MECHANISMS OF STOCHASTIC DIFFUSION IN OPTIMIZED HELICAL SYSTEMS

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Various mechanisms of stochastic diffusion of energetic ions in optimized stellarators are reviewed and analyzed. It follows from the carried out analysis that alpha losses caused by stochastic diffusion in the Wendelstein-line stellarators (Helias configurations) result mainly from successive orbit transformations of the transitioning particles.

1. INTRODUCTION

The lack of the axial symmetry is known to lead to the loss of all locally trapped α-particles in conventional stellarators. Therefore, one of the important aspects for stellarator optimization is to realize good collisionles alpha particle confinement. For quasi-symmetric configurations such as the quasi-helically symmetric and the quasi-axisymmetric configurations, good collisionless particle confinement can be realized with vacuum configurations. On the other hand, finite beta effects are required to improve collisionless alpha particle confinement in the Helias configurations [1,2]. Because "mirror" harmonic dominates in the spectrum of the magnetic field strength of the Helias, for sufficiently high beta it can be roughly expressed by $B = B(\psi, \phi)$, where ψ and ϕ are the toroidal flux labeling a flux surface and ϕ is toroidal angle in Boozer coordinates. This implies that the contours of the longitudinal adiabatic invariant for the locally trapped particles, $J = \oint v_{\|} dl$, are closed and weakly deflected from the magnetic surfaces. Thus Helias configuration, including stellarator W7-X which is under construction, represents example of the quasiomnigenous system.

While the "prompt" losses are avoided in the quasiomnigenous configuration, there are various sources of stochastic losses presented in such systems, and the purpose of the present overview is to describe these losses. Under "stochastic" we mean losses arising due to dynamical (collisionless) randomization of the particle motion as a result of adiabaticity breaking. There are two principally different mechanisms for adiabaticity breaking in the stellarator. The first one is associated with the possibility of collisionless orbit transformations between locally trapped and locally passing states of motion [3]. This mechanism and concomitant spatial diffusion will be considered in the next Section.

Adiabaticity breaking can be driven also by various resonances of the particle motion with the stellarator magnetic field. In Sec. III we consider possibility of destruction of the ripple-averaged component of the toroidal canonical momentum, which serves as an adiabatic invariant for locally passing (toroidally trapped) particles. The resulting diffusion is similar to the Goldston-White-Boozer (GWB) diffusion in the tokamak with toroidal field ripples [4–6]. Nonconservation of magnetic moment μ as a result of cyclotron resonance with harmonic of the field line curvature [7] and concomitant pitch-angle scattering are briefly discussed in Sec. IV. In Sec. V our conclusion is presented. The exposition is concise, with an emphases on physics and final results. The analysis in Sec. II, III is more detailed because corresponding stochastic losses can really pose the problem for Helias operation.

2. DIFFUSION OF TRANSITIONING PARTICLES

In the present section we study the diffusion arising due to the following. The adiabaticity of the particle motion in the phase space breaks down near the separatrix between the regions of locally trapped and locally passing orbits (see Fig.1). Because of this the adiabatic invariant J acquires a phase dependent jump each time when a particle crosses separatrix [8–10]. The phases of the motion do not correlate for successive transitions. Therefore, the multiple crossings of the separatrix are accompanied by the random walk of particles in the J space, resulting in spatial diffusion. Note that stochastic diffusion having the same nature may take place also in tokamaks, where, however, it plays a minor role, being associated with the presence of the ripple wells [11].

The corresponding diffusion coefficient can be written as follows:

$$D = \frac{\langle (\Delta r)^2 \rangle}{\tau}, \tag{1}$$

where Δr is the change of the particle radial coordinate caused by the orbit transformation, $\langle\rangle$means ensemble averaging, r is the effective flux surface radius defined by the equation $\psi = \bar{B}r^2/2, \bar{B}$ is the average vacuum magnetic field at the magnetic axis; τ is the characteristic time,

$$\tau = \frac{1}{2}(\tau^l + \tau^p), \tag{2}$$

τ^l and τ^p are the characteristic times of the particle motion in the locally trapped and passing states. The factor 1/2 takes into account that a particle crosses separatrix twice per full period of a hybrid passing-localized orbit. The time τ^l is essentially the precession time of a localized particle, whereas $\tau^p = (2/P)\int_{-\theta(\kappa^2=1)}^{\theta(\kappa^2=1)} d\theta/\dot{\theta}$ where $\dot{\theta}$ is the frequency of the poloidal motion of a passing particle., P is the probability of the orbit transformation resulting in the trap of the passing particle. The probability P and the radial jump caused by the separatrix crossing can be expressed in terms of the longitudinal invariant. All ingredients in Eq.(1) were calculated with using the bounce-averaged equations of the particle motion in the magnetic field

