

THE α -PARTICLES CONFINEMENT IN ZERO AND
FINITE β MIRROR-TYPE STELLARATORS

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Helical magnetic systems with poloidal direction of the lines $B = \text{constant}$ on the magnetic surfaces are investigated to clarify in more detail the connection between the shape of the magnetic surfaces and the topology of the $B = \text{constant}$ surfaces on the one hand and particle confinement on the other. The possibilities to fulfil the pseudosymmetry condition as well as the condition that the second adiabatic invariant J_{\parallel} forms closed contours are investigated numerically for almost zero and finite β values.

Introduction

As was shown in Ref. [1], the fulfilment of the quasi-isodynamicity (qi) condition in configurations with poloidal direction of lines $B = \text{constant}$ on the magnetic surfaces leads to good particle confinement. Only the particles that are near trapped-transition boundaries can leave the plasma volume after a large number of changes of direction in the motion. The existence of such particles is closely connected with the presence of local maxima of B , i.e. islands of lines $B = \text{constant}$ on the magnetic surfaces. The possibility to eliminate such particles is investigated numerically in the present paper. As a first step, the optimisation of the configuration with respect to pseudosymmetry (ps) [2] is undertaken and the confinement properties of the ps configuration is studied through the calculation of the second adiabatic invariant contours and more directly with the computation of the collisionless α -particles lost. It is shown that the fulfilment of the ps condition itself is not sufficient for good particle confinement. Therefore, as a next step, the optimisation toward the poloidal closure of the J_{\parallel} contours is performed. In addition, the effect of finite β on the shape of the magnetic field strength surfaces and the particle confinement is studied. Some results on calculations of neoclassical diffusion are presented for configurations considered.

Optimisation toward pseudosymmetry

The initial boundary for the optimisation toward ps was obtained from that of W7-X [3] by changing the number of periods from $N = 5$ to $N = 6$ and by exchanging the six-period bumpy magnetic field component with a three times periodic term. In this case the extrema of the magnetic field strength should be located at equivalent positions, all exhibiting small curvature of the magnetic axis. The pressure gradient was taken to be very small, $\beta \approx 0.05\%$. The behaviour of $B = \text{constant}$ lines in Boozer coordinates at the $1/3$ and $3/4$ of the minor plasma radius of the ps-optimised configuration is shown in Fig. 1. Direct calculations of collisionless α -particle loss have shown that the fulfilment of ps condition itself does not improve the particle confinement: the bulk of the reflected particle fraction is lost in a short time. Numerical calculations have shown that the contours of the second adiabatic invariant $J_{\parallel} = \int v_{\parallel} dl$ are open which leads to the particle loss.

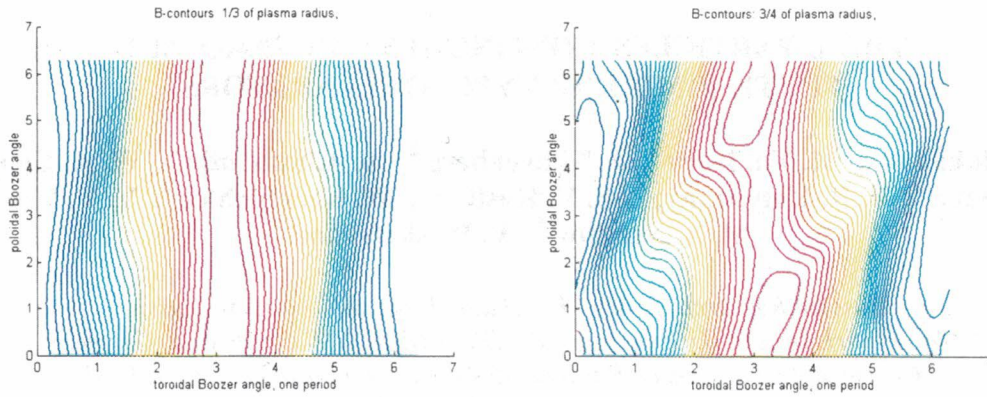


Fig. 1. B = constant lines on an inner (left) and an outer (right) magnetic surfaces, respectively, for a ps-optimised configuration.

