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ON THE EFFECTS OF NUCLEAR STRUCTURE AND THE COULOMB INTERACTION AT DIFFRACTION DEUTERON-NUCLEUS SCATTERING

V. I. Kovalchuk, V. K. Tartakovsky, O. I. Ivanova

Taras Shevchenko National University, Kyiv

Within diffraction model framework it has been proposed the method of cross section calculation of deuteron-nucleus scattering at intermediate energies. The deuteron wave function was chosen as Hülten one, the Coulomb interaction and nuclear surface diffuseness of targets were taken into account. The calculating cross sections of 700 MeV deuteron elastic scattering from ^{40}Ca and ^{58}Ni satisfactorily fit the experimental data.

1. Introduction

The modern diffraction theory of nuclear reactions is powerful instrument for investigating of various nuclear processes that allows to research both elastic and non-elastic reactions, excitation of nuclei, breakup ones etc. The diffraction phenomena arise when the De Broigle wave length of projectile become less than characteristic dimension of interaction region that corresponds to 10 – 15 MeV per nucleon and more (for medium and heavy targets). At intermediate energies, as is known [1], movement of scattering particle in nuclear field of a target nucleus can be considered as quasi-classic. Earlier, by using of diffraction approach, we performed [2] the preliminary calculations of differential cross sections of 700 MeV deuteron elastic scattering from ^{40}Ca. In that paper the deuteron wave function was chosen as Gaussian and the Coulomb interaction has not taken into account but these approximations were not gave an opportunity to reach a good fitting with the experiments. In the present paper as deuteron wave function we choose the Hülten one which has good asymptotic behavior both on small and large proton-neutron distances, besides the influence of the Coulomb interaction also takes into account.

2. Formalism

Assuming that targets have spherical form, in non-spin approximation and without taking into account of deuteron D-wave, let us start from general expression for amplitude of deuteron-nucleus elastic scattering [3]

$$F(\vec{q}) = \frac{ik}{2\pi} \int d^{(3)}\vec{r} \int d^{(2)}\vec{\rho}\, \psi_0^2(r) \exp(i\vec{q}\cdot\vec{\rho}) \left[\omega_1(\rho_1) + \omega_2(\rho_2) - \omega_1(\rho_1)\omega_2(\rho_2)\right], \quad \vec{\rho} = (\vec{\rho}_1 + \vec{\rho}_2)/2, \quad (1)$$

where $\vec{q} = \vec{k} - \vec{k}'$ is momentum transfer (all our further calculations are carried out for c.m. system and $\hbar = c = 1$), $\vec{k}(\vec{k}')$ is incident (scattered) momentum of deuteron, $\psi_0(r)$ is intrinsic wave function of relative motion of deuteron clusters. Here $\omega_j(\rho_j)$ are the nucleon-nucleus profile functions ($j = 1$ is neutron number, $j = 2$ corresponds to proton) which expressed in terms of impact parameter $\vec{\rho}_j = \{\rho_j, \varphi_j\}$ from scattering phases $\delta(\rho_j)$

$$\omega_j(\rho_j) = 1 - \exp[2i\delta(\rho_j)] .$$

From common reasons it is clear that while deuterons have relativistic energy, the Coulomb contribution to differential cross section of reaction will be insignificant excepting regions of small scattering angles and diffraction minima. However, the further calculations will show that the mentioned contribution may reaches essential values even for such energies of incident deuterons that leads to an improvement of the experiment fitting. But for simplicity, let us consider the deuteron-nucleus scattering without the Coulomb interaction.

