STELLARATOR FIELDS WITH SMALL PS CURRENT AT SMALL ROTATIONAL TRANSFORM

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One aspect of the optimization concept of stellarators is the reduction of the normalized Pfirsch-Schlüter current density $\left\langle j_{\parallel}^2 / j_{\perp}^2 \right\rangle^{1/2}$ to a reasonable level but obeying other side conditions, e.g., concerning small bootstrap currents, good stability properties, reasonable aspect ratio, etc. This problem is addressed in the present work. Various stellarator vacuum field are given analytically for $M = 2, 3, 5, 10, 12$ (M is the number of field periods around the torus) where the PS-current density is reduced by more than a factor of ten to rather small values around 0.3 even at small ɩ-values.

1. Introduction

One aspect of the optimization concept of stellarators is the reduction of the normalized Pfirsch-Schlüter current density $\langle j_{\parallel}^2/j_{\perp}^2\rangle^{1/2}$ to a resonable level but obeying other side conditions, e.g., concerning small bootstrap currents, favourable stability properties, resonable aspect ratio, good α-particle confinement, material properties and technical aspects [1,2,3].

Considering classical $\ell = 2$ stellarator fields, this ratio varies with the rotational transform ɩ according to $\langle j_{\parallel}^2/j_{\perp}^2\rangle^{1/2} = \sqrt{2}/ɩ$, i.e. the PS-current density is high at low ɩ-values.
In the present paper various stellarator vacuum fields are given analytically for $M = 1, 2, 5, 10$ (M is the number of field periods around the torus) where for some cases the PS-current density is reduced by more than a factor of ten to rather small values around 0.3 even at small ɩ-values of $ɩ = 0.2$.

The vacuum fields are given analytically by a finite set of Dommaschk potentials [4,5] that essentially consist of finite power series in the cylindrical coordinates R and Z for each toroidal harmonic. The singularities of the individual magnetic fields are located at $R = 0$ (main torus axis) and at infinity. In that respect they differ from field represented by spherical harmonics. For the present study, the essential physical properties of the configurations should be described by the low-order multipole fields so the data sets consists only of a small number of harmonics (typically eight to twelve amplitudes).

In terms of Dommaschk potentials $V_{m,l}(R,\phi,Z)$, the exact scalar potential of the vacuum field **B**

$$\mathbf{B} = \nabla(\phi + \sum V_{m,l}), \quad m = 0, M, 2M, 3M, \ldots; \quad l = 0, 1, 2, 3, \ldots$$

is given by the Dommaschk functions $D_{m,l}$, $N_{m,l-1}$:

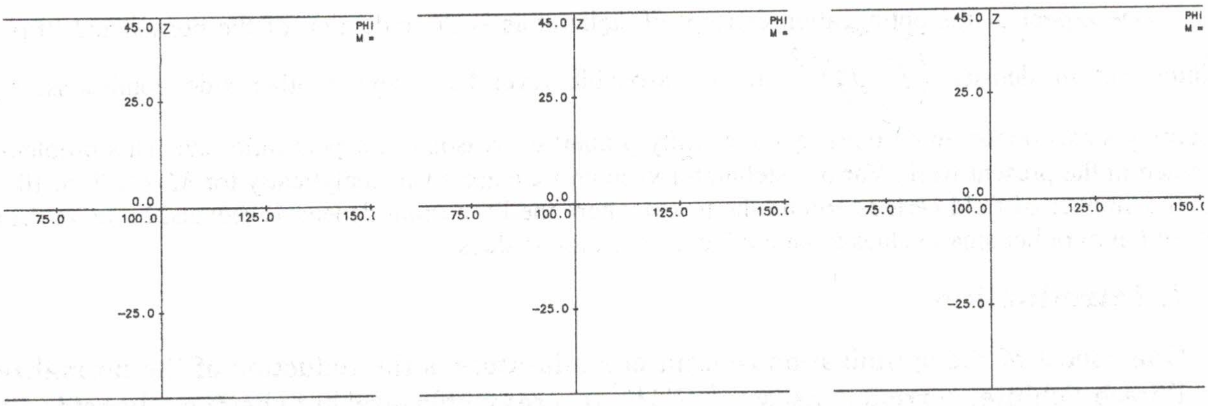
$$V_{m,l}(R,\phi,Z) = A_{m,l}^{(1)} D_{m,l} \sin(m\phi) + A_{m,l}^{(2)} N_{m,l-1} \cos(m\phi), \quad l = 0, 2, 4, \ldots$$