The variation of J_{\parallel} on a magnetic surface is connected with a displacement of the trapped particle orbit from the magnetic surface. This variation is characterised by a derivative $\partial J_{\parallel} / \partial \theta_0$, where $\theta_0 = \theta_B - \iota \zeta_B$, θ_B and ζ_B being the Boozer angular coordinates. In accordance with [4], the results are presented for $\partial J_{\parallel} / \partial \theta_0$ in a normalised form as a functional dependence between dimensionless parameters η and $\gamma = v_{\parallel i} / v_{\perp 0}$ with

$$\eta = \frac{R}{J_{\perp}} \frac{1}{\tau_b \langle |\nabla \Psi| \rangle} \frac{\partial J_{\parallel}}{\partial \theta_0}, \quad \frac{\partial J_{\parallel}}{\partial \theta_0} = \frac{e}{mc} \delta \Psi, \quad v_{\perp 0} = \sqrt{J_{\perp} B_0}.$$

Here $J_{\perp} = v_{\perp}^2 / B$, τ_b is the bounce period, $\delta \psi$ corresponds to the differential of ψ during τ_b . The normalisation of η is performed in such a way that for the standard stellarator magnetic field, the maximum η value, η_m , equals 0.5. The $v_{\parallel i}$ quantity is v_{\parallel} at the point of a local minimum of B and the γ parameter relates to the pitch angle at this point.

The computations are done in cylindrical coordinates (ρ, φ, z) with the magnetic field ($N = 3$ periods along the torus) presented as a superposition of a finite number (1300) of toroidal harmonic functions containing the associated Legendre functions. To find the decomposition coefficients a surface equation of the given boundary is used. The magnetic surfaces were calculated for the starting points of integration $(\rho = \rho_0, \varphi = 0, z = 0)$ with $\rho_0 = 12.00, 12.25, 12.50, 12.75, 13.00, 13.10$ and 13.123 ($\rho_0 = 13.20$ corresponds to the boundary surface). Between the boundary and the magnetic surface $\rho_0 = 13.123$ the behavior of the magnetic field lines is stochastic. Between the magnetic surfaces $\rho_0 = 13.10$ and $\rho_0 = 13.123$ small islands exist.

Fig. 2 shows the distribution of B along the magnetic field lines for the magnetic surfaces $\rho_0 = 12.0$ (inner, near the magnetic axis) and $\rho_0 = 12.75$ (rather close to the boundary). The n value corresponds to the number of integration steps (320 steps for one magnetic field period). Some of the minima of B are labelled with their numbers.

The results of η calculations are presented in Figs. 3 and 4. The curves are numbered in accordance to the numbers of the B minima. It is seen that non-zero value of η is realised in the γ interval $0 \leq \gamma \leq \gamma_{\max}$ which corresponds to the trapped particles. From the results follows that for the inner magnetic surface the η value is rather small for deeply trapped particles however for particles with moderate trapping and sufficiently big γ the maximum value of η is close to the analogous value for the standard stellarator. For the surface with $\rho_0 = 12.75$ big η values appear also for the deeply trapped particles.

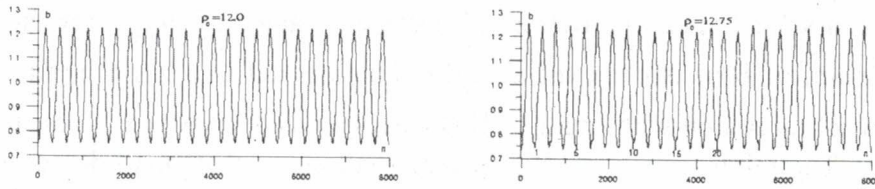


Fig. 2. Distributions of $b = B/B_0$ along the magnetic field lines.

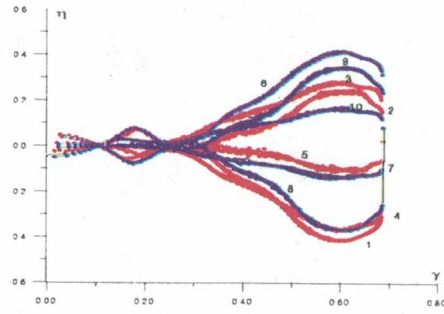


Fig. 3. η vs. γ for the surface $\rho_0 = 12.0$.

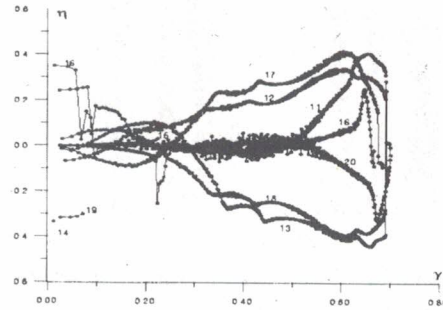


Fig. 4. η vs. γ for the surface $\rho_0 = 12.75$.

