

## AVERAGED DESCRIPTION OF 3D MHD EQUILIBRIUM \*

S. Yu. Medvedev <sup>1</sup>, V. V. Drozdov <sup>1</sup>, A. A. Ivanov <sup>1</sup>, A. A. Martynov <sup>1</sup>,  
Yu. Yu. Poshekhonov <sup>1</sup>, M. I. Mikhailov <sup>2</sup>

<sup>1</sup> *Keldysh Institute for Applied Mathematics, Moscow Russia*

<sup>2</sup> *Institute for Nuclear Fusion, Russian Research Centre "Kurchatov Institute", Moscow, Russia*

A general approach by S. A. Galkin et al. in 1991 to 2D description of MHD equilibrium and stability in 3D systems was proposed. The method requires a background 3D equilibrium with nested flux surfaces to generate the metric of a Riemannian space in which the background equilibrium is described by the 2D equation of Grad-Shafranov type. The equation can be solved then varying plasma profiles and shape to get approximate 3D equilibria. In the framework of the method both planar axis conventional stellarators and configurations with spatial magnetic axis can be studied. In the present report the formulation and numerical realization of the equilibrium problem for stellarators with planar axis is reviewed. The input background equilibria with nested flux surfaces are taken from vacuum magnetic field approximately described by analytic scalar potential.

**1 Background** Modelling of 3D equilibrium plasma configurations is a challenging task. Full 3D equilibrium codes PIES [1] and HINT [2] were employed for that during the last decade. The existence of magnetic islands and stochastic magnetic field regions makes the modelling time consuming, not very robust and flexible, and hardly useful for systematic equilibrium optimization and stability analysis. More tractable conventional model for 3D MHD equilibrium and stability studies is based on the nested magnetic surface approximation. The standard 3D nested flux surface equilibrium code is VMEC [3]. The 3D stability codes TERPSICHORE [4] and CAS3D [5] use the VMEC solution as an input. The variational formulation of the equilibrium problem is used in VMEC. Solution representation with a finite series of harmonics and higher harmonic damping by poloidal variable choice provide the needed regularization to find an approximate nested flux surface equilibrium. However the code convergence is sensitive to the choice of harmonic set and significantly deteriorates with increasing resolution.

For equilibrium and stability studies based on the averaging methods a lot of numerical codes have been developed: STEP in Princeton [6], FAR, RSTEQ, NAV, RST [7] in Oak Ridge and others. Comparison of the results determined with these 2D and 3D codes shows their good agreement for both equilibrium and stability in planar-axis stellarators [7, 8].

The approach to 2D description of MHD equilibrium and stability proposed in [9] is more general. The key idea is connected with the fact that all average methods mentioned above are based on some approximation (less or more accurate) of real 3D metric tensor through 2D one. It is equivalent to introducing Riemannian space  $\mathbb{R}^3$ , in which reference 3D equilibrium is close to symmetrical one. The first step in such interpretation was carried out by Degtyarev and Drozdov in [10, 11], where it was shown that for arbitrary 3D equilibrium (with nested magnetic surfaces at least) one can construct some formal 2D metric tensor and obtain 2D Grad-Shafranov type equation. The equation was obtained by averaging exact 3D equation. In fact, it is the exact zero 2D moment of equilibrium equation, like Kruskal-Kulsrud equation is the exact zero 1D moment.

**1.1 Scalar equations for 3D MHD equilibria description** By assuming the magnetic surfaces  $a(r) = const$  exist the ideal MHD equilibrium problem

$$\mathbf{j} \times \mathbf{B} - \nabla p = 0, \quad \mathbf{j} = \nabla \times \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0 \quad (1)$$

can be reduced to the field equations

---

\* **Acknowledgement** The work is supported in part by the Joint Research Project "Novel Approaches to Improve Confinement in 3-D Plasma Magnetic Systems" INTAS 99-00592.



$$\nabla \cdot \mathbf{B} = 0, \quad \mathbf{B} \cdot \nabla a = 0, \quad \nabla \cdot (\mathbf{B} \times \nabla a) = 0 \quad (2)$$

and to the force balance equation

$$|\nabla a|^{-2} (\mathbf{B} \times \nabla a) \cdot \nabla \times \mathbf{B} = dp(a)/da. \quad (3)$$

This system reflects the important property of equilibrium plasma configuration – the coincidence of magnetic and current surfaces with isobars.