$$V_{m,l}(R,\phi,Z) = A_{m,l}^{(1)} D_{m,l} \cos(m\phi) + A_{m,l}^{(2)} N_{m,l-1} \sin(m\phi), \quad l = 1, 3, 5, \ldots$$

where the notation of Refs.4,5 is used; here m is a sum index; (R,ϕ,Z) are ordinary cylindrical coordinates, ϕ is the toroidal angle and l the poloidal stellarator mode number; m describes the toroidal harmonics associated with the common period number M. The normalization used here is that the main toroidal field $B_\phi = 1/R$ is one at $R = 1$. A particular magnetic field configuration is given by the data set of the amplitudes $A_{m;l}^{(i)}$ and the vertical field B_{z0}. An identification number, upper and lower indices of $A_{m,l}^{(i)}$ are given in the first four columns of the data sets. In some data sets a scaling factor cc is given so that the actually used field coefficients are obtained by multiplying the listed field coefficients with the factor *cc*.

2. Results

The following field configuration has one field period around the torus ($M = 1$), aspect ratio $A = 4.2$, average magnetic well -6.4% and rotational transform $\iota(0) = 0.19$, $\iota(r)$ is almost constant over the cross section.



FELD-KONFIG. cc = 1. FZH941

MP,IP,RA,BZO / IDN,IDN,M,L,AP

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1	2	0	2	0.0280E-00
2	1	0	3	0.1870E-00
3	2	0	4	-0.9350E-00
4	1	1	0	-0.3270E-00
5	2	1	1	0.0935E-00
6	1	1	2	0.3370E-00
7	2	1	2	0.3000E-00
8	1	1	3	0.3740E+00
9	2	1	3	-0.3740E-00
10	1	2	2	-0.1400E-00
11	2	2	2	-0.1400E-00

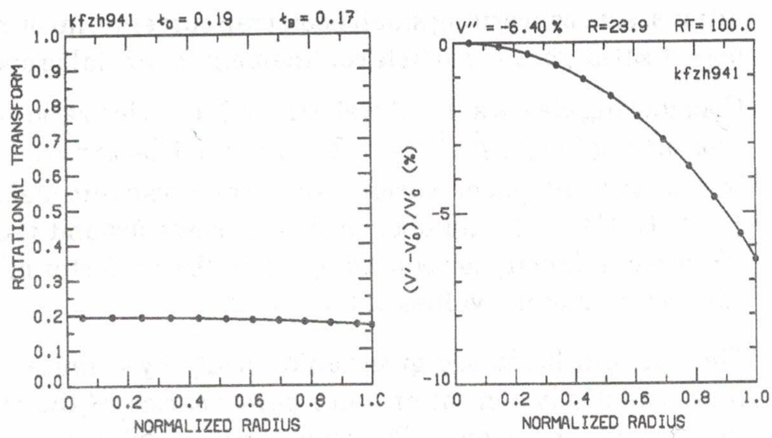


Fig.1. Poincaré plot of magnetic surfaces for an M=1 stellarator field at $\phi = 0$ and $\phi = 180^o$ (half of a field period); field coefficients for the Dommaschk potentials, rotational transform $\iota(r)$ and V' as functions of the normalized minor radius are also show.

