.

The function $S_L(s)$ in Eq. (7) gives the moving-surface contribution to the response. With the simple interaction (6) this function can be evaluated explicitly as [8]

$$S_L(s)=-\frac{R^6}{1-\kappa_L R_L^0(s)}\frac{[\chi_L^0(s)+\kappa_L\varrho_0 R^L R_L^0(s)]^2}{[C_L-\chi_L(s)][1-\kappa_L R_L^0(s)]+\kappa_L R^6[\chi_L^0(s)+\varrho_0 R^L]^2}, \tag{11}$$

with $C_L=\sigma R^2(L-1)(L+2)+(C_L)_{coul}$, $\sigma\approx 1\,\mathrm{MeV\,fm^{-2}}$ is the surface tension parameter obtained from the mass formula, $(C_L)_{coul}$ gives the Coulomb contribution to the restoring force and $\varrho_0=A/\frac{4\pi}{3}R^3$ is the equilibrium density. The functions $\chi_L^0(s)$ and $\chi_L(s)$ are given by [7]

$$\chi_L^0(s)=\frac{9A}{4\pi}\frac{1}{R^3}\sum_{n=-\infty}^{+\infty}\sum_{N=-L}^{N=L}(C_{LN})^2\int_0^1 dx x^2 s_{nN}(x)\frac{(-)^n Q_{nN}^{(L)}(x)}{s+i\varepsilon-s_{nN}(x)}, \tag{12}$$

and

$$\chi_L(s)=-\frac{9A}{2\pi}\varepsilon_F(s+i\varepsilon)\sum_{nN}(C_{LN})^2\int_0^1 dx x^2\frac{1}{s+i\varepsilon-s_{nN}(x)}, \tag{13}$$

their structure is similar to that of the zero-order propagator (9).

We refer to the papers [5 - 8] for further details on the formalism and discuss here only the main points.

Equation (11) is the main result in the present context. Together with Eqs.(6) and (7), this equation gives a unified expression of the isoscalar response function, including both the low- and high-energy collective excitations. By comparing the fixed- and moving-surface response functions, we can appreciate the effects due to the coupling between the motion of individual nucleons and the surface vibrations.

3. Isoscalar collective modes in heavy nuclei

We apply our theory to study isoscalar collective excitations in heavy spherical nuclei. The strength function $S_L(E)$ associated with the response function (7) is defined as ($E = \hbar\omega$)

$$S_L(E) = -\frac{1}{\pi}\,\mathrm{Im}\,\tilde{R}_L(E). \tag{14}$$

We discuss here the isoscalar monopole, quadrupole and octupole strength distributions. We determine the strength κ_L of the residual interaction (6) phenomenologically, by requiring that the peak of the high-energy resonance agrees with the experimental value of the giant multipole resonance energy.

First we study the isoscalar monopole response of spherical nuclei. In the case of the monopole excitations the radial dependence of the external field should be taken as r^2, so that the response function (7) is slightly modified at $L=0$. In Fig. 1 the monopole strength function $S_{L=0}(E)$ is shown in different approximations. The zero-order monopole strength (9) (the dotted curve in Fig.1), which is similar to the quantum single-particle strength, is distributed at the energies higher than a threshold energy $\hbar\omega_{min}$. This comes from the fact that there is a gap in the monopole single-particle frequency spectrum which expands from $s_{min} = \pi$ (or in frequency units $\omega_{min} = \pi v_F/R$) to infinity, see Eq. (10) at $N=0$ and $n=1$. The moving-surface zero-order response (the dashed curve in Fig.1), given by Eq. (7) at $\kappa_{L=0} = 0$, has one resonance situated inside of the gap. The moving-surface assumption corresponds to taking into account an attractive effective interaction in the surface region which leads to a collective monopole mode with frequency smaller than ω_{min} [6]. In the case of a system with $A = 208$ nucleons and $R = 1{,}2A^{1/3}$ shown in Fig. 1 the monopole resonanse energy is equal to $\hbar\omega = 14{,}8$ MeV.