Separate consideration of the set of equations (2) allows one to conclude that equilibrium magnetic field is determined in fact only by shape of magnetic surfaces. Namely, the following statement is valid (see [10, 12] for example).

**Statement.** For any a priori given family of nested toroidal surfaces  $a(r) = const$  the full set of solutions of (2) can be represented by the linear combinations

$$\mathbf{B} = \Phi' \nabla a \times \nabla \theta_\psi + \Psi' \nabla a \times \nabla \zeta_\psi, \quad \mathbf{B} = J \nabla_a \theta_F + F \nabla_a \zeta_F \quad (4)$$

while each summand satisfies (2).

Here

$$(\cdot)' = d(\cdot)/da, \quad \nabla_a(\cdot) = \mathbf{n} \times \nabla(\cdot) \times \mathbf{n}, \quad \mathbf{n} = \nabla a / |\nabla a|,$$

and the pairs of the coefficients  $\Phi(a), \Psi(a)$  and  $J(a), F(a)$  are arbitrary and refer to toroidal and external poloidal (helicoidal) fluxes or currents. The pairs of the basis vectors  $\nabla a \times \nabla \theta_\psi, \nabla a \times \nabla \zeta_\psi$  and  $\nabla_a \theta_F, \nabla_a \zeta_F$  are particular solutions of (2). They are generated by the cyclic functions  $\theta_{\psi,F}, \zeta_{\psi,F} \in [0, 1) \times [0, 1)$ , which can be interpreted as poloidal (helicoidal) and toroidal angles, satisfying equations

$$L_\psi \theta_\psi = L_\psi \zeta_\psi = 0, \quad L_\psi = \nabla \cdot |\nabla a|^2 \nabla_a(\cdot), \quad L_F \theta_F = L_F \zeta_F = 0, \quad L_F = \nabla \cdot \nabla_a(\cdot). \quad (5)$$

From this, it is seen that  $\theta_{\psi,F}, \zeta_{\psi,F}$  depend on the shape of magnetic surfaces only, while each summand in the flux (contravariant) field representation or in the current (covariant) representation (4) satisfies all the properties of (2). The relations between the coefficients and basis vectors in (4), (5) can be written as

$$\begin{aligned} J &= -\alpha_{22} \Psi' + \alpha_{23} \Phi', & \alpha_{22} \nabla_a \theta_F + \alpha_{23} \nabla_a \zeta_F &= -\nabla a \times \nabla_a \zeta_\psi, \\ F &= -\alpha_{23} \Psi' + \alpha_{33} \Phi', & \alpha_{23} \nabla_a \theta_F + \alpha_{33} \nabla_a \zeta_F &= \nabla a \times \nabla_a \theta_\psi. \end{aligned} \quad (6)$$

Here matrix elements  $\alpha_{ik}(a) = \mathbf{e}_i^F \cdot \mathbf{e}_k^\psi / \sqrt{g_\psi}$  depend on the shape of magnetic surfaces and volume  $V(a)$  inside magnetic surfaces  $a = const$  only, for example,

$$V'(a) \alpha_{22}(a) = (\nabla V \times \zeta_\psi) \cdot (\nabla V \times \zeta_F) / (\nabla V \cdot \nabla \theta_F \times \zeta_F),$$

$\mathbf{e}_i$  - contravariant basis vectors and  $\sqrt{g}$  - Jacobian of flux coordinate system  $(a, \theta, \zeta)$ .

Using other combinations of particular solutions of (2) a pair of mixed representations (in terms of flux and current) of magnetic field is can obtained:

$$\mathbf{B} = \nabla \Psi \times \mathbf{b}_3^F + F \mathbf{b}_3^\psi, \quad \mathbf{B} = \nabla \Phi \times \mathbf{b}_2^F + F \mathbf{b}_2^\psi, \quad (7)$$

where, as before, each component satisfies equations (2). The vectors  $\mathbf{b}_{2,2}^{\psi,F}$  are tangential to magnetic surfaces. They can be written as linear combinations of  $(\nabla_a \theta_{\psi,F}, \nabla_a \zeta_{\psi,F})$ , for example,

$$\mathbf{b}_3^\psi = \nabla_a \zeta_F + \frac{\alpha_{23}}{\alpha_{33}} \nabla_a \theta_F = \mathbf{e}_3^\psi / (\mathbf{e}_3^F \cdot \mathbf{e}_3^\psi), \quad \mathbf{b}_3^F = \nabla_a \zeta_\psi + \frac{\alpha_{23}}{\alpha_{33}} \nabla_a \theta_\psi = \mathbf{e}_3^F / (\mathbf{e}_3^F \cdot \mathbf{e}_3^\psi). \quad (8)$$

