THE IMAGINARY PART OF THE NUCLEON-NUCLEUS OPTICAL POTENTIAL IN HOT NUCLEI

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In the present paper, we describe a semiclassical method of calculation of the imaginary part of the nucleon-nucleus optical potential, W, in *hot* nuclei. A practical means to account for the temperature dependence of W in statistical particle-emission calculations is also discussed.

1. Introduction

The concept of the nucleon-nucleus optical potential $U+iW$ provides the proven means to go beyond the pure shell-model description. This concept was introduced in 1952 to model the continuum single-particle states, $E > 0$. Later on, it was extended to describe the strength functions of the bound single-particle states, $E = E_F - 0$, and the spreading widths of the hole states $E < E_F$ (E_F is the Fermi energy).

The formal relation of W to the mass operator (e.g. see [1]) established in 1959 [2] paved the way for microscopic calculations of W in terms of the nucleon-nucleon interaction in nuclear matter and in finite nuclei. For a review, see Refs. [3] and [4, 5], respectively.

On formal replacement of the step-function occupation numbers with the temperature-dependent Fermi function, the microscopic expression for W can be used to calculate the imaginary part of the nucleon-nucleus optical potential in hot nuclei. The most important application this quantity finds in microscopic models of friction coefficients of slow collective motion [6, 7].

Calculations of temperature-dependent W in nuclear matter [3, 6, 8] and in finite nuclei [7 - 10] have been performed. The strongest variation of W with the temperature T are found at particle energies close to E_F. According to Ref.[7] owing to the temperature dependence of W, the quadrupole friction coefficients increase from zero at $T \approx 0$ to the wall formula value at $T \approx 3$ MeV.

In the present work, we study the T-dependence of W at particle energies $E = 0\text{-}10$ MeV typical for nucleons evaporated from hot nuclei. It is evident from general considerations that the T-dependence of W at these energies is much weaker than at $E \approx E_F$. On the other hand, evaporative properties are easier to study experimentally than the dissipative ones.

Sections 2 and 3 of this work present the derivation of semiclassical microscopic expression for W in hot nuclei. In Section 4 we perform numerical studies of radial, particle-energy and temperature dependencies of W. A practical means to account for T-dependence of W in statistical particle-emission calculations are discussed in the last section. The parameters involved in computations of W are described in the appendix.

2. Quantum expression for W

In the following the two-body interaction is assumed to be diagonal in spin-isospin quantum numbers s

$$\langle s_1 s_2 | v(\vec{r} - \vec{r}') | s_3 s_4 \rangle = \delta_{s_1 s_3} \delta_{s_2 s_4} v(\vec{r} - \vec{r}') \tag{1}$$

and $v(\vec{r} - \vec{r}')$ is supposed to be a zero-range one

$$v(\vec{r} - \vec{r}') = V_0(\vec{r}) \delta(\vec{r} - \vec{r}') . \tag{2}$$

With these assumptions, the general expression for W (see [1], p. 131, 206, 207) takes the form (we put $\hbar = 1$)

$$W^{(E)}(\vec{r},\vec{r}') = -3\pi \sum_{11'234} \varphi_1(\vec{r})v_{1234}v_{4321'}\varphi_{1'}^*(\vec{r}')\delta(E + E_2 - E_3 - E_4)(n_2\bar{n}_3\bar{n}_4 + \bar{n}_2 n_3 n_4), \qquad (3)$$

where E is the energy of the particle. The $\varphi_i(\vec{r})$ and E_i are the eigenstates and eigenvalues of the single-particle Hamiltonian

$$\hat{H} = -\frac{1}{2m}\nabla^2 + U(\vec{r}),$$

where m is the nucleon mass and $U(\vec{r})$ is the one-body potential. Moreover,

$$v_{1234} = \int d\vec{r}\,\varphi_1^*(\vec{r})\varphi_2^*(\vec{r})V_0(\vec{r})\varphi_3(\vec{r})\varphi_4(\vec{r}) \qquad (4)$$

are the matrix elements of the nucleon-nucleon interaction,

$$n_i = \frac{1}{1 + e^{(E_i - \lambda)/T}} \qquad (5)$$

are the Fermi-gas occupation numbers at temperature T and chemical potential λ and $\bar{n}_i = 1 - n_i$.