$$B = \bar{B}[1 + \epsilon_0(\psi) + \epsilon_m(\psi)\cos N\phi - \epsilon_h(\psi)\cos(\theta - N\phi) - \epsilon_t(\psi)\cos\theta], \tag{3}$$

where ψ, θ, ϕ are the magnetic flux coordinates with ψ the toroidal magnetic flux; ϵ_0 describes the change of the vacuum magnetic field due to finite β; ϵ_m, ϵ_h, and ϵ_t are the amplitudes of the mirror, helical, and toroidal harmonics, respectively, ϵ_m being dominant in the plasma core; $N \gg 1$ is the number of the field periods along the large azimuth of the torus. As a result, a diffusion coefficient was obtained. Its magnitude for $\epsilon_t \ll \epsilon_h \ll \epsilon_m$, which is the case in the plasma core of the Helias, can be evaluated as

$$D \simeq \frac{4}{\pi}\frac{R^2\omega_B\rho_B^4}{N^2ar^3}\frac{\epsilon_h^2}{\epsilon_m}\epsilon_0', \tag{4}$$

where ω_B is the energetic ion gyrofrequency, ρ_B is the gyroradius; a and R are the minor and major radius of the torus, respectively; $\epsilon_0' \equiv d\epsilon_0/dx$ with $x = r/a$. Equation (4) is relevant to particles with the pitch-angle parameter $\alpha \equiv E/\mu B - 1 \sim \epsilon_m + \epsilon_0$.

The condition that an energetic ion will be lost because of diffusion (rather than displaced within the plasma) is $\tau_d \ll \tau_s$, where τ_s is the characteristic diffusion time, and τ_d is the diffusion time defined by

$$\tau_d(r) \sim \frac{(a-r)^2}{D}. \tag{5}$$

It is of importance to know the fraction of transitioning particles, which is essentially the stochastic-diffusion-induced loss fraction of alpha particles when the condition $\tau_d \ll \tau_s$ is satisfied. This quantity relevant to a flux surface is given by

$$\nu(r) = \sqrt{\frac{\alpha_{max}}{1+\alpha_{max}}} - \sqrt{\frac{\alpha_{min}}{1+\alpha_{min}}}, \tag{6}$$

where $\alpha_{min} = \epsilon_m + \epsilon_0 - \epsilon_h - \epsilon_t$, $\alpha_{max} = \epsilon_m + \epsilon_0 + \epsilon_h + \epsilon_t$.

Now we calculate numerically the diffusion coefficient, the fraction of transitioning particles and the diffusion time, using corresponding equilibrium data. The obtained dependence of τ_d on α at $r/a = 0.5$ is presented in Fig.2. We observe that the dependence of τ_d on α has a rather flat minimum around $\alpha = \epsilon_m + \epsilon_0$ where D was expected to be maximum; the magnitude of τ_d is in qualitative agreement with analytical estimates. These results are also in agreement with the confinement time and the fraction of lost particles calculated numerically using a guiding center code [13,3].

3. GOLDSTON-WHITE-BOOZER DIFFUSION IN HELIAS

In the previous section it was shown that repeated orbit transformations result in stochastic diffusion of the transitioning particles. There is, however, additional mechanism of the stochastic loss for this population. Transitioning particles in the locally passing state, similar to toroidally trapped particles in tokamaks, experience toroidal precession superimposed on the bounce motion along magnetic field line. When resonance condition $N\omega_p = l\omega_b$ is realized with $\omega_{p(b)}$ the precession (bounce) frequency, N the number of field periods, and l the integer, the adiabaticity of the motion breaks down (ripple-averaged toroidal component of the canonical angular momentum is no longer conserved). When the drift islands, which are formed around neighbouring resonances, overlap, then the motion become stochastic [14]. This mechanism is similar to the Goldston-White-Boozer (GWB) diffusion well known in tokamaks [4]. For the "standard" stellarator configuration it was considered in Ref. [15]. The purpose of the present section is to demonstrate that GWB diffusion plays minor role in Helias configuration.