Hence for constructing the specific field  $\mathbf{B}$ , which provides the plasma MHD equilib-

rium configuration, it is sufficient to know only the shape of magnetic surfaces and the distribution of any pair from fluxes and currents over these surfaces. Substitution of the magnetic field (in any form) into the force-balance equation (3) yields the equation for function  $a(\mathbf{r})$  and, hence, complements (4)-(8) to a close system of equations.

Such a method with  $\mathbf{B}$  in the form (7) generates three-dimensional analog of the two-dimensional Grad-Shafranov equilibrium equation

$$\begin{aligned}
 -\mathbf{j} \cdot \left( \mathbf{b}_3^F + F \frac{\mathbf{b}_3^\psi \times \nabla \Psi}{|\nabla \Psi|^2} \right) &\equiv \nabla \cdot (|\mathbf{b}_3^F|^2 \nabla \Psi) + |\mathbf{b}_3^\psi|^2 F \frac{dF}{d\psi} - F \mathbf{b}_3^F \cdot \nabla \times \mathbf{b}_3^\psi - \\
 (\mathbf{b}_3^F \times \nabla \times \mathbf{b}_3^F) \cdot \nabla \Psi + (\mathbf{b}_3^F \times \mathbf{b}_3^\psi) \cdot \nabla \Psi \frac{dF}{d\psi} + & \\
 F \frac{\mathbf{b}_3^\psi \times \nabla \Psi}{|\nabla \Psi|^2} \cdot \nabla \times (\mathbf{b}_3^F \times \nabla \Psi) - F^2 \frac{\mathbf{b}_3^\psi \times \nabla \Psi}{|\nabla \Psi|^2} \cdot \nabla \times \mathbf{b}_3^\psi &= -\frac{dp}{d\Psi}.
 \end{aligned} \quad (9)$$

For axisymmetric systems the vectors  $\mathbf{b}_3^{F,\psi}$  do not depend on a shape of magnetic surfaces and can be found in an explicit form:  $\mathbf{b}_3^F = \mathbf{b}_3^\psi = \nabla \phi$ .

The flux representation (4) leads to the equilibrium equations

$$\nabla \cdot (\nabla \Lambda \times \mathbf{B}) = -\frac{dp}{da}, \quad \Lambda = \Phi' \theta_\psi + \Psi' \zeta_\psi. \quad (10)$$

Equation (10) together with some modification of (4), (5) is, in fact, a base for existing 3D equilibria codes BETA [13], VMEC [3], POLAR-3D [14].

Using the representation of the current density in arbitrary flux coordinate system

$$\mathbf{j} = \nabla J \times \nabla \theta + \nabla F \times \zeta - \nabla \nu \times \nabla a$$

it is straightforward to obtain one dimensional zero moment of equilibrium equation (10) – the Kruskal-Kulsrud equation

$$p'V' = J'\Psi' - F'\Phi'.$$

**1.2 2D Grad-Shafranov type equation for 3D plasma equilibrium as the exact zero two-dimensional moment for magnetohydrostatics** The following statement was formulated in [10, 11]:

**Statement.** For any 3D plasma equilibrium (with nested flux surfaces at least) there exist coordinate system  $(x^1, x^2, \zeta)$  and corresponding Riemannian space  $\mathbb{R}^3$  in which the following conditions are satisfied: i. metric tensor  $\hat{g}_{ik}(\mathbf{r})$  is two-dimensional:  $\frac{\partial}{\partial \zeta} \hat{g}_{ik} = 0$ ; ii. poloidal (helical) flux function  $\Psi$  is two-dimensional  $\Psi = \Psi(x^1, x^2)$  and it is the solution of 2D Grad-Shafranov type equation

$$\hat{\nabla} \cdot \left( \frac{\hat{\nabla} \Psi}{\hat{g}_{33}} \right) + \frac{F}{\hat{g}_{33}} \frac{dF}{d\Psi} - \hat{\nabla} \cdot \left( \frac{\hat{\mathbf{e}}_3 \times \hat{\mathbf{e}}^3}{\hat{g}_{33}} \right) = -\alpha \frac{dp}{d\Psi}; \quad (11)$$

iii. magnetic field takes the form  $\mathbf{B} = (\nabla \Psi \times \mathbf{e}_3 + B_3 \mathbf{e}_3) / g_{33}$ , with  $\mathbf{e}_3 = \partial \mathbf{r} / \partial \zeta$ ,  $\langle B_3 \rangle_\zeta = F(\Psi)$ .

Here  $\mathbb{R}^3$  is generated by the metric tensor

$$\hat{g}_{ik} = \sqrt{\hat{g}} \langle \frac{g_{ik}}{\sqrt{\hat{g}}} \rangle_\zeta, \quad \hat{g} = \det \hat{g}_{ik} = \det^{-2} \langle \frac{g_{ik}}{\sqrt{\hat{g}}} \rangle_\zeta, \quad \hat{g}^{ik} = \hat{G}_{ik} / \hat{g}, \quad (12)$$

and

$$\alpha = \alpha(x^1, x^2) = \langle \sqrt{\hat{g}} \rangle_\zeta / \sqrt{\hat{g}},$$



$\hat{\nabla}$  is an  $\nabla$ -operator in  $\mathbb{R}^3$ . In these formulas  $\langle f \rangle_\zeta = \frac{1}{\zeta_{max}} \int_0^{\zeta_{max}} f(x^1, x^2, \zeta) d\zeta$ .

For coordinate transformation in  $\mathbb{R}^3$   $x^i \rightarrow x_*^i$  one can use general transition formulas

$$(\hat{g}_{ik})_* = \sum_{m,n} \hat{g}_{mn} \frac{\partial x^m}{\partial x_*^i} \frac{\partial x^n}{\partial x_*^k}, \quad (\sqrt{\hat{g}})_* = \sqrt{\hat{g}} \frac{\mathcal{D}(x^i)}{\mathcal{D}(x_*^i)}.$$

Therefore, each coordinate system, connected with  $(x^1, x^2, \zeta)$  by 2D transformation  $x_*^{1,2} = x_*^{1,2}(x^1, x^2)$ ,  $\zeta_* = \zeta - \lambda(x^1, x^2)$  generates the same  $\mathbb{R}^3$ . For such coordinate transformations the equation (11) is invariant and can be written in an explicit 2D form:

$$\frac{1}{\sqrt{\hat{g}}} \sum_{i,k=1}^2 \frac{\partial}{\partial x^i} \left( \frac{\hat{G}_{ik}}{\hat{g}_{33}\sqrt{\hat{g}}} \frac{\partial \Psi}{\partial x^k} \right) = -\alpha \frac{dp}{d\Psi} - \frac{F}{\hat{g}_{33}} \frac{dF}{d\Psi} + \frac{1}{\sqrt{\hat{g}}} \left( \frac{\partial}{\partial x^1} \left( \frac{\hat{g}_{23}}{\hat{g}_{33}} \right) - \frac{\partial}{\partial x^2} \left( \frac{\hat{g}_{13}}{\hat{g}_{33}} \right) \right) F. \quad (13)$$

Equation (11) or (13) can be obtained by averaging the exact 3D equilibrium equation (9) if the reference coordinate system is chosen as following  $x^{1,2} = x^{1,2}(a, \theta_\psi)$ ,  $\zeta = \zeta_\psi - \lambda(a, \theta_\psi)$ .

**2 Averaged 3D equilibrium modelling** An example of the application of the method to the equilibrium and stability modelling of the LHD device and comparison with other code results can be found in [15]. The background vacuum equilibrium for the averaged 2D treatment was obtained there by the POLAR-3D code. Here another possibility of the background configuration choice is explored – the approximate vacuum equilibrium given by the scalar potentials of helically symmetric magnetic fields [16].

