EFFECT OF DESTABILIZED ALFVÉN EIGENMODES ON ALPHA PARTICLES IN A HELIAS REACTOR

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The upper limits of the local energy losses of circulating α-particles caused by the various instabilities of Alfvén eigenmodes in a four-period Helias reactor are evaluated. It is found that certain destabilized Alfvén eigenmodes will affect only alphas with the energy well below 3,5 MeV, which seems to open a possibility to remove the helium ash by exciting the corresponding Alfvén eigenmodes by either energetic particles or an antenna system.

1. INTRODUCTION

It was predicted at the end of 60's that energetic α-particles can result in destabilization of Alfvén waves in a fusion reactor [1]. Later, this topic attracted a great attention of both theorists and experimentalists. It was found experimentally on tokamaks and stellarators that Alfvén instabilities (AI) driven by energetic ions can lead to the loss of a significant fraction of these ions. Linear and non-linear theories of Alfven instabilities were developed, and transport of the energetic ions induced by AI's was studied. However, most of developed theories are relevant to tokamaks. Furthermore, no attempts to study theoretically the effect of AIs on the confinement of energetic ions in stellarators were done yet. This fact motivated the fulfillment of this work aimed at evaluating the AI-induced losses of alpha particles in a Helias reactor.

Generally speaking, in order to find the effect of AIs on energetic ions one has to study various destabilizing and stabilizing mechanisms, determine the spatial structure and the wave numbers of destabilized waves, and consider the non-linear stage of the instabilities. There is, however, another way which is simple but sufficient to evaluate the *maximum possible* energy loss of alphas caused by AIs. This way is based on the assumption that the amplitudes of the destabilized waves are so large that all the energetic ions which enter the resonance region in the velocity space are lost from the AI localization region. Based on the mentioned assumption, alpha losses in tokamaks were evaluated in Refs. [2,3]. Here, we will apply this approach to evaluate the alpha loss and the energy of the particles affected by the destabilized AEs in a four-period Helias reactor [4]. We restrict ourselves to studying the influence of AIs on the circulating particles and a certain group of transitioning particles ("quasi-circulating" particles), which constitute the majority of the energetic alpha population.

2. EVALUATION OF ALPHA LOSS FRACTION

The magnetic field strength of the Helias configurations contains several main Fourier harmonics:

$$B = \bar{B}[1 + \epsilon_m cos N\varphi - \epsilon_h cos(\vartheta - N\varphi) - \epsilon_t cos\vartheta + \epsilon_0], \tag{1}$$

where ϵ_m is the mirror harmonic, which dominates; ϵ_h is the helical harmonic; ϵ_t is the toroidal harmonic, ϵ_h is the diamagnetic harmonic, see Fig. 1. Therefore, in the Helias systems, in contrast to tokamaks, there exist several sideband resonances, which can strongly change the growth rates and the conditions of instabilities [5]. However, most important resonances, at least when well-localized modes are destabilized, are associated only with the helicity and toroidicity. For most pitch angles, the toroidicity-induced resonance curve lies either higher or coincides with the corresponding helicity-induced one [5]. Therefore, we evaluate alpha losses in an approximation which describes only losses caused by the toroidicity-induced resonance. This approximation considerably simplifies the analysis and, on the other hand, it will enable to make a reasonable estimate of the energy loss.

Thus, we assume that the destabilized waves affect alpha particles through the following resonance:

$$\omega \equiv |k_\parallel| v_A = \left| k_\parallel R \iota^{-1} \pm 1 \right| \omega_\vartheta \, , \tag{2}$$

where

$$k_\parallel \approx (N\nu_0 - \mu_0 \iota_*) \frac{v_{A*}}{2R_0}, \quad \iota_* = \frac{2n + \nu_0 N}{2m + \mu_0}. \tag{3}$$