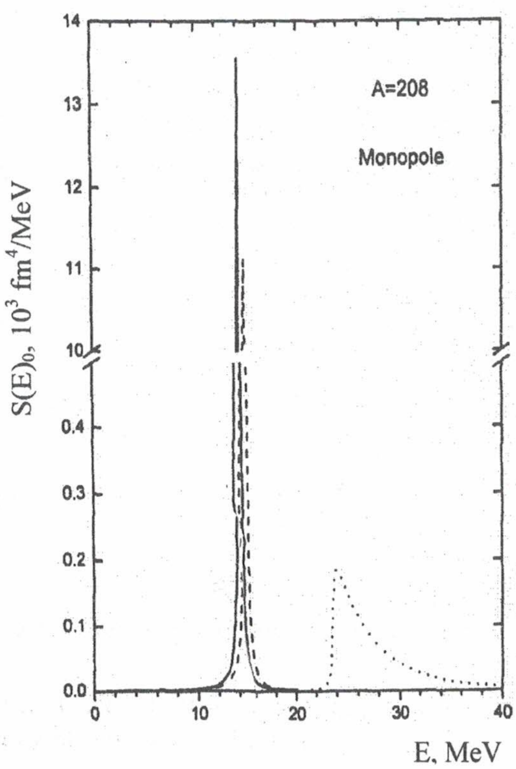

Fig. 1. Monopole strength function for a hypothetical nucleus of $A = 208$ nucleons. The dotted curve shows the zero-order approximation, the dashed curve instead shows the collective response evaluated in the fixed-surface approximation. The full curve gives the moving-surface response.

The found monopole mode exhausts about 99% of the energy- weighted sum rule associated with the monopole operator that for a system with a sharp surface is given by [9]

$$\int_0^{\infty} dE\, E\, S(E) = \frac{3}{10\pi}\frac{\hbar^2}{m} AR^2 . \qquad (15)$$

We get that, already in the moving-surface zero-order approximation; we have achived a resonable reproduction of the energy of the isoscalar giant monopole resonance (IGMR) in nucleus ^{208}Pb which amounts to $E^{IGMR} \approx 14,2\,\mathrm{MeV}$ [10]. Moreover the moving-surface zero-order approximation gives a good reproduction of the systematic behaviour of the energy of the IGMR as a function of the mass number A given by $E^{IGMR} \approx 82A^{-1/3}\,\mathrm{MeV}$ [1]. By using the equation for the eigenfrequencies of the collective modes in the moving-surface zero-order approximation given by

$$C_L - \chi_L(s) = 0 \qquad (16)$$

we can find that the energy of the first monopole mode amounts to $s_1 = 0,62$ or in energy units $\hbar\omega \approx 88A^{-1/3}\,\mathrm{MeV}$. Finally, the solid curve in Fig.1 displays the monopole response given by Eq. (7) with including the residual interaction of the monopole-monopole type

$$v_{L=0}(r,r') = \kappa_{L=0} r^2 r'^2 . \qquad (17)$$

The strength $\kappa_{L=0}$ of the interaction, chosen in order to reproduce the experimental value of the IGMR energy in ^{208}Pb, is $\kappa_{L=0} \approx -2 \cdot 10^{-4}$ MeVfm^{-4}. The giant resonance found in the moving-surface zero-order approximation (the dashed curve) is shifted to smaller energy that is equal to the IGMR energy in ^{208}Pb.

We notice that the width of the IGMR is underestimated by our theory. The Landau damping of the monopole mode given by the response function (14) is absent and the width shown in Fig. 1 (the dashed and solid curves) is only due to the smearing parameter $\varepsilon = 0,1\,\mathrm{MeV}$ that was used in the calculations. This result agrees with the one obtained in the quantum RPA calculations for the heavy nuclei, where the fragmentation width (the Landau damping) is small or wholly absent, depending on the used isoscalar interaction [11, 12]. The description of the widths of the collective modes in nuclei must also includes the contribution from the two-body collisions. Taking into account of the collisional damping makes it possible to explain about 30% of the observed width of the IGMR [13, 14]. The description of the full width of the IGMR was achieved recently in paper [4] by involving both the collisional damping and non-linear effects associated with coupling between the motion of nucleons and the surface vibrations of different multipolarity .