**2.1 Vacuum equilibrium from analytic scalar potential** The local polar coordinate system  $(r, \phi, \omega)$  is introduced connected to the cylindrical coordinates  $(R, \phi, Z)$  by the transformation

$$R = R_0 + r \cos \omega, \quad Z = r \sin \omega, \quad \phi = \phi, \quad (14)$$

where  $R = R_0$  is the circular stellarator axis. Then a flux coordinate system of the vacuum magnetic field  $(\rho, u, \phi)$  can be found in the following form:

$$r = \rho + \delta(\rho, u, \phi), \quad \omega = u + \lambda(\rho, u, \phi), \quad \phi = \phi, \quad (15)$$

where for the small corrections  $\delta, \lambda$  the approximate expressions are valid:

$$\delta = h^2 \frac{R_0}{B_0} \frac{\partial \hat{\Phi}_{st}}{\partial \rho}, \quad \lambda = -\frac{h^2}{\rho^2} \frac{R_0}{B_0} \frac{\partial \hat{\Phi}_{st}}{\partial u}, \quad (16)$$

with  $h = 1 - \frac{1}{R_0} \rho \cos u$ . The function  $\hat{\Phi}_{st} = \int \tilde{\Phi}_{st} d\phi$  is the primitive function of the sum of the scalar potentials of the helically symmetric vacuum fields with different helicity:

$$\Phi_{st} = B_0 R_0 \phi + \tilde{\Phi}_{st}, \quad \tilde{\Phi}_{st} = \sum_{l,m} B_0 \epsilon_{lm} I_l \left( \frac{m\rho}{R_0} \right) \sin(lu - m\phi), \quad (17)$$

where  $B_0$  is the value of magnetic field at the stellarator axis,  $I_l$  are modified Bessel functions. In particular, for a single harmonic the (16) can be rewritten as

$$\delta = h^2 \epsilon \frac{l R_0}{m} I_l' \left( \frac{m\rho}{R_0} \right) \cos(lu - m\phi), \quad \lambda = -h^2 \epsilon \frac{l^2 R_0^2}{m^2 \rho^2} I_l \left( \frac{m\rho}{R_0} \right) \sin(lu - m\phi). \quad (18)$$

**2.2 Numerical procedure** The first step in the averaged 3D equilibrium modelling is a choice of the background configuration. The vacuum magnetic field of the WVII-A and Heliotron-E stellarator in the single helical magnetic field harmonic approximation are described by the parameters [16]:  $R_0/\rho_{max} = 20$ ,  $l = 2$ ,  $m = 5$ ,  $\epsilon = 0.43$ , and  $R_0/\rho_{max} = 10$ ,  $l = 2$ ,  $m = 19$ ,  $\epsilon = 0.32$ , respectively. The magnetic surfaces were constructed using (18).

At the second step the magnetic surface coordinates were used as an input to the module which computes the "natural" flux coordinate system (4) solving a series of two-dimensional equations (5) on magnetic surfaces and produces the coefficients  $\alpha_{ik}$  (6) which connect the fluxes and currents, and the averaged metric coefficients (12). For the vacuum case ( $F = const$ ,  $J = 0$ ) the fluxes  $\Psi$  and  $\Phi$  and the rotational transform  $\iota = -\Psi'/\Phi'$  profile as well as all magnetic field can be found using only the coordinates of magnetic surfaces as input.

Fig.1 and Fig.2 demonstrate  $|\mathbf{B}|^2$  level lines on magnetic surfaces and in the natural flux coordinates reconstructed from the magnetic surface geometry for the WVII-A and the Heliotron-E cases.