The frequency of the poloidal motion of the particles equals to $\iota v_\parallel / R_0$ for the well-circulating particles and tends to zero for the marginally circulating ones. Therefore, the resonance particle velocity, v_r, determined by Eq. (2) is a monotonically growing function of the pitch-angle variable λ in the region $(0, \lambda_{max})$, where λ_{max} is the maximum possible magnitude of λ for the particles interacting with the waves through the considered resonance. Taking into account that λ is approximately conserved during the particle slowing down, we can write the fraction of the energy lost from a given flux surface as [2]:

$$\nu_\epsilon = \frac{W_-(r)}{W_+(r)} = \frac{\int_{\Delta v, \Delta\lambda} \overline{d^3v}\, S_\alpha v_r^2}{\int \overline{d^3v}\, S_\alpha v^2} \, , \tag{4}$$

where $W_+(r)$ and $W_-(r)$ are the produced and lost power at the flux surface with the radius r, correspondingly; $S_\alpha \equiv S_\alpha(r, \mathbf{v})$ represents the source of the energetic particles [in particular, $S_\alpha = n_i^2 \langle \sigma v \rangle \delta(v - v_\alpha)/4$ for α-particles]; $n_i^2 \langle \sigma v \rangle /4$ characterizes the rate of alpha production; n_i is the bulk ion density; the bar means the flux surface averaging. The integral in the numerator of Eq. (4) is taken over the region in the (v, λ) space from which the energetic ions can reach the resonance curve $v_r(\lambda)$ during the collisional slowing down. If we take into account only the ϵ_m and ϵ_0 harmonics, we can easily find:

$$\omega_\vartheta = \frac{2\pi}{\tau_\vartheta}, \quad \tau_\vartheta = \frac{4R\mathbf{K}(\kappa^{-1})}{\iota v \kappa \sqrt{2\lambda_l \tilde{\epsilon}_m}} \, , \tag{5}$$

where $\mathbf{K}(\kappa^{-1})$ is the complete elliptic integral of the first kind, κ is the trapping parameter $(\kappa > 1)$ given by

$$\kappa^2 = \frac{\lambda^{-1} - \epsilon_0 + \epsilon_m - 1}{2\epsilon_m} = \frac{\lambda_l^{-1}(r) + \tilde{\epsilon}_m - 1}{2\tilde{\epsilon}_m} \, , \tag{6}$$

$\lambda_l(r) = \mu \hat{B}(r)/\mathcal{E}$ is the local pitch-angle parameter, $\hat{B}(r) = \bar{B}[1 + \epsilon_0(r)]$ is the average magnetic field at the flux surface with the radius r, $\tilde{\epsilon}_m = \epsilon_m/(1 + \epsilon_0)$. Equation (5) is valid for $\lambda_l \leq (1 + \tilde{\epsilon}_m)^{-1}$. It follows from Eq. (6) that the plasma diamagnetism leads to decrease of κ, and, because $d\epsilon_0(r)/dr > 0$, it can result in the transformation of a circulating particle moving outwards into a trapped one.

Taking into account more Fourier harmonics complicates the picture. Indeed, the maximum magnetic field strength is $B_{max} = B(\vartheta = \pi, \varphi = 0) = \hat{B}(1 + \tilde{\epsilon}_m + |\tilde{\epsilon}_h| + |\tilde{\epsilon}_t)|$. This implies that the circulating particles are characterized by $\lambda_l \leq (1 + \tilde{\epsilon}_m + |\tilde{\epsilon}_h| + |\tilde{\epsilon}_t|)^{-1}$ $(\tilde{\epsilon}_{h,t} = \epsilon_{h,t}/(1 + \epsilon_0))$, i.e., adding the helical and toroidal Fourier harmonics reduces the region of circulating particles. But, on the other hand, a particle moving along a field line can pass through the point $(\vartheta = \pi, \varphi = 0)$ only when a part of the field line passed by the particle for the considered time goes through that point. Otherwise, the particles with $\lambda_l = (1 + \tilde{\epsilon}_m + |\tilde{\epsilon}_h| + |\tilde{\epsilon}_t|)^{-1}$ will not be reflected. Furthermore, a particle moving through $(\vartheta = 0, \varphi = 0)$ will not be reflected even when $\lambda_l \leq (1 + \tilde{\epsilon}_m - |\tilde{\epsilon}_h| - |\tilde{\epsilon}_t|)^{-1}$. Therefore, the resonance (2) can be applicable also to particles with $(1 + \tilde{\epsilon}_m + |\tilde{\epsilon}_h| + |\tilde{\epsilon}_t|)^{-1} \leq \lambda_l \leq (1 + \tilde{\epsilon}_m - |\tilde{\epsilon}_h| - |\tilde{\epsilon}_t|)^{-1}$. These are transitioning particles. Eventually they are reflected either due to the motion along the field line or due to precession. After the reflection they can become locally trapped particles or remain locally passing ones. Equation (2) is applicable not to all locally passing particles but only to those ones which have $\tau_\theta \ll \tau_b$, where τ_b is the bounce period. The latter can be referred to as quasi-circulating particles.