Line 1 in Fig. 5 shows the radial (with s the normalised flux variable) dependence of the effective ripple, $\mathcal{E}_{eff}^{3/2}$, for the ps-optimised configuration. This quantity characterises the strength of the $1/\nu$ transport [5]. For a corresponding standard stellarator the $\mathcal{E}_{eff}^{3/2}$ value turns out to be $0.01 \div 0.03$. So, the results obtained are only slightly better than those for the standard stellarator.

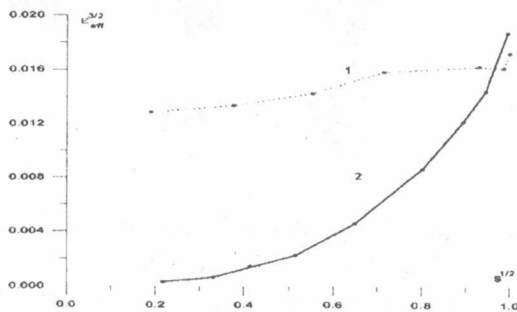


Fig. 5. Effective ripple amplitude, for small β ps-optimised (line 1) and $\beta = 5\%$ $J_{||}$ optimised configurations.

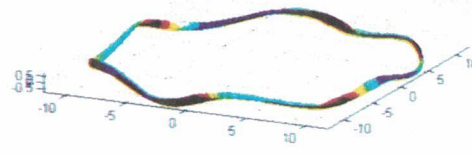


Fig. 6. Inner magnetic surface for the configuration optimised with respect to ps and closure of $J_{||}$ contours.

Optimisation toward closure of the $J_{||}$ contours and finite β effect

In addition to the ps condition, the requirement of closure of the $J_{||}$ contours was implemented in the optimisation procedure. As in the previous step, the β value was $\beta \approx 0.05\%$. Due to the different penalty function, the optimisation leads to a configuration with a dominating six-period bumpy component of the field strength and a different geometry of the magnetic axis

(see Fig. 6). It is seen from Fig. 7 that in this configuration the lines $B = \text{constant}$ have a similar form for almost all values of B , in contrast to the initial one (see Fig. 1). Fig. 8 demonstrates the behaviour of J_{\parallel} contours for trapped particles with different values of B_{reflect} for the part of the period near the minimum of B . It is observed that the function $\langle J_{\parallel} \rangle_{\theta}$ has a minimum near the magnetic axis for this configuration with small β . This corresponds to convex $B = \text{constant}$ surfaces, as is seen from Fig. 9. The minimum of $\langle J_{\parallel} \rangle_{\theta}$ is very shallow, so that small deviations from the qi condition can create open J_{\parallel} contours.

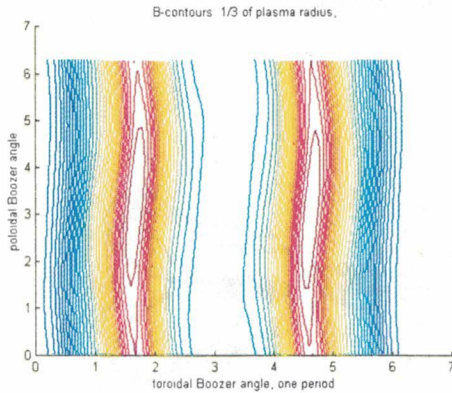


Fig. 7. $B = \text{constant}$ lines on the inner magnetic surface for the configuration shown in Fig. 6.

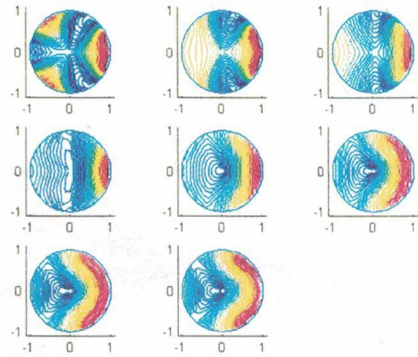


Fig. 8. J_{\parallel} contours for the configuration shown in Fig. 6 with low β . The value of B_{reflect} increases from the top left to the bottom right diagrams. The closed J_{\parallel} contours are characterised by a minimum near the magnetic axis.

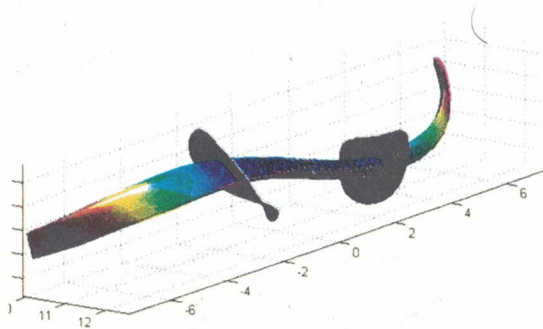


Fig. 9. Inner magnetic surface and surfaces $B = \text{constant}$ for the low β . The surfaces $B = \text{constant}$ are slightly convex.