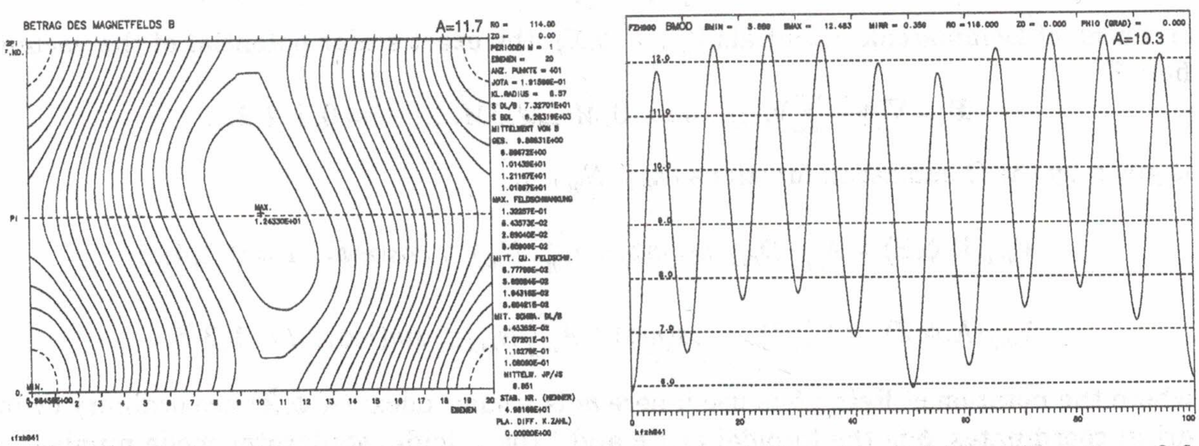
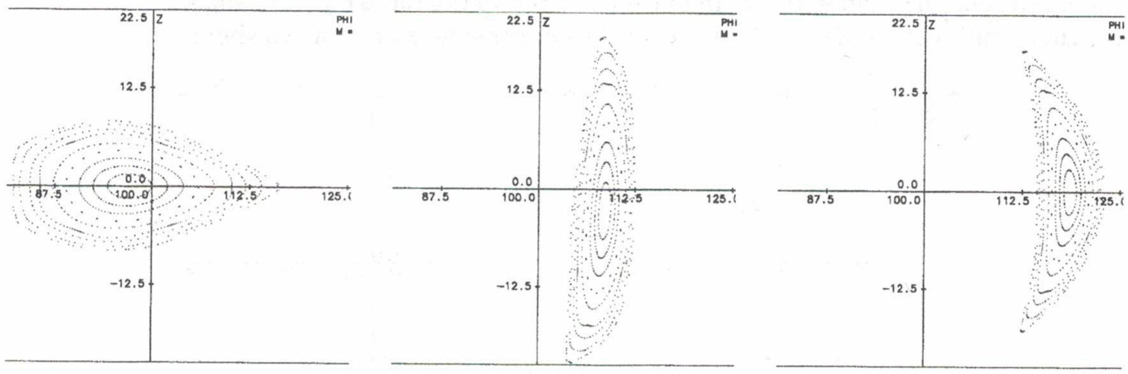


Fig.2. Contours of absolute value of B on a magnetic surface ($A = 11.7$) as function of the toroidal and poloidal coordinates. The right graph shows B along a field line as function of the toroidal angle along ten field periods ($A = 10.3$). The mirror ratio at magnetic axis is $mirr = (B_{max} - B_{min})/(B_{max} + B_{min}) = 0.29$.

The PS-current density is 6.7 which is rather large. However other $M = 1$ configurations have been found where the PS-value is decreased to 3.8 while the rotational transform is increased to $\iota = 0.27$ ($A = 12$, $V'' = -1\%$, $mirr = 0.11$).

The second example is a configuration with two field periods around the torus ($M = 2$), aspect ratio $A = 10.2$, marginal magnetic well (see Fig.3), rotational transform $\iota(0) = 0.30$, $\iota(r)$ is almost constant over the cross section.



