THE ASYMMETRY CURRENT IN STELLARATORS

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An analysis of drift trajectories of charged particles in tokamak leads us to the conclusion on the possibility of a new longitudinal electric current which appears, in contrast to the bootstrap current, to be proportional to plasma pressure. The qualitative difference in drift trajectories of particles, which move in co- and counter- direction with respect to magnetic field produces velocity-space asymmetry of the trapping boundary of charged particles. As a result the new electric current generates. We named this current an asymmetry current.

The approximate formula for the asymmetry current in tokamaks and stellarators, which is valid for the entire plasma column, including the near-axis region, is obtained. It is shown that the density of asymmetry current is maximal near magnetic axis and decreases at plasma periphery.

The possibility of the experimental detecting of the asymmetry current in stellarators is discussed.

It is known that in tokamaks besides Spitzer longitudinal current, which is induced by electric field, the bootstrap current exist. The bootstrap current is a current parallel to the toroidal magnetic field that is driven by the radial pressure gradient.

In the paper [1] it was shown that in a trap with closed magnetic configuration the new toroidal electric current, named asymmetry current, must exist due to the velocity-space asymmetry of the trapping boundary of charge particles. The driving force of the asymmetry current is the toroidal magnetic field gradient and plasma pressure. This current has its maximal value near the vicinity of a trap magnetic axis.

Practically the existence of this current is connected with the peculiarity of drift trajectories of charge particles in toroidal magnetic field.

Let us begin the discussion from the tokamak case. When analyze particle motion in a tokamak, we use the conditions for three quantities – the total energy, the magnetic moment, and toroidal canonical momentum to be conserved. The magnetic surfaces with circular cross-sections and $\boldsymbol{\varepsilon = r / R << 1}$ will be used.

In this case the drift trajectories of charge particles are described by expression

$$\varepsilon^2 - \varepsilon_s^2 + \sigma_s \varsigma \sqrt{G + \varepsilon_s \cos\theta_s} = \sigma_v \varsigma \sqrt{G + \varepsilon \cos\theta}\,, \tag{1}$$

where ε_s and θ_s are the point on the magnetic surface coordinates, $\sigma_s = \pm 1$ is the sign of the parallel to the magnetic field velocity in the point with coordinates ε_s, θ_s, $G = 1 - {\mu B_0}/{E}$, μ is a particle magnetic moment, $\sigma_v = \pm 1$ is the sign of the parallel to the magnetic field velocity in the point with coordinates ε, θ on the particle drift trajectory, $\varsigma = 2\rho_L q / R$, ρ_L is Larmor radius, q is safety factor.

The analysis of this equation shows that it is some range in $\boldsymbol{G}$ values when particles which move in one direction are trapped and particles which move in opposite direction are untrapped (See Fig. 1). To find this range let us define the trapped particles as the particles which have $v_{II} = 0$ on the trajectory, where v_{II} is parallel to the magnetic field component of particle velocity. The point coordinates in which $v_{II} = 0$ $(\varepsilon_0, \theta_0)$ can be find from the equations

$$\varepsilon_0^2 - \varepsilon_s^2 = -\sigma_s \varsigma \sqrt{G + \varepsilon_s \cos\theta_s} \tag{2}$$

$$\cos\theta_0 = -\frac{G}{\varepsilon_0} \tag{3}$$

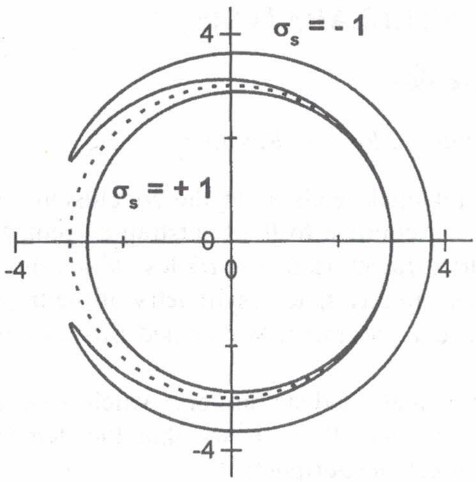

Fig. 1 Trajectories of trapped and untrapped particles with the same parameters $E, G, \varepsilon_s, \theta_s$.