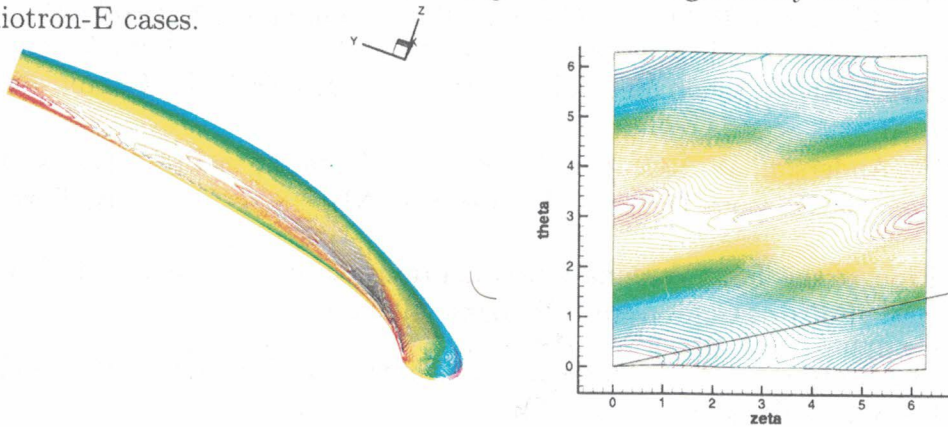


Fig.1  $|\mathbf{B}|^2$  level lines on the boundary magnetic surface and plasma cross section in real space and in flux coordinates  $\theta_\psi, \zeta_\psi$  for the WVII-A case. Dashed line shows the magnetic field line slope.

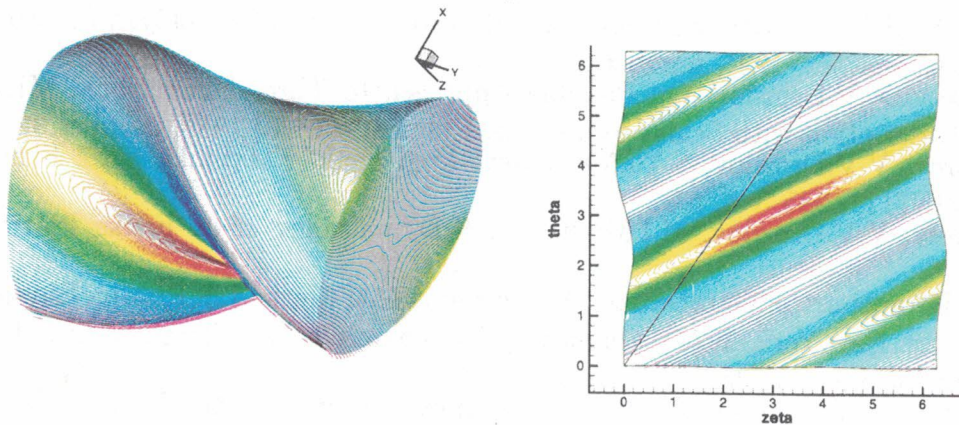


Fig.2  $|\mathbf{B}|^2$  level lines on the boundary magnetic surface and plasma cross section in real space and in flux coordinates  $\theta_\psi, \zeta_\psi$  for the Heliotron-E case. Dashed line shows the magnetic field line slope.

The third step is a solution of the equation (13). The first natural test is a reproduction of the background equilibrium in the Riemannian space – the Grad-Shafranov type of equation should satisfy if  $\Psi(x^1, x^2) = \Psi(a)$ .



Finally the the magnetic field in real space space should be reconstructed from the solution of (13) and the coordinate transformation  $\mathbf{r} = \mathbf{r}(\Psi, \theta_\psi, \zeta_\psi)$  given by harmonics of the magnetic surface coordinates, for example.

**3 Future work** The averaged equilibrium treatment proposed in [9] on the base of analytic vacuum magnetic field background configurations is under development. The calculations of zero net current equilibria with finite  $\beta$  on the base of averaged equation (13) and the reconstruction of the equilibrium magnetic field in real space would make possible robust optimization of the stellarator equilibria varying the set of helical field harmonics and plasma profiles.