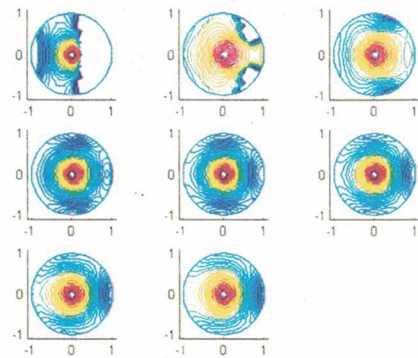


Fig. 10. J_{\parallel} contours for the configuration shown in Fig. 6 with $\beta = 5\%$. The closed J_{\parallel} contours are characterised by a maximum near the magnetic axis.

In Fig. 10, the contours of J_{\parallel} are shown for $\beta = 5\%$. It is seen that now J_{\parallel} has a maximum near the magnetic axis. It corresponds to the creation of an absolute minimum of B due to the diamagnetic effect and to the transition from convex to concave surfaces $B = \text{constant}$ for moderate values of B (Fig. 11). Here, the maximum of J_{\parallel} is strong, so that even large deviations from qi can conserve the closure of the J_{\parallel} contours. The result of $\epsilon_{\text{eff}}^{3/2}$ calculations for $\beta = 5\%$ is shown in Fig. 5, too, line 2.

The transition from minimum of J_{\parallel} near the magnetic axis for small β to a maximum of this quantity for $\beta \approx 5\%$ can lead to the deterioration of particle confinement for intermediate β values, when $\langle J_{\parallel} \rangle_{\theta}$ becomes independent on the plasma radius. This really occurs, as is seen from Fig. 12. Further optimisation is required for systems with finite β to clarify the possibility to confine all reflected particles in configurations without a local maximum of B on the magnetic surfaces.

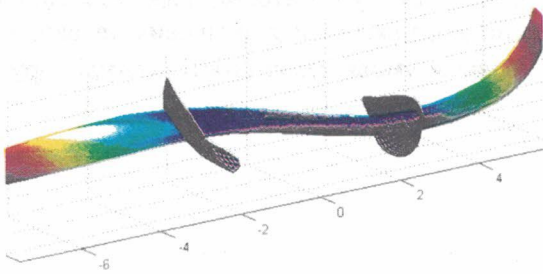


Fig. 11. Inner magnetic surface and surfaces $B = \text{constant}$ for $\beta = 5\%$. The surfaces $B = \text{constant}$ are concave.

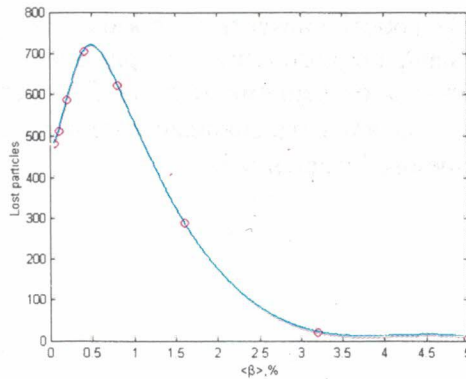


Fig. 12. Effect of β on the collisionless particle confinement. Increased losses correspond to the transition from minimum of J_{\parallel} near the magnetic axis to maximum.

Conclusions

Numerical investigations have shown that the ps condition can be fulfilled with high accuracy in the whole plasma volume. The fulfilment of the ps condition itself is not enough for improvement of particle confinement. The closure of the J_{\parallel} contours is defined both by radial dependence of $\langle J_{\parallel} \rangle_{\theta}$ and by the accuracy of the fulfilment of the qi condition.

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УТРИМАННЯ α -ЧАСТИНОК У СТЕЛАРАТОРАХ ДЗЕРКАЛЬНОГО ТИПУ З НУЛЬОВИМ ТА СКІНЧЕНИМ β

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В роботі вивчається зв'язок між формою магнітних поверхонь та топологією поверхонь $B = \text{constant}$, з одного боку, та утриманням частинок, з іншого боку, у гвинтових магнітних системах із полоїдальним напрямком ліній $B = \text{constant}$. Чисельно досліджено для нульових та скінчених значень β можливість виконання умов псевдосиметрії, а також умови замкнутості контурів другого адіабатичного інваріанту $J_{||}$.