In Fig. 2 we display the quadrupole strength function (L = 2 in Eq. (7)) obtained for $A = 208$ nucleons by using different approximations [8]. The dotted curve is obtained from the zero-order response function (9), it is similar to the quantum response evaluated in the single-particle approximation. The dashed curve is obtained from the collective fixed-surface response function (8). Comparison with the dotted curve clearly shows the effects of collectivity. The collective fixed-surface response has one giant quadrupole peak. Our result for this peak is very similar to that of the recent random-pase approximation (RPA) calculations of [15]. However, contrary to the RPA calculations, there is no signal of a low-energy peak in the fixed-surface response function. The solid curve instead shows the moving-surface response given by Eq. (7). Now a broad bump appears in the low-energy part of the response and a narrower peak is still present at the giant resonance energy. Of course the details of the low-energy excitations are determined by quantum effects, nonetheless the present semiclassical approach does reproduce the average behaviour of this systematic feature of the quadrupole response.

We finally notice that the width of the giant quadrupole resonance is underestimated by our approach, this is a well known limit of all mean-field calculations that include only Landau damping. A more realistic estimate of the giant-resonance width would require including a collision term into our kinetic equation.

In Fig. 3 we show the octupole strength function ($L = 3$ in Eq. (7)) [16]. The zero-order octupole strength function (dotted curve) is concentrated in two regions around 8 and 24 MeV. In this respect our semiclassical response is strikingly similar to the quantum response, which is concentrated in the $1\hbar\omega$ and $3\hbar\omega$ regions. This concentration of strength is quite remarkable because our equilibrium phase-space density, which is taken to be of the Thomas-Fermi type, does not include any shell effect, however we still obtain a strength distribution that is very similar to the one usually interpreted in terms of transitions between different shells.

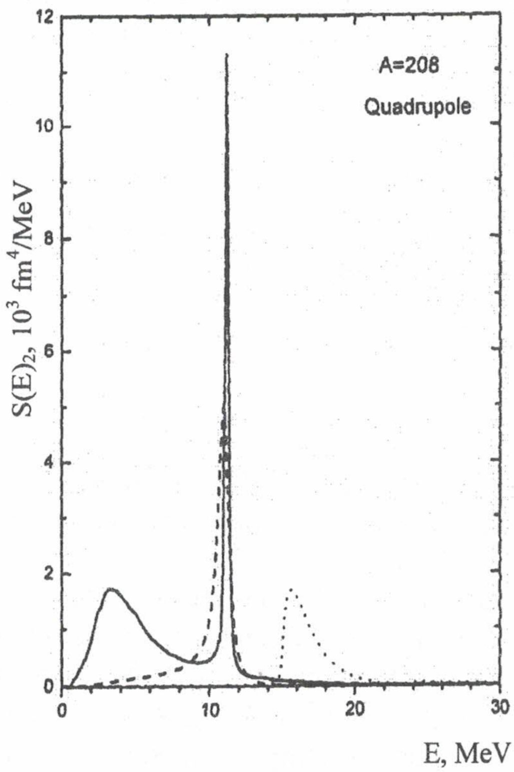

Fig. 2. The same as in Fig. 1 for the quadrupole strength function.

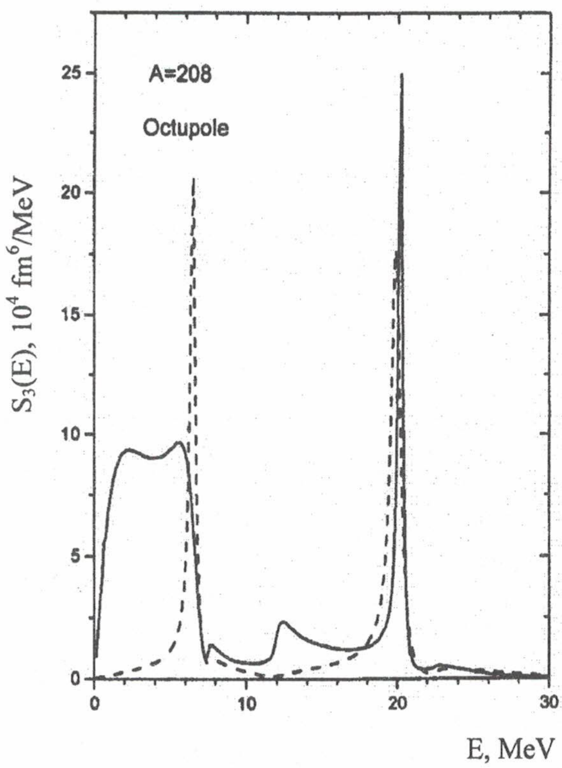

Fig. 3. The same as in Fig. 1 for the octupole strength function.