The last trapped trajectory is when $\theta_0 = \pi$ and from (2) and (3) we have

$$G_{\pm} = \sqrt{\varepsilon_s^2 - \sigma_s \varsigma \sqrt{G_{\pm} + \varepsilon_s \cos\theta_s}} \tag{4}$$

To solve this equation we use the successive approximation method and for first approximation we choose $G_{\pm} = \varepsilon_s$. As result we obtain

$$G_+ = 0 \text{ and } G_- = \varsigma^{2/3} \quad \text{for} \quad \varepsilon_s = 0 \tag{5}$$

$$G_{\pm} \approx \varepsilon_s - \sigma_s \varsigma \frac{\sqrt{1+\cos\theta_s}}{2\sqrt{\varepsilon_s}} \quad \text{for} \quad \varepsilon_s > \varsigma^{2/3}. \tag{6}$$

From (5) and (6) one can see that the trapping boundary is asymmetric in phase space.

So the full range of G variations can be divided on three ranges:
1) $-\varepsilon_s \cos\theta_s \le G < G_+$, 2) $G_+ \le G \le G_-$, 3) $G_- < G \le 1$. In the first range all particles are trapped, in the second one the particles which move such a way that rotation transformation rise ($\sigma_s = +1$) are untrapped and particles which move in the opposite direction ($\sigma_s = -1$) are trapped, and in the third range all particles are untrapped. It is not difficult to see that as the trapped particles do not take part in current transport untrapped particles from the second range produce the toroidal current, which we named as asymmetry current.

The density of untrapped particles in the second range is $\Delta n \sim \varsigma n / \varepsilon_s$, and parallel velocity is $v_{II} \sim \sqrt{\varepsilon_s}\, v_T$, where v_T is thermal velocity. So for asymmetry current we have

$$j_A \sim e\Delta n v_{II} \sim \frac{\varsigma e n v_T}{\sqrt{\varepsilon_s}} \sim \sqrt{\varepsilon_s} \frac{cT}{B_\theta} \frac{n}{R}, \tag{7}$$

where B_θ is the poloidal magnetic field. One can see that the asymmetry current density is maximal near trap magnetic axes .

To estimate this current by different way let us use the equation for parallel momentum balance. The untrapped particles which move in different directions exchange by momentum with velocity $\sqrt{\varepsilon_s}\, v\, m \frac{cT}{eB_\theta} \frac{n}{R}$, and all untrapped particles give momentum to the trapped particles with velocity $\sqrt{\varepsilon_s}\, v\, m\, n\, u_{II}$, where v is collision frequency, u_{II} is the average parallel velocity. Consequently, the equation for the parallel velocity build up in the passing domain is

for ions

$$m_i n_i \frac{du_{IIi}}{dt} = \mu_i \left(\frac{cT_i}{eB_\theta} \frac{1}{R} - u_{IIi} \right) + l_{ei} (u_{IIe} - u_{IIi}), \tag{8}$$

where $\mu_i = \sqrt{\varepsilon_s}\, v_i n_i m_i$ is the ion viscosity coefficient, and $l_{ei} = m_e n_e v_{ei}$ is the electron-ion friction coefficient.

for electrons

$$m_e n_e \frac{du_{IIe}}{dt} = \mu_e \left(\frac{cT_e}{eB_\theta}\frac{1}{R} - u_{IIe}\right) - l_{ei}(u_{IIe} - u_{IIi}), \tag{9}$$

where $\mu_e \approx \sqrt{\varepsilon_s}\, m_e n_e \nu_{ei}$ is the electron viscosity coefficient (we have neglected here the electron-electron collision effect for simplicity). If ε is small $\mu_e << l_{ei}$ and for $n_e = n_i$ we have

$$j_A = en(u_{IIi} - u_{IIe}) \sim \frac{\varepsilon_s}{\sqrt{\varepsilon_s} + m_e/m_i}\frac{\mu_e}{\mu_e + l_{ei}}\frac{cn}{B_\theta R}(T_i + T_e) = \frac{ne(\varsigma_e v_{Te} + \varsigma_i v_{Ti})}{\sqrt{\varepsilon_s} + m_e/m_i}. \tag{10}$$