In derivation of Eq. (3) we took into account that for the spin-isospin independent zero-range interactions the summation over s_2, s_3, s_4 in the general expression for W [1] is reduced to finding the sum

$$\sum_{s_2 s_3 s_4} \langle s_1 s_2 | s_3 s_4 \rangle \langle s_4 s_3 | (1 - P^{(s)}) | s_2 s_1' \rangle = 3 \langle s_1 | s_1' \rangle,$$

where $P^{(s)}$ is the spin-isospin exchange operator. The omission of the exchange term (as done in Ref.[4]) would give the factor 4 instead of 3 in the above expression.

Making use of the standard presentation of the delta function $\delta(\varepsilon) = (2\pi)^{-1}\int dt\, e^{i\varepsilon t}$, we can present $W^{(E)}(\vec{r},\vec{r}')$ as follows

$$W^{(E)}(\vec{r},\vec{r}') = -3\pi V_0(\vec{r})V_0(\vec{r}')\int\frac{dt}{2\pi}\left\{e^{iEt}G^p(\vec{r}',\vec{r})\left[G^h(\vec{r},\vec{r}')\right]^2 + e^{-iEt}G^h(\vec{r}',\vec{r})\left[G^p(\vec{r},\vec{r}')\right]^2\right\}, \quad (6)$$

where G^p and G^h are determined by

$$G^p(\vec{r},\vec{r}') = \sum_i \varphi_i(\vec{r})e^{iE_i t}n_i\varphi_i^*(\vec{r}'), \qquad G^h(\vec{r},\vec{r}') = \sum_i \varphi_i(\vec{r})e^{-iE_i t}\bar{n}_i\varphi_i^*(\vec{r}').$$

3. Classical approximation

The Weyl symbol of $W^{(E)}(\vec{r},\vec{r}')$ is defined by (e.g. see [15, 16])

$$W_W^{(E)}(\vec{K},\vec{R}) = \int d\vec{x}\,e^{i\vec{K}\vec{x}}W^{(E)}\left(\vec{R} - \frac{\vec{x}}{2}, \vec{R} + \frac{\vec{x}}{2}\right). \qquad (7)$$

As shown in Refs. [11-14], the quantity $W_W^{(E)}(\vec{K},\vec{R})$ taken at $K = \sqrt{2m(E - U(R))}$, provides the best local equivalent of the nonlocal potential $W^{(E)}(\vec{r},\vec{r}')$. In spherical nuclei this quantity will not depend on the directions of $\vec{K}$ and $\vec{R}$ and is denoted as $W_T(E,R)$.

Let us find the classical approximations for the functions G^p and G^h, following the method suggested in Ref. [4]. For that purpose, it is convenient to present them in the form

$$G^p(\vec{r},\vec{r}') = \langle\vec{r}|\hat{G}^p|\vec{r}'\rangle, \qquad G^h(\vec{r},\vec{r}') = \langle\vec{r}|\hat{G}^h|\vec{r}'\rangle \tag{8}$$

with

$$\hat{G}^p = n(\hat{H})e^{i\hat{H}t}, \qquad \hat{G}^h = \bar{n}(\hat{H})e^{-i\hat{H}t} .$$

Here, $n(\hat{H})$ is obtained from n_i by replacing E_i with $\hat{H}$.