We can clearly see that the collective fixed-surface response given by Eq. (8) (dashed curve) has two sharp peaks around 20 Mev and 6 - 7 Mev. The experimentally observed [1] concentration of isoscalar octupole strength in the two regions usually denoted by HEOR (high energy octupole resonance) and LEOR (low energy octupole resonance) is qualitatively reproduced, however the considerable strength experimentally observed at lower energy (low-lying collective states) is absent from the fixed-surface response function. The most relevant change induced by the moving surface (solid curve in Fig. 3) is the large double hump appearing at low energy. This feature is in qualitative agreement both with experiment [1] and with the result of RPA-type calculations (see e.g. [17]). We interpret this low-energy double hump as a superposition of surface vibration and LEOR.

The moving-surface octupole response of Fig. 3 displays also a novel resonance-like structure between the LEOR and the HEOR (at about 13 MeV for a system of $A = 208$ nucleons).

4. Conclusions

A unified description of the low- and high-energy isoscalar collective response has been achieved by using appropriate boundary conditions for the fluctuations of the phase-space density described by the linearized Vlasov equation. The response functions obtained in this way give a good qualitative description of all the main features of the isoscalar response in heavy nuclei, i. e. low-lying quadrupole and octupole collective modes, plus monopole, quadrupole and octupole giant resonances.

As a further remark we would like to add that the problem of which boundary conditions to use in the linearized Vlasov equation for finite systems, rather than being a limitation of the approach, may be seen as a richness of the theory: different boundary conditions allows us to study different physical properties of the system.

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ЄДИНИЙ НАПІВКЛАСИЧНИЙ ОПИС ІЗОСКАЛЯРНИХ КОЛЕКТИВНИХ ЗБУДЖЕНЬ В ЯДРАХ

В. І. Абросімов

Напівкласична модель, що спирається на розв'язок кінетичного рівняння Власова для скінченних систем з рухомою поверхнею, використовується для вивчення ізоскалярних колективних мод у важких сферичних ядрах. Завдяки врахуванню зв'язку між коливаннями поверхні й рухом нуклонів у рамках цієї моделі вдається отримати єдиний опис гігантських резонансів та колективних мод з малою енергією збудження. Отримано аналітичний вираз для ізоскалярної мультипольної функції відгуку, використовуючи сепарабельне наближення для залишкової взаємодії між нуклонами. Знайдена функція відгуку відтворює спостережувані властивості ізоскалярного (монопольного, квадрупольного та октупольного) відгуку у важких ядрах. Незважаючи на те, що оболонкові ефекти не враховуються явно в моделі, отримані напівкласичні ізоскалярні мультипольні функції відгуку подібні до квантових. Це обумовлено тим, що існує тісний зв'язок між класичними траєкторіями та оболонковою структурою.

ЕДИНОЕ ПОЛУКЛАССИЧЕСКОЕ ОПИСАНИЕ ИЗОСКАЛЯРНЫХ КОЛЛЕКТИВНЫХ ВОЗБУЖДЕНИЙ В ЯДРАХ

В. И. Абросимов

Полуклассическая модель, которая исходит из решения кинетического уравнения Власова для конечных систем с подвижной поверхностью, используется для изучения изоскалярных коллективных мод в тяжелых сферических ядрах. Благодаря учету связи между колебаниями поверхности и движением нуклонов в рамках этой модели удается получить единое описание гигантских резонансов и низколежащих коллективных мод. Получено аналитическое выражение для изоскалярной мультипольной функции отклика, используя сепарабельное приближение для остаточного взаимодействия между нуклонами. Найденная функция отклика воспроизводит наблюдаемые свойства изоскалярного (монопольного, квадрупольного и октупольного) отклика тяжелых ядер. Хотя оболочечные эффекты не учитываются явно в модели, полученные полуклассические изоскалярные мультипольные функции отклика похожи на квантовые. Это происходит потому, что имеется тесная взаимосвязь между классическими траекториями и оболочечной структурой.

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