To obtain the value of numerical coefficient in (10) let us write the expression for the flux-surface averaged asymmetry current

$$< j_A > = \frac{1}{2\pi}\int_0^{2\pi} d\theta_s \int_0^{\infty} f dE \int_{-v}^{v} v_{II} dv_{II}\,. \tag{11}$$

The trapped particles carry no net current, because they do not move toroidally except for the toroidal (precession) drift, which is a smaller effect than the asymmetry current. So the distribution function of untrapped particles which carry asymmetry current may be approximated by

$$f_+ = f_M H(v_{II} - v_+) \qquad \text{for} \qquad \sigma_s = +1$$
$$f_- = f_M H(v_{II} - v_-) \qquad \text{for} \qquad \sigma_s = -1 \tag{12}$$

where

$$v_+ = 0 \quad \text{and} \quad v_- = -v\varsigma^{1/3} \qquad \text{for} \qquad \varepsilon_s = 0$$
$$v_\pm = \pm v\sqrt{G_\pm + \varepsilon_s \cos\theta_s} \qquad \text{for} \qquad \varepsilon_s > \varsigma^{2/3} \tag{13}$$

f_M is Maxwellian distribution, and $H(x) = 0$ if $x < 0$ or 1 if $x > 0$.

From Eqs. 11 - 13 we have

$$< j_A > = \frac{1}{4\pi}\int_0^{2\pi} d\theta_s \int_0^{\infty} \left(v_-^2 - v_+^2\right) f_M dE = \frac{1}{2\pi\, m}\int_0^{2\pi} d\theta_s \int_0^{\infty} (G_- - G_+) f_M E dE =$$
$$= \begin{cases} 0.34\varsigma_T^{2/3} nev_T\,, & \varepsilon_s = 0 \\ \dfrac{0.34\varsigma_T nev_T}{\sqrt{\varepsilon_s}}\,, & \varepsilon_s > \varsigma^{2/3} \end{cases}, \tag{14}$$

where $\varsigma_T = \varsigma(v_T)$.

From (14) we have possibility to obtain the expression, which is valid for all plasma columns

$$< j_A > \approx \frac{0.34\varsigma_T nev_T}{\sqrt{\varsigma_T^{2/3} + \varepsilon_s}} \tag{15}$$

For a stellarator we use the model

$$B = B_0(1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(l\theta + M\varphi)) \tag{16}$$

for the magnetic field strength. Here ε_t is the inverse aspect ratio, ε_h is the ripple amplitude, θ and φ are the poloidal and toroidal angles, l is multipolarity and M is toroidal period number. In this case we have

$$G_+ = 0 \quad \text{and} \quad G_- = \varsigma^{2/3} \qquad \text{for} \quad \varepsilon_s = 0 \tag{17}$$

$$G_\pm \approx \varepsilon_s - \sigma_s \varsigma \frac{\sqrt{1 + \cos\theta_s + \frac{\varepsilon_{hs}}{\varepsilon_s}\cos(l\theta_s + M\varphi)}}{2\sqrt{\varepsilon_s}} \qquad \text{for} \quad \varepsilon_s > \varsigma^{2/3} \tag{18}$$

A stellarator is non-axisymmetric trap and for obtaining the asymmetry current value we need to fulfill averaging through toroidal and poloidal angles. The numerical calculations show that for $0 \le \varepsilon_h / \varepsilon_t \le 2$, $2 \le l \le 4$ and $4 \le M \le 10$ for the asymmetry current estimations we can use Eq. 4 if we change the safety factor value q by $1/\mu_{IM}$ where μ_{IM} - is rotation transform.