Denoting the Weyl symbols of $\hat{G}^{p,h}$ as $G_W^{p,h}(\vec{K},\vec{R})$, we can write

$$G^{p,h}(\vec{r},\vec{r}') = \frac{1}{(2\pi)^3}\int d\vec{K}e^{i\vec{K}(\vec{r}-\vec{r}')}G_W^{p,h}\left(\vec{K},\frac{\vec{r}+\vec{r}'}{2}\right). \tag{9}$$

The classical expressions for $G^{p,h}(\vec{r},\vec{r}')$ are obtained by introducing in r.h.s. of Eq. (9) the classical expressions for $G_W^{p,h}(\vec{K},\vec{R})$. The latter are given by

$$G_W^p = n(H)e^{iHt}, \qquad G_W^h = \bar{n}(H)e^{-iHt} , \tag{10}$$

where

$$H = H_{\vec{K}\vec{R}} = \frac{K^2}{2m} + U(R)$$

is the classical energy of a particle with momentum $\vec{K}$ and coordinate $\vec{R}$.

Calculating $W^{(E)}(\vec{r},\vec{r}')$ defined in (6) with $G^{p,h}$ from (9), (10) and inserting the result into (7), we find the classical expression for the local imaginary potential:

$$W_T(E,R) = -3\pi(2\pi)^{-6}[V_0(R)]^2\int d\vec{K}_2 d\vec{K}_3 d\vec{K}_4 \Delta(R,\vec{K}+\vec{K}_2-\vec{K}_3-\vec{K}_4)\times$$
$$\times \delta(E+H_2-H_3-H_4)[n(H_2)\bar{n}(H_3)\bar{n}(H_4)+\bar{n}(H_2)n(H_3)n(H_4)], \tag{11}$$

where $K = \sqrt{2m(E-U(R))}$, $H_j = H_{\vec{K}_j\vec{R}}$ with $j=2,3,4$ and

$$\Delta(R,\vec{q}) = \frac{1}{(2\pi)^3[V_0(R)]^2}\int d\vec{x}e^{i\vec{q}\vec{x}}V_0\left(\vec{R}+\frac{\vec{x}}{2}\right)V_0\left(\vec{R}-\frac{\vec{x}}{2}\right).$$

It can be shown that $\Delta(R,\vec{q}) \approx \delta(\vec{q})$ for $q \sim \sqrt{2m(E_F-U(0))}$. Following Refs. [5, 10], we adopt this equality at all q. With this assumption our $-2W_T(E,R)$ becomes equivalent to the semiclassical one-particle spreading width in hot infinite systems obtained in Refs. [17, 18] by the Green function method. The lack of the complete coincidence is caused by the fact that in Refs. [17, 18] the spin (and isospin) degrees of freedom are ignored and that we passed to a zero-range force.

The Weyl symbols of $\hat{G}^p = n(\hat{H})e^{i\hat{H}t}$ and $\hat{G}^h = \bar{n}(\hat{H})e^{-i\hat{H}t}$, besides the leading classical terms, contain the terms proportional to the gradients of $U(R)$, which were neglected above. Since these terms can be large in the surface region, Eq.(11) will not be reliable there, especially at $T=0$.

At $\Delta(R,\vec{q}) = \delta(\vec{q})$, the integrations over $\vec{K}_4$ and the directions of $\vec{K}_3$ and $\vec{K}_2$ in Eq. (11) may be performed explicitly. Replacing m with the effective mass $m^*(R)$, we obtain

$$W_T(E_1,R) = -3(2\pi)^{-3} [V_0(R)]^2 [m^*(R)]^3 \int dE_2 dE_3 F(E_1,E_2,E_3) \times$$

$$\times [n(E_2)\bar{n}(E_3)\bar{n}(E_1+E_2-E_3) + \bar{n}(E_2)n(E_3)n(E_1+E_2-E_3)], \quad (12)$$

where

$$F(E_1,E_2,E_3) = \frac{1}{2P_1}\left(\sqrt{P_1^2+P_2^2+2P_1P_2\alpha} - \sqrt{P_1^2+P_2^2-2P_1P_2\alpha}\right),$$

$$\alpha = \frac{P_3\sqrt{P_1^2+P_2^2-P_3^2}}{P_1P_2} \ , \ \text{if} \ (P_3-P_1)(P_3-P_2) > 0 \ ,$$