As is known [4], quasi-classic phase $\delta_N(\rho_j)$ of scattered nucleon can be expressed from nucleon-nucleus potential $V_N(r)$

$$\delta_N(\rho_j) = -\frac{1}{v}\int_0^\infty ds\, V_N(r), \quad r = \sqrt{\rho_j^2 + s^2}\ , \tag{2}$$

where v is the relative speed of incident nucleon and target nucleus on infinite distance between them. At relativistic kinetic energies of nucleon E potential $V_N(r)$ can be presented as [3]

$$V_N(r) \equiv V_N(r,E) = -\frac{2\pi}{E+M} f(0)\rho_N(r), \quad \int d^{(3)}\vec{r}\rho_N(r) = A\ , \tag{3}$$

where M is the nucleon mass, $\rho_N(r)$ is the density of nuclear substance of target which has A nucleons (we supposed that $A \gg 1$), $f(0)$ is the amplitude of 0-angle scattering

$$f(0) = \frac{k_N \sigma_{tot}}{4\pi}(i+\gamma)\ ,$$

where k_N is the nucleon momentum, $\sigma_{tot} \equiv \sigma_{tot}(E)$ is the total NN cross section, $\gamma \equiv \gamma(E) = \mathrm{Re} f(0)/\mathrm{Im} f(0)$ is the real parameter. As radial distribution $\rho_N(r)$ we use expression [5, 6]

$$\rho_N(r) = \rho_{N0}\left(1 + \exp\frac{r-R}{\Delta}\right)^{-\xi}, \quad R = r_0 A^{1/3} + \Delta \cdot \ln \xi\ ,$$

where ρ_{N0} is normalization constant, Δ is the diffuseness of target surface. The potential of a similar kind (3) was found as useful during the analysis of experimental data on ^{4}He elastic scattering from nuclei both at low [7], and at intermediate energies [8] thus the geometry of usual Woods-Saxon potential was modified in a surface layer of a target that corresponded to introduction of asymmetric parameter $\xi \neq 1$. The similar potential was used also in [9] for calculations of some general nuclear characteristics within the framework of microscopic model. Earlier, within diffraction model framework [10], we have shown also that introduction of additional parameter ξ in Woods-Saxon potential allows us to improve agreement with experiments on elastic nucleon-nucleus scattering.

After integration on polar angle φ_1 (or φ_2), it is possible to present (1) as

$$F(q) = ik\Bigg\{\Phi(-q/2)\int_0^\infty d\rho_1\, \rho_1\, \omega_1(\rho_1) J_0(q\rho_1) + \Phi(q/2)\int_0^\infty d\rho_2\, \rho_2\, \omega_2(\rho_2) J_0(q\rho_2) -$$

$$- \int_{-\infty}^\infty dz \int_0^\infty d\rho_1\, \rho_1\, \omega_1(\rho_1) \int_0^{2\pi} d\varphi_{12} \int_0^\infty d\rho_2\, \rho_2\, \omega_2(\rho_2) \psi_0^2\left(\sqrt{z^2 + (\vec{\rho}_1 - \vec{\rho}_2)^2}\right) J_0\left(\frac{q}{2}\cdot\left|\vec{\rho}_1 + \vec{\rho}_2\right|\right)\Bigg\}, \tag{4}$$

where φ_{12} is the angle between $\vec{\rho}_1$ and $\vec{\rho}_2$, $\Phi(g)$ is the structural form-factor of deuteron

$$\Phi(\vec{g}) = \int d^{(3)}\vec{r}\, \psi_0^2(r) \exp(i\vec{g}\cdot\vec{r}), \quad \Phi(0) = 1\ .$$

If $\psi_0(r)$ is choosed as Gaussian

$$\psi_0(r) \equiv \psi_0^G(r) = \left(\frac{2\lambda}{\pi}\right)^{3/4} \exp[-\lambda r^2] \ , \tag{5}$$

then quadruple integral in (4) can be reduced analytically to the triple one, corresponding form-factor is

$$\Phi^G(g) = \exp(-g^2/8\lambda) \ .$$

Nevertheless, for choosing of $\psi_0(r)$ as Hülten function

$$\psi_0(r) \equiv \psi_0^H(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}} \frac{\exp(-\alpha r)-\exp(-\beta r)}{r}, \ \alpha = \sqrt{M\varepsilon}, \ \beta \cong 7\alpha, \ \varepsilon = 2.23 \ \text{MeV} \tag{6}$$