We calculate the energy losses of (i) circulating particles; (ii) circulating plus quasi-circulating particles. Correspondingly, we take $0 \leq \lambda_l \leq (1 + \tilde{\epsilon}_m + |\tilde{\epsilon}_h| + |\tilde{\epsilon}_t|)^{-1}$ and $0 \leq \lambda_l < (1 + \tilde{\epsilon}_m - |\tilde{\epsilon}_h| - |\tilde{\epsilon}_t|)^{-1}$. In the latter case we overestimate the influence of AIs on alphas through the resonance given by Eq. (2) because this resonance is valid not for all the transitioning particles. To calculate ω_θ, we use Eqs. (5) and (6) with $\tilde{\epsilon}_m$ replaced by the effective magnitude $\epsilon_{eff} = \tilde{\epsilon}_m + |\tilde{\epsilon}_h| + |\tilde{\epsilon}_t|$ or $\epsilon_{eff} = \tilde{\epsilon}_m - |\tilde{\epsilon}_h| - |\tilde{\epsilon}_t|$, respectively.

At first, let us consider the case when a resonance curve lies close to v_α. Then marginally circulating particles are not affected by the waves; therefore [2],

$$\nu_\epsilon = \frac{v_{r0}^2}{2v_\alpha^2}\int_0^{\lambda_{max}} \frac{d\lambda_l}{(1-\lambda_l)^{3/2}} = \frac{v_{r0}}{v_\alpha}\left(1 - \frac{v_{r0}}{v_\alpha}\right), \tag{7}$$

where $v_{r0} \equiv v_r(\lambda_l = 0)$, $\lambda_{max} = 1 - v_{r0}^2/v_\alpha^2$. The loss fraction given by Eq. (7) cannot exceed 25% and tends to 0 when $v_{r0} \to v_\alpha$. The ratio v_{r0}/v_α depends on both plasma parameters and the kind of the destabilized waves. This ratio and the resonance curves for the Helias reactor are shown in Fig. 2.

Now we calculate ν_ϵ numerically. The results are presented in Figs. 3, 4. It follows from these figures that the destabilization of MAE modes would be the most dangerous, as it could result in the loss of about 35% of the whole α-particle energy. Uncertainties associated with the behavior of transitioning particles become essential for $r/a \geq 0.4$, where the differences between the losses shown in Figs. 3 and 4 constitute about 5%. The smallest of the shown losses is associated with the destabilization of TAE modes; however, this loss is still considerable (more than 20%). On the other hand, the HAE_{21} modes, which are located in the largest frequency gap of the optimized stellarators [6] can result in the loss much less than those shown in Figs. 3 and 4, affecting only particles with the energy $\mathcal{E} \ll 3.5\,\mathrm{MeV}$. This conclusion can be drawn from Fig. 2, from which it follows that α-particles can reach only the lower branch of the HAE_{21} resonance curve located well below v_α.

The obtained results are relevant to the local losses, i.e., to the losses from a flux surface. A necessary condition for AEs to transport the particles over a distance Δr is that the resonant wave-particle interaction persists during the transportation. But the increase of v_r with the radius tends to destroy the interaction of the particles moving outwards. Therefore, the finite width of the resonances must be taken into account to answer the question of whether the wave-particle interaction can take place in the whole region Δr. The magnitude of Δr is the largest for a given resonance width when the plasma density is flat, which is expected to be the case in the plasma core of a Helias reactor.