$$\alpha = 1 \ , \qquad \text{if} \ (P_3-P_1)(P_3-P_2) < 0 \ , \qquad (13)$$

$$P_j = \sqrt{2m^*(R)(E_j-U(R))} \ , \qquad j=1,2,3$$

and

$$n(E) = \frac{1}{1+e^{(E-\lambda)/T}} \ , \qquad \bar{n}(E) = 1 - n(E) \ .$$

At $T=0$, the chemical potential λ converts into the Fermi energy E_F. The integrals over E_2 and E_3 in Eq.(12) in this case can be found analytically. For instance, at $E_1 > E_F$ (and $T=0$), Eq. (12) simplifies to

$$W_0(E_1,R) = -(2\pi)^{-3}[V_0(R)]^2 m^*(R)\frac{1}{10P_1}\left\{(5P_1^2-7P_F^2)P_F^3 + 2(2P_F^2-P_1^2)^{\frac{5}{2}}\vartheta(2P_F^2-P_1^2)\right\}, \quad (14)$$

where $\vartheta(x)$ is the unit step function and

$$P_F = \sqrt{2m^*(R)(E_F-U(R))} \ .$$

Equations (12), (13), (14) agree with the results given in Refs. [5, 10].

4. Illustrative examples

In Fig. 1 we survey the temperature and radial dependence of the absorptive potential $W_T(E,R)$ in the n+^{208}Pb system at $E=5$ MeV. The parameters used in the calculation are given in Appendix. The figure shows that the increase of absorption with growing temperature is quite considerable at this energy.

The rise of absorption in the surface region seen in the figure is caused by the fact that the effective mass increases from its balk value of about $0.7m$ to the value of m outside the nucleus. Note that $m^*(R)$ enters the expression for W in the third power. The factor $[V_0(R)]^2$, which is an increasing function of R, is also contributing to the enhancement of absorption in the surface region.

As seen from Fig. 1, the microscopic absorptive potential at $T=0$ significantly differs, by shape and magnitude, from the phenomenological neutron absorptive potentials by Becchetti and Greenlees [19] and Wilmore and Hodgson [20]. In part, this is related to the fact that we employ a simplified Skyrme interaction, without terms with the derivatives $\nabla_1 - \nabla_2$ acting on the wave function either on the right or on the left (e.g. see [8]). As shown in Ref.[5], full Skyrme interactions, keeping such terms, provide much better agreement with the phenomenological

absorptive potentials. Another reason for deviations of Eqs. (12), (13), (14) from phenomenological potentials is that we omitted quantum corrections.
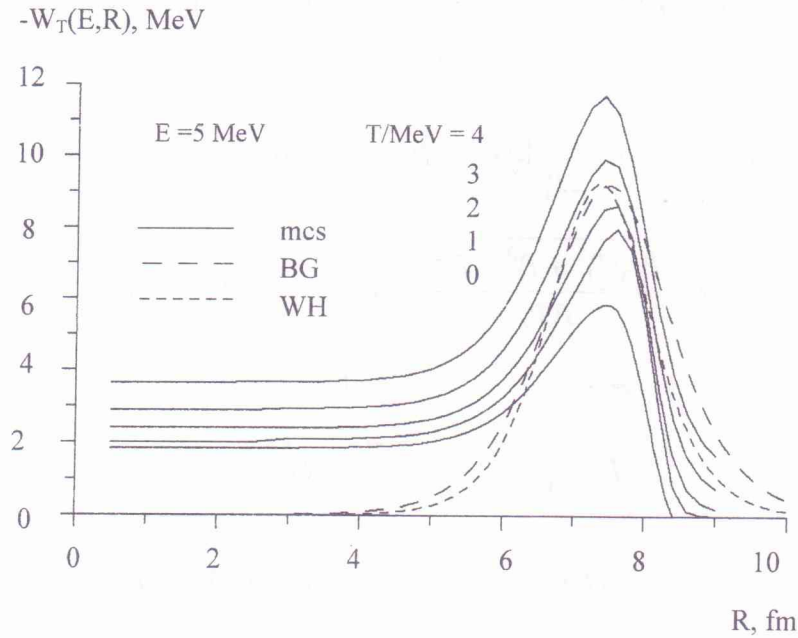
$-W_T(E,R)$, MeV