FELD-KONFIG. cc=0.75 FZH176
MP,IP,RA,BZO / IDN,IDN,M,L,AP
02 20 1.00E+00 +0.0189E-00

1	2	0	2	-0.02000E-00
2	1	0	3	0.70000E-00
3	2	0	4	-0.53000E-00
4	1	0	5	-7.75000E-00
5	2	0	6	-1.24400E-02
6	1	2	0	-0.27000E-00
7	1	2	1	-0.03500E-00
8	2	2	1	0.25000E-00
9	1	2	2	0.70800E-00
10	2	2	2	0.34800E-00
11	1	2	3	2.21300E-00
12	2	2	3	-1.46000E-00
13	2	2	4	2.55000E-00
14	1	4	0	-0.00430E-00
15	1	4	1	0.01200E-00
16	2	4	1	-0.01700E-00
17	1	4	2	0.57800E-00
18	2	4	2	0.57100E-00
19	1	4	3	-0.00000E-00
20	2	4	3	2.00000E-00

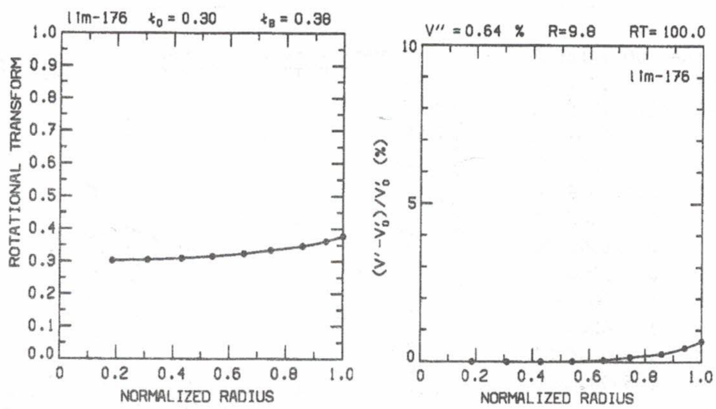


Fig.3. Poincaré plot of magnetic surfaces for an M=2 stellarator field at $\phi = 0$ and $\phi = 90^o$; field coefficients, rotational transform $\iota(r)$ and V' are also show.

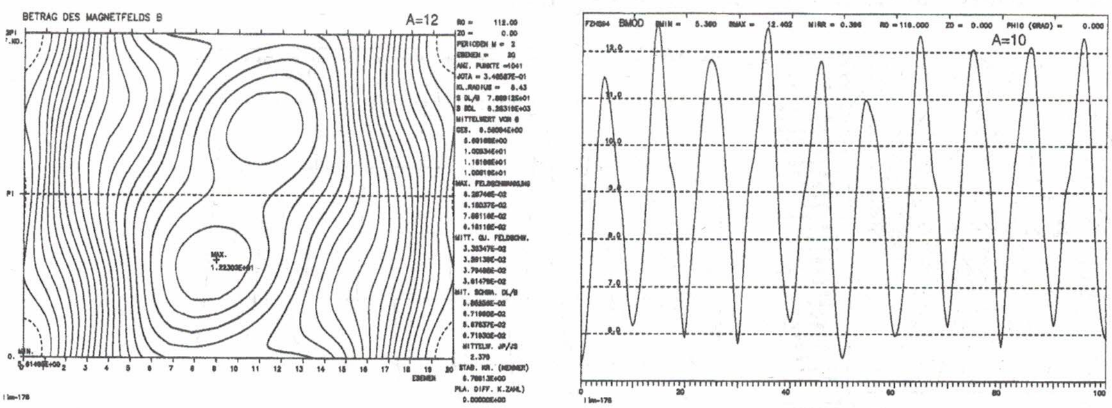
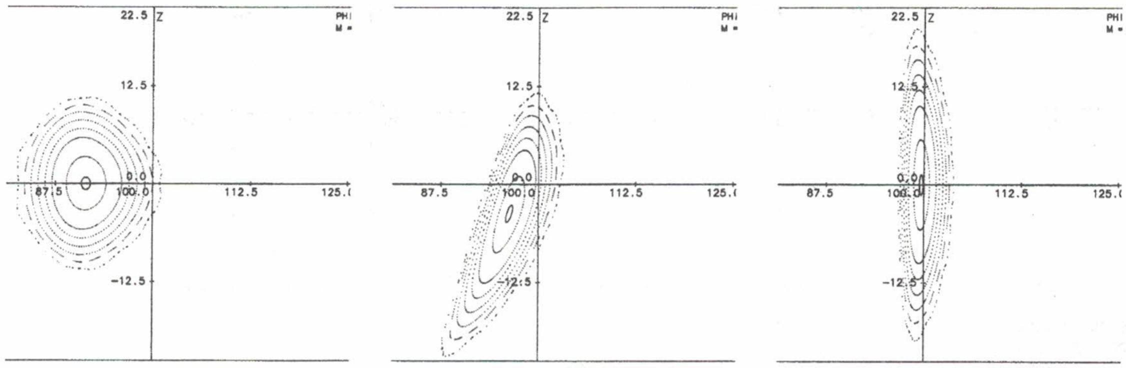