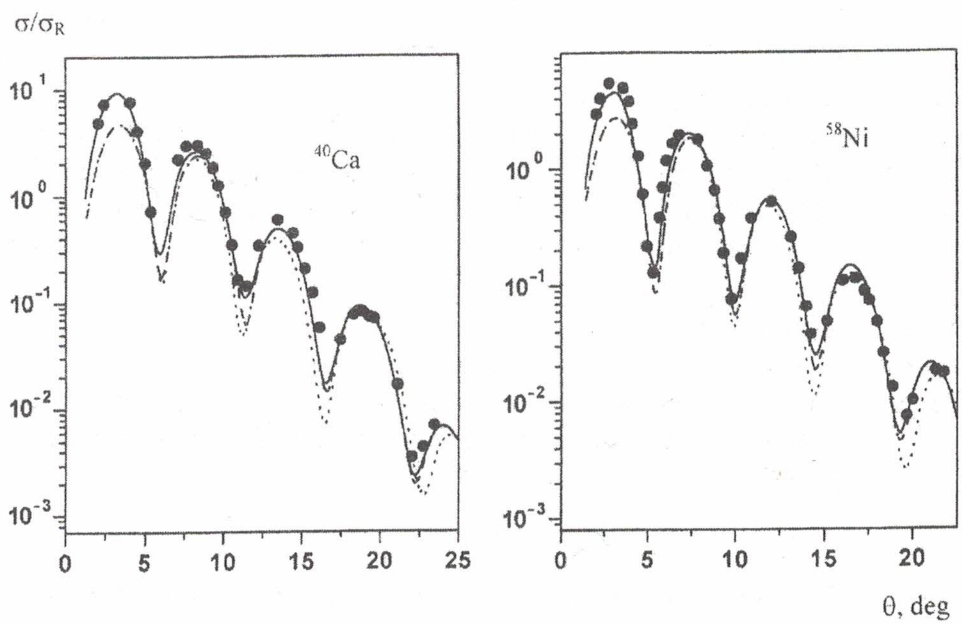
we have to carry out quadruple numerical integration, though the form-factor is calculated explicitly

$$\Phi^H(g) = \frac{\beta(\beta+\alpha)}{(\beta-\alpha)^2\zeta} \text{arctg}\left\{\frac{(\beta-\alpha)^2(\beta+\alpha)\zeta}{\beta(\beta+\alpha)^2+\alpha(3\alpha^2+2\alpha\beta+3\beta^2)\zeta^2+4\alpha^3\zeta^4}\right\}, \quad \zeta = \frac{g}{2\alpha} \ .$$

3. Calculation results and discussion

The computed relations σ/σ_R for 700 MeV deuterons elastic scattering from ^{40}Ca and ^{58}Ni are shown in Figure. There are $\sigma = |F(q)|^2$ is the differential cross section and $\sigma_R = (2kn)^2/q^4$ is the Rutherford one ($n = Ze^2/v$ is the Coulomb parameter, Ze is the charge of target nucleus, $q = 2k\sin(\theta/2)$ is the module of momentum transfer, θ is the scattering angle).

The dot curves were calculated by deuteron wave function (5) and the dash ones by (6). The chi square method was used as criterion of the fitting, thus optimal values of parameters of potential (3) were found as in the Table.



Relations σ/σ_R for deuteron-nucleus elastic scattering at 700 MeV energy of projectiles. Experimental data (bold points) were taken from [11].