Taking into account that "mirror" harmonic dominates in the spectrum of the Helias magnetic field strength, we obtain the following expressions for the toroidal precession frequency and bounce period of the locally passing particle in the Helias configuration:

$$\omega_p \simeq \frac{c}{e\iota}\mu\bar{B}\left[\frac{\partial}{\partial\psi}(\epsilon_h + \epsilon_t)\cos\theta_b - \frac{\partial\epsilon_0}{\partial\psi}\right], \tag{7}$$

$$\tau_b = \frac{R_0}{\pi\iota}\left(\frac{\mu\bar{B}\epsilon_m}{m}\right)^{-1/2}\left[\theta_b \ln\frac{64\epsilon_m}{\epsilon_h + \epsilon_t} + \sum_{k=1}^{\infty}\frac{\sin(2k\theta_b)}{k^2}\right], \tag{8}$$

where θ_b is the bounce angle given by

$$\cos\theta_b = \frac{\epsilon_m + \epsilon_0 - \alpha}{\epsilon_h + \epsilon_t}.$$

Resonance lines $N\omega_p\tau_b = 2\pi l$ for $3.5MeV$ alpha particle in Helias configuration are shown in Fig.3 on the (α, x) plane for parameters the same as in previous section. Negative l values are the consequence of the precession reversal due to diamagnetic effect (see Eq.(7)). Note the gradual increase of the distance between neighbouring resonances with x for fixed α. This indicates that resonance overlap and associated stochasticity are possible only in the central part of the Helias, where high $|l|$ resonance lines are condensated near the separatrix with passing particles (upper dashed curve in Fig.3).

To determine the stochasticity condition, we should construct the double-step map relating phases of motion and radial shifts for resonant particles on different half-periods of the bounce oscillations [4,15]. Under condition $\epsilon_m \gg \epsilon_{h,t}$ (weak magnetic field corrugation) we obtain the following condition for "resonance overlap":

$$K \equiv \frac{1}{\pi\iota}\frac{R_0}{r}(\epsilon_h + \epsilon_t)\sqrt{\frac{2}{\epsilon_m}}\rho_B[\cos\theta_b(r)]'\ln\frac{16\epsilon_m e}{(\epsilon_t + \epsilon_h)\pi\iota|\sin\theta_b|} > 2, \tag{9}$$

where prime denotes derivative with respect to r, and $\ln e = 1$.

Figure 4 shows corresponding regions on the (x, α) plane. The lower shaded area is artificial, because according to Fig.3 there are no resonances here. Only the small fraction of transitioning particles, located near the separatrix with passing ones and at $r/a < 0.3$, is stochastic. Thus we can conclude that GWB diffusion is negligible in Helias.

4. CYCLOTRON INTERACTION OF α-PARTICLES WITH HELICAL RIPPLES

In the stellarator the curvature of magnetic field may contain the oscillatory terms $\propto \exp[i(n\theta - l\phi)]$ with toroidal mode numbers l considerably exceeding the number of the field periods. For $l \gg N$ the specific cyclotron interaction of the α-particles with rippled field become possible [7]. Because rippled field has zero frequency, the condition for cyclotron resonance takes the form

$$n\dot{\theta} - l\dot{\phi} \pm \omega_B = 0, \tag{10}$$

where n is the poloidal mode number of the corresponding curvature harmonic.

While resonant interaction considered in Sec.III results in breaking of the adiabatic invariant J_p, the cyclotron interaction is related with Larmor gyration and thus leads to the adiabaticity breaking for the magnetic moment μ. Because energy is conserved, the result will be the anomalous pitch-angle scattering, which greatly exceeds neoclassical one.

However, this interaction is absent in Helias configuration. Indeed, resonance condition (10) can be rewritten in the form

$$v_{\|}(B) = \frac{\omega_B R_0}{l - \iota n}. \tag{11}$$

From (11) it follows that the cyclotron interaction is only possible for toroidal mode numbers which satisfy $l > R_0/\rho_B \sim 300$ in Helias. Because $l/N \geq 60$, such harmonics are practically absent in Fourier decomposition of the Helias field curvature κ, which can be easily seen from the general expression for the latter:

$$\kappa = \left| \nabla(p + \frac{B^2}{2}) - \vec{B}(\vec{b} \cdot \nabla)B \right| /B^2, \vec{b} = \frac{\vec{B}}{B}, \tag{12}$$

where p is the plasma pressure. Nevertheless, this diffusion can be essential in compact stellarators [16].

5. CONCLUSION

A general conclusion which follows from our overview is that the stochastic diffusion of transitioning particles associated with repeated orbit transformations may represent the dominant mechanism of the loss of energetic ions in optimized stellarators. The dependence of the obtained diffusion coefficient on plasma parameters and the relatively large diffusion time indicate that the loss region and the loss fraction of energetic ions in Helias configuration can be minimized by shaping the plasma temperature and density profiles so that they satisfy certain requirements.