Fig. 1. Microscopic absorptive potentials in^{208}Pb at $E = 5$ MeV, $T = 0, 1, 2, 3, 4$ MeV are shown as solid lines; Becchetti – Gneenlees and Wilmore – Hodgson potentials as long-dashed and short-dashed lines, respectively.

From the inspection of Eq. (12) for $W_T(E,R)$, it is expected that the general trend of $W_T(E,R)$ as a function of temperature can be described by the factor

$$f_E(T) = g(E,T) / g(E,0),$$

where the function

$$g(E,T) = \int dE_2 dE_3 \left[n(E_2)\bar{n}(E_3)\bar{n}(E + E_2 - E_3) + \bar{n}(E_2)n(E_3)n(E + E_2 - E_3) \right]$$

does not depend on the 2-body interaction.

To verify this expectation we compare in Fig.2 the function $f_E(T)$ with the ratio $W_T(E,R)/W_0(E,R)$ as functions of T. The comparison is made inside the nucleus ($R = 0$) and in the surface region $R = R_{max}$, where $W_T(E,R)$ reaches its maximum. According to Fig. 1, $R_{max} \approx 7.5$ fm. The calculations are performed at $E = 2$ and 5 MeV.

From Fig. 2 one can see that at fixed T, the temperature-enhancement factor $f_E(T)$ is greater for the lower value of E. Such behaviour of $f_E(T)$ is easily to understand from physical considerations (e.g. see [21]). One also observes that *inside* the nucleus the temperature dependence of $W_T(E,R)/W_0(E,R)$ is nicely reproduced by $f_E(T)$ in the whole range of temperatures, whereas a good description of this ratio in the *surface* region is achieved only at high temperatures ($T > 2$ MeV).

5. Conclusion

The range of temperatures $T = 0 - 4$ MeV chosen above is typical for nuclei emerging at different stages of particle-emission cascades accompanying heavy-ion fusion reactions at beam energies below 10 - 20 MeV/u. The transmission coefficients of nucleons used in modeling such cascades are usually calculated with W extracted from reactions on cold, ground-state nuclei. This

contradicts to the principle of the detailed balance, according to which the target nucleus should have the same temperature as the residual nucleus.
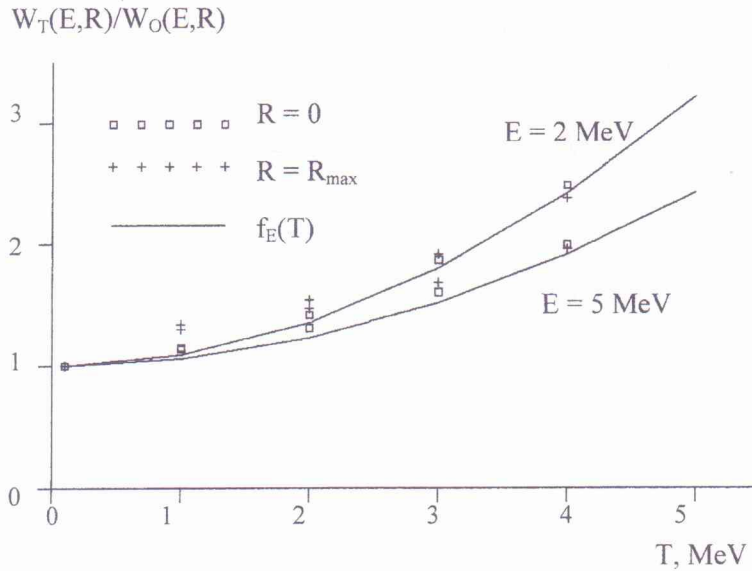

Fig. 2. The ratio $W_T(E,R)/W_0(E,R)$ in ^{208}Pb at $R = 0$ and $R = 7,5$ fm for $E = 2$ and 5 MeV as a function of temperature in comparison with the function $f_E(T)$ defined in the text.