Fig.4. Contours of B on a magnetic surface ($A = 12.0$) The right graph shows B along a field line over ten field periods ($A = 10.0$). The mirror ratio at magnetic axis is $mirr = 0.30$.

The PS-current density is reduced to 2.3; the variation of the minima of B along a field line is now considerably smaller than in the previous case; this property might also have favourable effects on the neoclassical particle transport. Similar $M = 2$ configurations have been found with even smaller PS-currents ($PS = 1.8$) at higher values of $\iota = 0.47$ (constant over the cross section), small aspect ratio $A = 6.9$, magnetic well $V'' = -1\%$, $mirr = 0.17$; however the modulation of the minima of B along a field line is increased. Some of those parameter values are in the range which are accessible in the Wendelstein-7-AS device (see Fig.9).

The third example is a configuration with five field periods around the torus ($M = 5$), aspect ratio $A = 11$, magnetic hill 5.7%, $\iota(0) = 0.37$, $\iota(a) = 0.51$ i.e. considerable positive shear.



FELD-KONFIG. cc=1.00 kfzh667
MP,IP,RA,BZO / ID IDN,M,L,AP)

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1	2	0	2	0.12000E-00
2	1	0	3	1.60000E-00
3	2	0	4	-9.60000E-00
4	1	5	0	-0.03440E-00
5	1	5	1	-0.12800E-00
6	2	5	1	0.12800E-00
7	1	5	2	1.16000E-00
8	2	5	2	1.16000E-00
9	1	5	3	-0.91000E-00
10	2	5	3	0.88000E-00

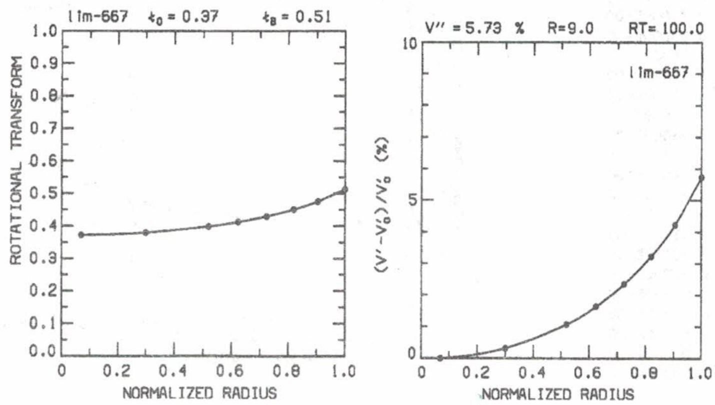


Fig.5. Poincaré plot of magnetic surfaces for an M=5 stellarator field at $\phi = 0$ and $\phi = 36^o$; field coefficients, rotational transform $\iota(r)$ and V' are also show.