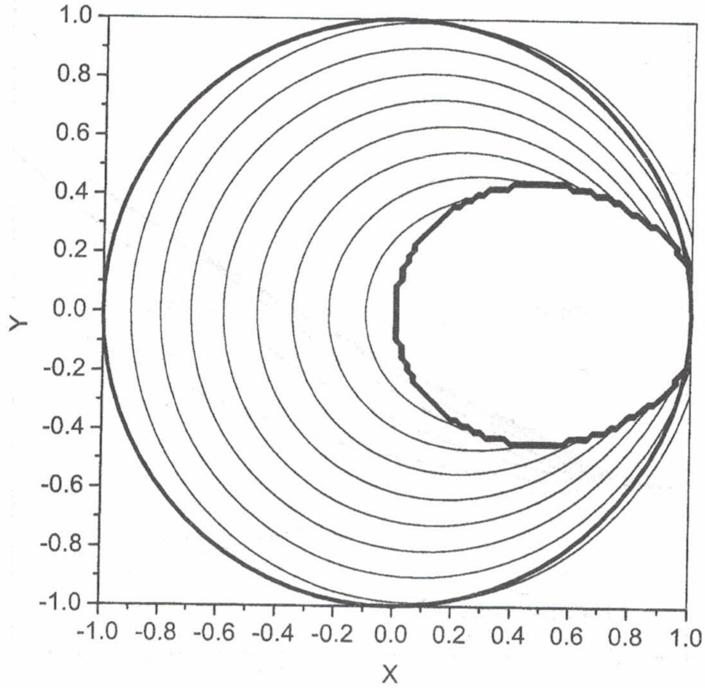
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
Y
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
X

FIG. 1.

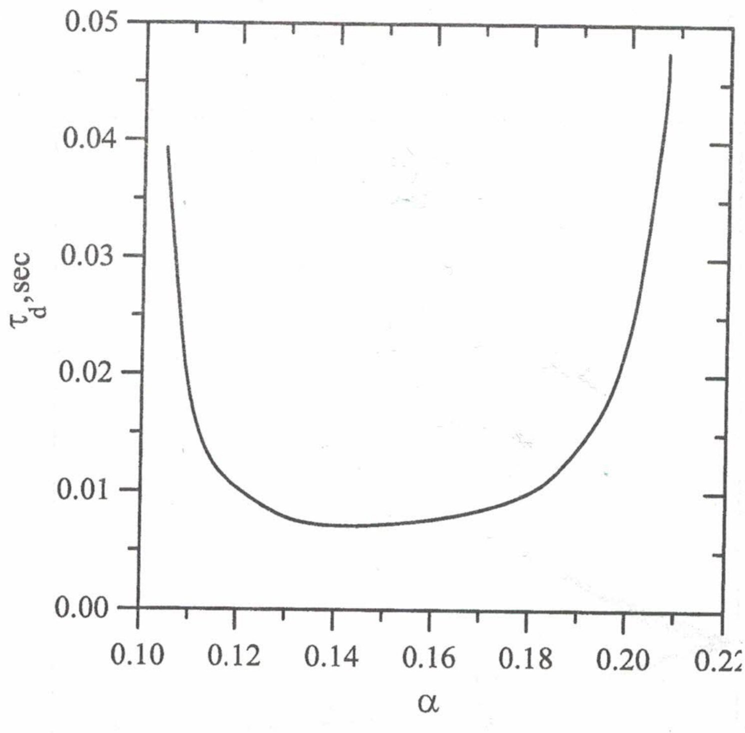
0.05
0.04
0.03
0.02
0.01
0.00
τ_d,sec
0.10 0.12 0.14 0.16 0.18 0.20
α

FIG. 2.