One may expect that the $f_E(T)$ function will be capable to reproduce the temperature trend of $W_T(E,R)/W_0(E,R)$ not only in the simple model, used above, but also in more sophisticated calculations which are consistent at $T = 0$ with the phenomenological absorptive potentials $W_{exp}(E,R)$. Therefore we recommend to introduce in the statistical calculations the absorptive potentials equal to $f_E(T)W_{exp}(E,R)$. This would allow to assess the degree to which the temperature effects in W are experimentally observable.

Acknowledgements

Appendix

The two-body and one-body potentials, effective mass and chemical potential used in our calculations are taken from Refs.[5,10]. The function $V_0(R)$ in Eq. (2) for the nucleon-nucleon potential is chosen in the form

$$V_0(R) = t_0 + t_3\rho(R),$$

$$t_0 = -1020\,\text{MeV}\times\text{fm}^3, \quad t_3 = 2404\,\text{MeV}\times\text{fm}^6,$$

$$\rho(R) = \rho_0\left(1 + e^{(R-R_d)/a_d}\right)^{-1}, \quad \rho_0 = 0.17\,\text{fm}^{-3},$$

$$R_d = \left(1.12A^{\frac{1}{3}} - 0.86A^{-\frac{1}{3}}\right)\text{fm}, \quad a_d = 0.54\,\text{fm}.$$

The mean field potential is given by

$$U(R) = \frac{m}{m^*(R)} \frac{U_0}{1 + e^{(R-R_w)/a_w}},$$

$$U_0 = \left(-45.51 + 17.68\frac{N-Z}{A}\right)\text{MeV},$$

where $R_w = 1.27A^{\frac{1}{3}}\,\text{fm}, \quad a_w = 0.67\,\text{fm}.$

The effective mass is taken in the form

$$\frac{m^*(R)}{m} = 1 - \frac{1 - \frac{m_0^*}{m}}{1 + e^{(R-R_w)/a_w}},$$

where m_0^* is the effective mass in the center of the nucleus $m_0^* = 0.7m$.

The chemical potential λ is determined from the usual equation

$$\int_B^{E_0} dE g(E) n(E) = A$$

where $g(E)$ is the single-particle level density, $B = U(0)$ is the minimum of $U(R)$, E_0 is the upper limit in the integrals over energy. For $E < 0$, $g(E) = g_c(E)$, where

$$g_c(E) = \frac{8}{\pi}\int dR R^2 m^*(R)\sqrt{2m^*(R)(E - U(R))}.$$

For $E > 0$, $g(E) = g_c(0)\sqrt{1 - E/B}$. Throughout this work, $E_0 = 40$ MeV is assumed.

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УЯВНА ЧАСТИНА НУКЛОН-ЯДЕРНОГО ОПТИЧНОГО ПОТЕНЦІАЛУ В НАГРІТИХ ЯДРАХ

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Описано напівкласичний метод розрахунку уявної частини нуклон-ядерного оптичного потенціалу W у нагрітих ядрах. Обговорюються практичні способи врахування температурної залежності W у статистичних розрахунках емісії частинок.

МНИМАЯ ЧАСТЬ НУКЛОН-ЯДЕРНОГО ОПТИЧЕСКОГО ПОТЕНЦИАЛА В НАГРЕТЫХ ЯДРАХ

В. П. Алешин

Описан полуклассический метод расчета мнимой части нуклон-ядерного оптического потенциала W в нагретых ядрах. Обсуждаются практические способы учета температурной зависимости W в статистических расчетах эмиссии частиц.