In Fig.2 one can see the current density distribution experimental data obtained in the stellarator Proto-Cleo [2] and the theoretical estimations – trace 1 demonstrates the asymmetry current and trace 2 demonstrate the bootstrap current, solid line is full noninductive current. The asymmetry current value is obtained for $\mu_{eff} = 2$ near magnetic axes (in Proto-Cleo $l = 3$) and we accepted that asymmetry current dependence on collision frequency is the same as in bootstrap current. One can see that it is some qualitative accordance between experimental and theoretical data.

A stellarator is the system without Ohmic current and so it is not difficult, in principle, to find any noninductive current in it. In Fig. 3 one can see the calculated dependence of rotation

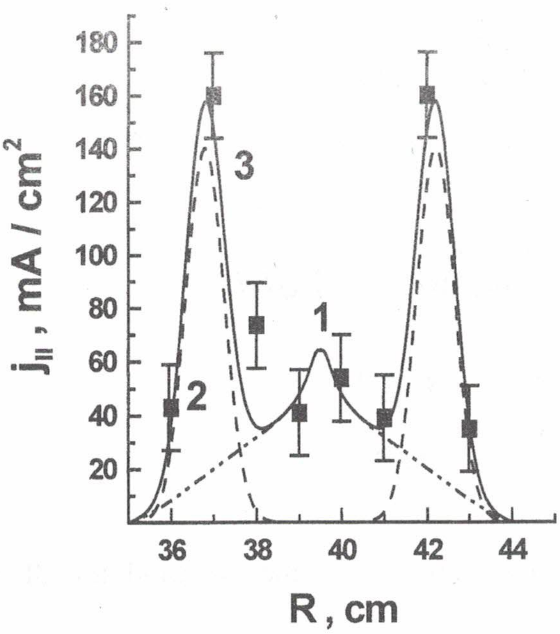

Fig. 2 Experimental measurements (points) and theoretical estimations. *1* – asymmetry current, *2* – bootstrap current, solid line – full non-inductive current.

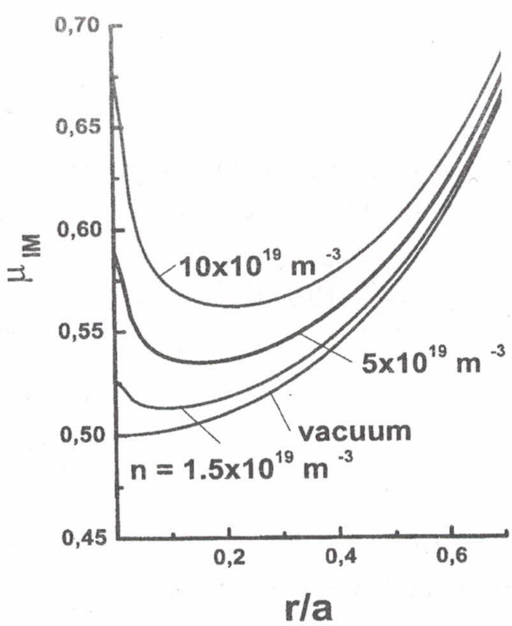

Fig. 3 Dependence of rotation transform on plasma density in LHD stellarator.

transform on plasma density in LHD stellarator [3]. The change of μ_{IM} in comparison with vacuum case is produced by asymmetry current.

We believe that such rotation transform modification near magnetic axis can be find in the future experiments.

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СТРУМ АСИМЕТРІЇ У СТЕЛАРАТОРАХ

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Аналіз дрейфових траєкторій заряджених частинок у токамаці приводить нас до висновку про можливість нового поздовжнього електричного струму, який , на відміну від бутстреп струму, не є пропорційним тиску плазми. Якісні відмінності дрейфових траєкторій частинок, що рухаються у напрямку магнітного поля та проти нього, створюють асиметрію границі між захопленими та пролітними частинками у просторі швидкостей. Як наслідок, виникає новий електричний струм. Ми назвали цей струм струмом асиметрії. Одержано наближений вираз для струму асиметрії в токамаках і стелараторах, який є справедливим для будь-якої області плазми, включаючи приосьову. Показано, що густина струму асиметрії є максимальною біля магнітної осі і зменшується до периферії плазми. Обговорюється можливість експериментального спостереження струму асиметрії в стелараторах.