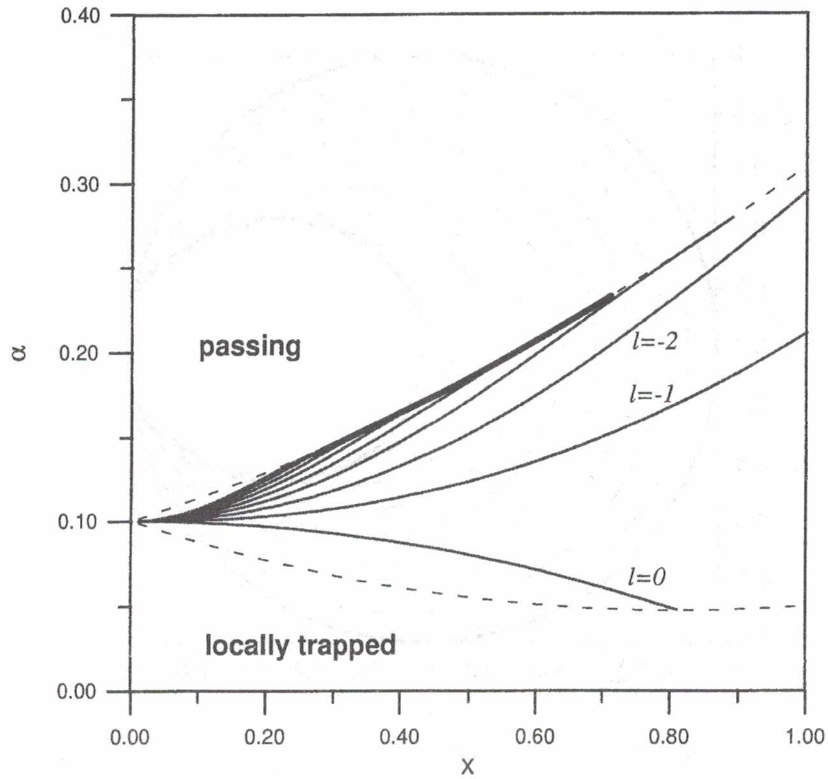
0.40
0.30
0.20
0.10
0.00
α
passing
locally trapped
l=-2
l=-1
l=0
0.00
0.20
0.40
0.60
0.80
1.00
X

FIG. 3.

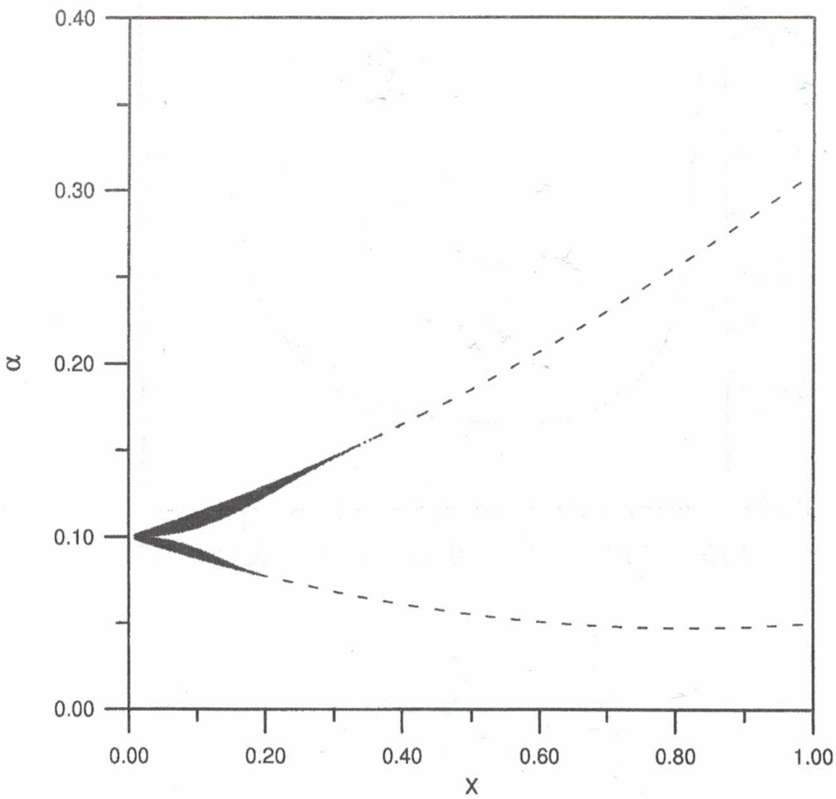
0.40
0.30
0.20
0.10
0.00
α
0.00
0.20
0.40
0.60
0.80
1.00
X

FIG. 4.

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МЕХАНІЗМИ СТОХАСТИЧНОЇ ДИФУЗІЇ В ГВИНТОВИХ СИСТЕМАХ

Я. І. Колесниченко, В. В. Луценко, В. В. Марченко, Г. Вобіґ

Розглянуто можливі механізми стохастичної дифузії надтеплових альфа-частинок в оптимізованих стелараторах. Показано, що послідовні трансформації орбіт перехідних частинок являють собою домінуючий механізм втрат у Геліас-конфігурації.