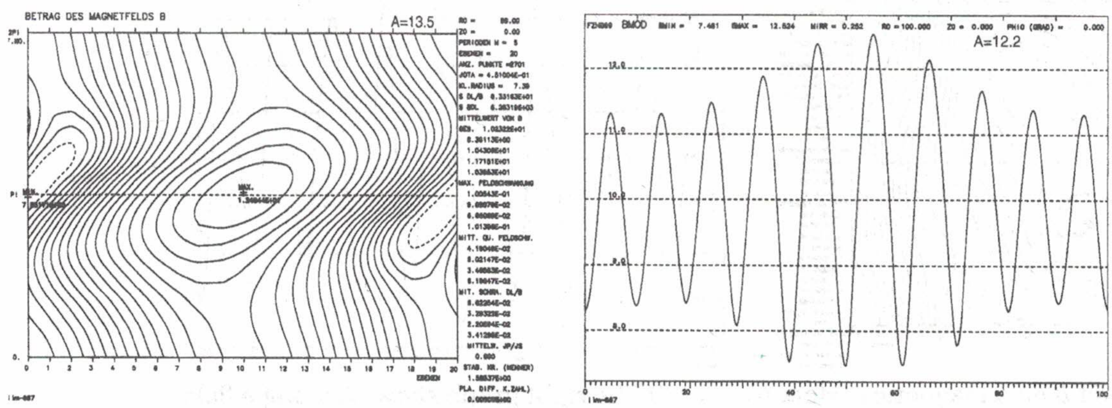
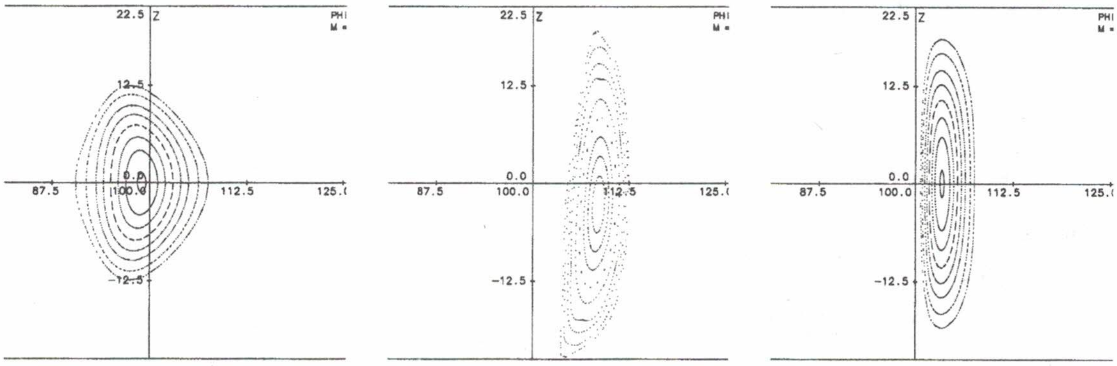


Fig.6. Contours of B on a magnetic surface ($A = 13.5$). The right graph shows B along a field line over ten field periods ($A = 12.2$). The mirror ratio at magnetic axis is $mirr = 0.19$.

The PS-current density is now reduced to 0.64; the mirror ratio on magnetic axis is 0.19; unfortunately the variation of the minima of B along a field line is somewhat increased as shown in Fig.6 (right graph). On the other hand similar $M = 5$ configurations have been found (various dashed curves in Fig.9) where the variation of the minima of B along a field line is considarbly decreased but the PS-currents are somewhat higher ($PS = 0.76$, $\iota(0) = 0.44$, $\iota(a) = 0.48$, $A = 11.5$, magnetic hill $V'' = 5.2\%$, $mirr = 0.32$); the one dashed curve with triangles refers to an $M = 5$ configuration with marginal magnetic well.