Parameters	Targets and types of deuteron wave functions			
	^{40}Ca		^{58}Ni	
	Gaussian (dot curve)	Hülten (dash curve)	Gaussian (dot curve)	Hülten (dash curve)
γ	0,25	0,25	0,24	0,24
ξ	2,27	2,27	2,30	2,30
r_0, Fm	1,11	1,04	1,11	1,07
Δ, Fm	0,42	0,69	0,40	0,65

The values of σ_{tot} in (3) have been taken from [11 and references therein], value of structural parameter λ in (5) defined as $\lambda = (3/16)\cdot\langle r_d^2\rangle^{-1}$ where $\langle r_d^2\rangle^{1/2}$ is root mean square radius of deuteron [12, 13]. From comparison of behaviour of dot and dash curves on Figure it follows that usage of Hülten wave function allows us to reach the best fitting with experiments, especially for region of third and fourth diffraction minima. Therefore all the further calculations (with taking into account the Coulomb interaction) were carried out for Hülten wave function of deuteron.

The Coulomb interaction can be included if we shall add the Coulomb phase

$$\delta_C(\rho_j) = \frac{Ze}{v_j}\ln\left(\frac{1}{2}k\rho_j\right), \quad kR >> 1$$

to the $\delta_N(\rho_j)$ of proton ($j = 2$) [14]. It is necessary to notice, that as is known in general [15], the resulting phase for two potentials of interaction (the nuclear potential and the Coulomb one) does not equal to the sum of phases for each of potentials separately. But for intermediate energies of projectiles, when it is possible to use quasi-classic approximation, such a procedure of simple algebraic addition of phases justifies itself because scattering phase linearly depends on potential of interaction. The approach proposing here leads one to essential simplifications of calculations in comparison with methods which were used in [16]. The solid curves on the Figure, which correspond to the including of the Coulomb interaction, are computed for the same set of parameters (Table) by using the Hülten wave function of deuteron. Features of behaviour for these curves consist in increasing of cross sections values, especially for regions of first diffraction maxima ($\theta \le 7^\circ$) and filling of diffraction minima.

Thus satisfactory description of experimental differential cross sections of deuteron-nucleus elastic scattering at intermediate energies can be reached if it is taken into account the following:

1) the correct radial asymptotic behaviour of intrinsic wave function of deuteron;
2) the real part of nucleon-nucleus potential together with its imagine one (that correspond to $\gamma \ne 0$);
3) the diffuseness of target surface;
4) the Coulomb interaction of deuteron with nucleus.

The relative simplicity of the method has been proposed above gives an opportunity to use it for consideration of interaction with nuclei not only for deuterons but also another weakly-bounded projectiles.

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ПРО ВПЛИВ ЯДЕРНОЇ СТРУКТУРИ ТА КУЛОНОВОЇ ВЗАЄМОДІЇ НА ДИФРАКЦІЙНЕ РОЗСІЯННЯ ДЕЙТРОНІВ ЯДРАМИ

В. І. Ковальчук, В. К. Тартаковський, О. І. Іванова

У рамках дифракційної моделі запропоновано метод розрахунку перерізів дейтрон-ядерного розсіяння при проміжних енергіях падаючих частинок. Дейтронна хвильова функція вибиралася у вигляді хюльтенівської залежності, враховано також дифузність поверхні ядра-мішені та кулонову взаємодію. Розраховані диференціальні перерізи пружного розсіяння дейтронів з енергіями 700 МеВ на ядрах ^{40}Ca, ^{58}Ni задовільно узгоджуються з експериментальними даними.

О ВЛИЯНИИ ЯДЕРНОЙ СТРУКТУРЫ И КУЛОНОВСКОГО ВЗАИМОДЕЙСТВИЯ НА ДИФРАКЦИОННОЕ РАССЕЯНИЕ ДЕЙТРОНОВ ЯДРАМИ

В. И. Ковальчук, В. К. Тартаковский, О. И. Иванова

В рамках дифракционной модели предложен метод расчета сечений дейтрон-ядерного рассеяния при промежуточных энергиях падающих частиц. Волновая функция дейтрона выбиралась в виде хюльтеновской зависимости, учтены также диффузность поверхности ядра-мишени и кулоновское взаимодействие. Рассчитанные дифференциальные сечения упругого рассеяния дейтронов с энергиями 700 МэВ на ядрах ^{40}Ca, ^{58}Ni удовлетворительно согласуются с экспериментальными данными.

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