## References

- [1] A.Rieman, H.Greenside, *Comput. Phys. Commun.* **43**(1986)157
- [2] T.Hayashi, in *Proc. of the Joint Varenna-Lausanna International Workshop on theory of fusion plasmas*, (Editrice Compositori, Bologna, 1988) 11
- [3] S.P.Hirshman, W.I. van Rij, P. Merkel, *Comput. Phys. Commun.* **43** (1986) 143
- [4] D.V.Anderson, W.A.Cooper, U.Schwenn, R.Gruber, in *Proc. of the Joint Varenna-Lausanna International Workshop on theory of fusion plasmas*, (Editrice Compositori, Bologna, 1988) 93  
U.Schwenn, D.V.Anderson, W.A.Cooper, R.Gruber, S.Merazzi, in *Proc. 17th Conf. on Contr. Fusion and Plasma Heating*, Amsterdam (NL) 1990, Vol. 14B, Part II (1990) 931
- [5] C.Schwab, in *Proc. of the Joint Varenna-Lausanna International Workshop on theory of fusion plasmas*, (Editrice Compositori, Bologna, 1988) 85
- [6] J.L.Johnson, *Comput. Phys. Reports* **4**(1986) 37
- [7] V.E.Lynch, B.A.Carreras, L.A.Charlton, L.Garcia, T.C.Hender, H.R.Hicks, J.A.Holmes, *J. Comput.Phys.*, **66**(1986) 411  
L.A.Charlton, J.A.Holmes, H.R.Hicks, V.E.Lynch, B.A.Carreras, *J. Comput.Phys.*, **63** (1986) 107
- [8] F.Herrnegger, P.Merkel, J.L.Johnson, *J. Comput. Phys.*, **66** (1986) 445
- [9] S.A.Galkin, V.V.DrozdoV, A.A.Martynov. *Variational Approach to Constructing of Approximate Models for MHD Equilibrium and Stability*. Preprint Keldysh Institute of Applied Mathematics, 1991, number 52
- [10] L.M.Degtyarev, V.V.DrozdoV. *On Possible Approach to Three-Dimensional MHD Equilibrium Description with Scalar Equations*. Preprint KIAM, 1984, number 32
- [11] L.M.Degtyarev, V.V.DrozdoV, S.Yu.Medvedev. *Itogi nauki i tekhniki. Ser.: Fizika Plazmy. M.:VINITI*, 1985, v.6, p.81
- [12] L.M.Degtyarev, V.V.DrozdoV, M.I.Mikhailov, V.D.Shafranov *et al.* *Fizika Plazmy* **11**(1985)39
- [13] F.Bauer, O.Betancourt, P.Garabedian, *A Computational Method in Plasma Physics* (Springer-Verlag, New York, 1978). *Magnetohydrodynamic Equilibrium and Stability of Stellarators* (Springer-Verlag, New York, 1978).
- [14] L.M.Degtyarev, V.V.DrozdoV, Yu.Yu.Poshekhonov, Preprint of the KIAM, USSR Academy of Sciences, 1987, numbers 102,137,159,182.  
L.M.Degtyarev, V.V.DrozdoV, Yu.Yu.Poshekhonov, in *Proc. 14th Europ. Conf. on Contr. Fusion and Plasma Phys.*, Madrid, 1987, v. 11D, part 1, 377.
- [15] Y.Nakamura *et al.* Comparison of the Calculations of the Stability Properties of a Specific Stellarator Equilibrium with Different MHD Stability Codes. *J. Comput. Phys.* **128**(1996)43
- [16] V.D.Pustovitov, V.D.Shafranov. In *Reviews of Plasma Physics*. Ed. B.B.Kadomtsev. Consultants Bureau, N.Y., vol.15, 1990

**УСЕРЕДНЕНИЙ ОПИС ТРИВИМІРНОЇ МГД РІВНОВАГИ**

**С. Ю. Медведєв, В. В. Дроздов, А. А. Іванов,  
А. А. Мартинов, Ю. Ю. Пошехонов, М. І. Михайлов**

Загальний підхід до двовимірного опису МГД рівноваги та стійкості в тривимірних системах запропоновано С. А. Галкіним та ін. у 1991 р. Метод вимагає задання базової тривимірної рівноваги із вкладеними магнітними поверхнями для отримання метрики Ріманового простору, в якому рівновага описується двовимірним рівнянням типу Греда – Шафранова. Останнє може бути розв'язаним шляхом варіації профілів та форми плазми з метою отримання приблизної тривимірної рівноваги. У рамках методу можна вивчати як традиційні плосковісні стеларатори, так і конфігурації із просторовими магнітними осями. У цій роботі розглянуто формулювання і чисельне розв'язання задачі рівноваги для стелараторів із плоскою віссю. Базову рівноважну конфігурацію із вкладеними магнітними поверхнями береться з вакуумного магнітного поля, яке наближено описується аналітичним скалярним потенціалом.