The fourth example is a configuration with ten field periods ($M = 10$), aspect ratio $A = 11$, magnetic hill 9.7%, $\iota(0) = 0.28$, $\iota(a) = 0.59$ i.e. rather high shear and small PS-current density $PS = 0.37$.



FELD-KONFIG. cc=0.95 FZH145
MP,IP,RA,BZ0 / ID,IDN,M,L,AP)

10	10	1.00E+00	0.02560E-00	
1	2	0	2	0.25600E-00
2	1	0	3	1.68000E-00
3	2	0	4	-2.48000E+01
4	1	10	0	-0.02400E-00
5	1	10	1	-0.08000E-00
6	2	10	1	0.28000E-00
7	1	10	2	1.64000E-00
8	2	10	2	1.64000E-00
9	1	10	3	-1.80000E-00
10	2	10	3	+1.80000E-00

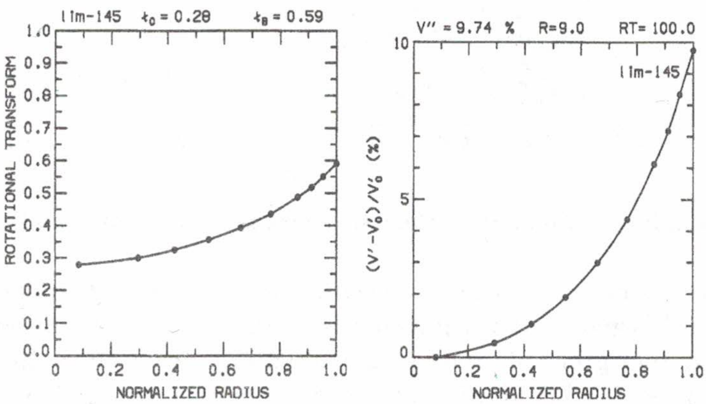


Fig.7. Poincaré plot of magnetic surfaces for an M=10 stellarator field at $\phi = 0$ and $\phi = 18\,^{o}$; field coefficients, rotational transform $\iota(r)$ and V' are also show.

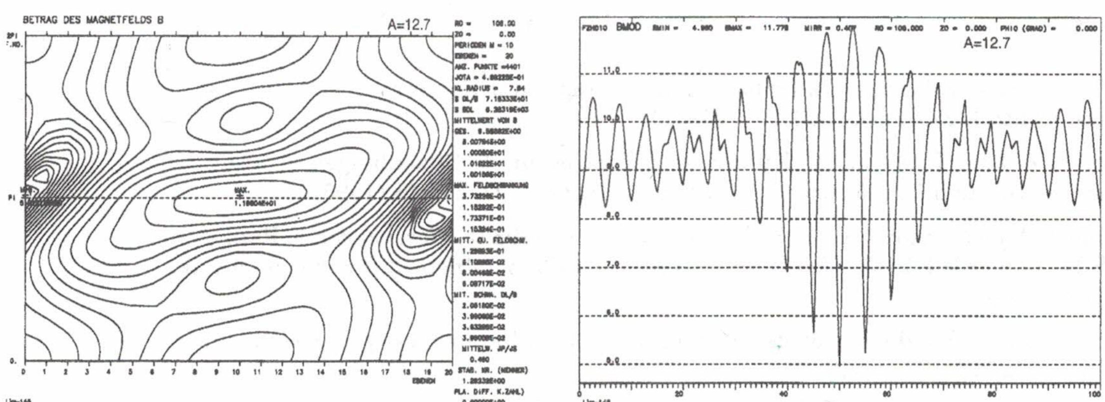


Fig.8. Contours of B on a magnetic surface ($A = 12.7$) (left); B along a field line over 20 periods ($A = 12.7$).

The mirror ratio at magnetic axis is $mirr = 0.20$. The PS-current density is reduced to samll values $PS = 0.37$; however the variation of the minima of B along a field line is considerably increased i.e. it needs further optimization to remove that disadvantage.

3. Summary

The Fig.9 summarizes the results about the PS-current density what have been reported in this paper together with results for some additional configurations including $M = 12$ fields (label m12). For comparison it shows also known results about experimental devices such as Wendelstein-7-AS ($M = 5$), Wendelstein-7-X ($M = 5$) and Heliotron-E ($M = 19$).

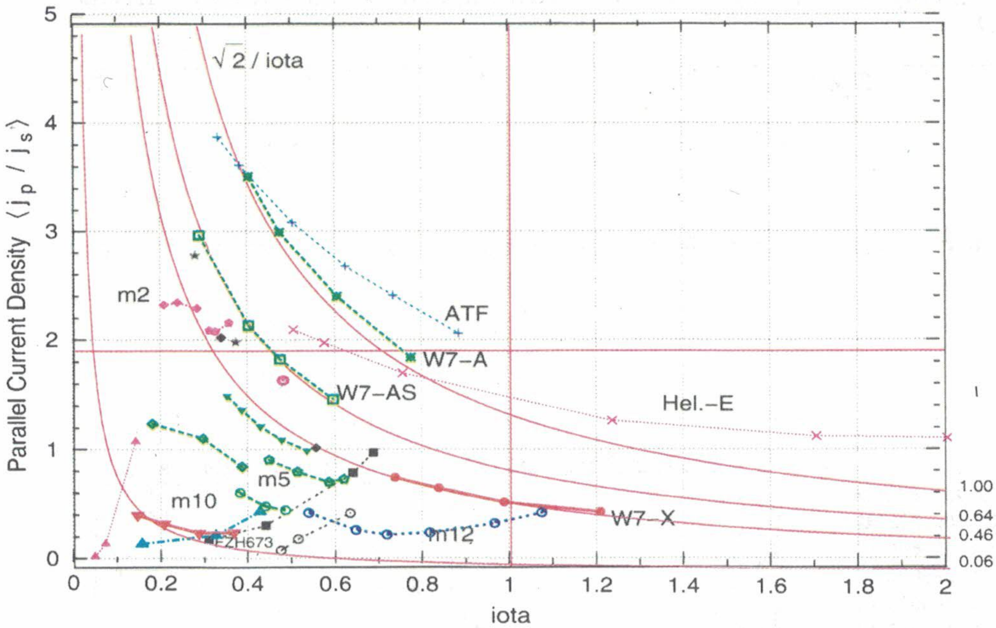


Fig.9. Parallel current density versus ι for various stellarator fields.

Some analytic curves $k\sqrt{2}/\iota$ are also shown which fit to the Wendelstein data (the value of the paramter k is given on the right boarder of the figure). The various $M = 5$ configurations (four dashed lines grouped around label $m5$) differ also in the value for V''. The PS-values of some of these $M = 5$ fields are in the range what is envisaged for Wendelstein-7-X which is under construction in Greifswald. The smallest values for the Pfirsch-Schlüter current density have been obtained for $M = 10$ configurations: $PS = 0.17$, $\iota(0) = 0.04$.

4. References

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МАГНІТНЕ ПОЛЕ СТЕЛАРАТОРІВ З МАЛИМ СТРУМОМ ПФІРША-ШЛЮТЕРА ТА МАЛИМ ОБЕРТАЛЬНИМ ПЕРЕТВОРЕННЯМ

Ф. Гернеґґер

Важливим питанням оптимізації стелараторів є мінімізація нормалізованого струму Пфірша-Шлютера $\left\langle j_{\parallel}^2 / j_{\perp}^2 \right\rangle^{1/2}$ з одночасним виконанням деяких інших умов, таких як забезпечення малості бутсреп-струму, достатньої стійкості плазми, прийнятого аспектного відношення тощо. Цю проблему розглянуто в представленій роботі, де наведено рівноважне поле стелараторів з M = 2, 3, 5, 10, 12 (M – число періодів поля), в яких струм Пфірша-Шлютера зменшено більш як на порядок навіть при малих значеннях обертального перетворення.