A specific feature of weak-shear systems is that their AEs are localized in the plasma core (at the periphery, where the plasma density is strongly decreasing, the waves are damped because of the continuum damping). Therefore, the most probable consequence of the excitation of AIs in a Helias reactor is the expulsion of α-particles from the plasma core to the periphery where they can be either thermalized or lost. The losses in the periphery can be induced by various mechanisms; for instance, the transitioning particles can be lost because of the stochastic diffusion predicted in Ref. [7]). In the latter case an AI will be a trigger of the alpha losses. But even when the AIs do not result in alpha loss they will deteriorate the plasma energy balance by moving fast alphas to the periphery.

On the other hand, the transport of fast alphas to the plasma periphery is not necessarily harmful. The destabilized waves that resonate with alphas of relatively low energy (e.g, HAE_{21} instability) will influence the particles only after they have already transferred most of their energy to the plasma. Such instabilities could play a positive role by removing the helium ash from the plasma core. If the corresponding eigenmodes are stable, it would be worth to excite them by an external antenna. Note that a possibility to destabilize TAE modes by using an antenna system was demonstrated in experiments on the JET tokamak [8].

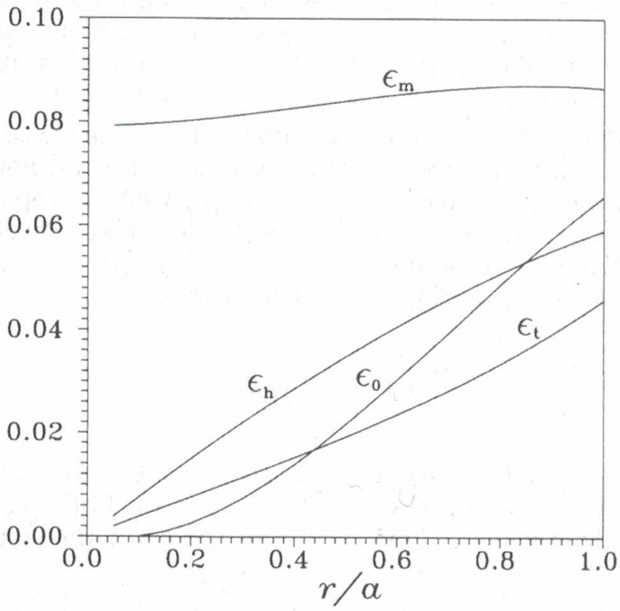

FIG. 1. Fourier harmonics of the magnetic field strength in a 4-period Helias reactor [4]. ϵ_0, ϵ_h, ϵ_m and ϵ_t are defined by Eq. (1).

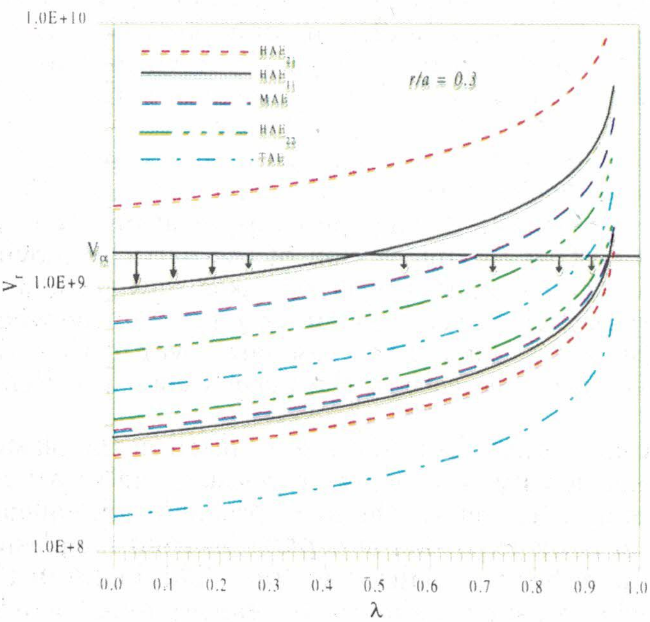

FIG. 2. The region of production of α-particles ($v = v_\alpha$) and the resonance curves determined by Eqs. (2), (3) for $r/a = 0.3$, $\epsilon_{eff} = \epsilon_m - \epsilon_h - \epsilon_t$, and various Alfvén eigenmodes. The arrows show the direction of the alpha motion due to collisional slowing down. The particles that reach the resonance curves are assumed to be lost.

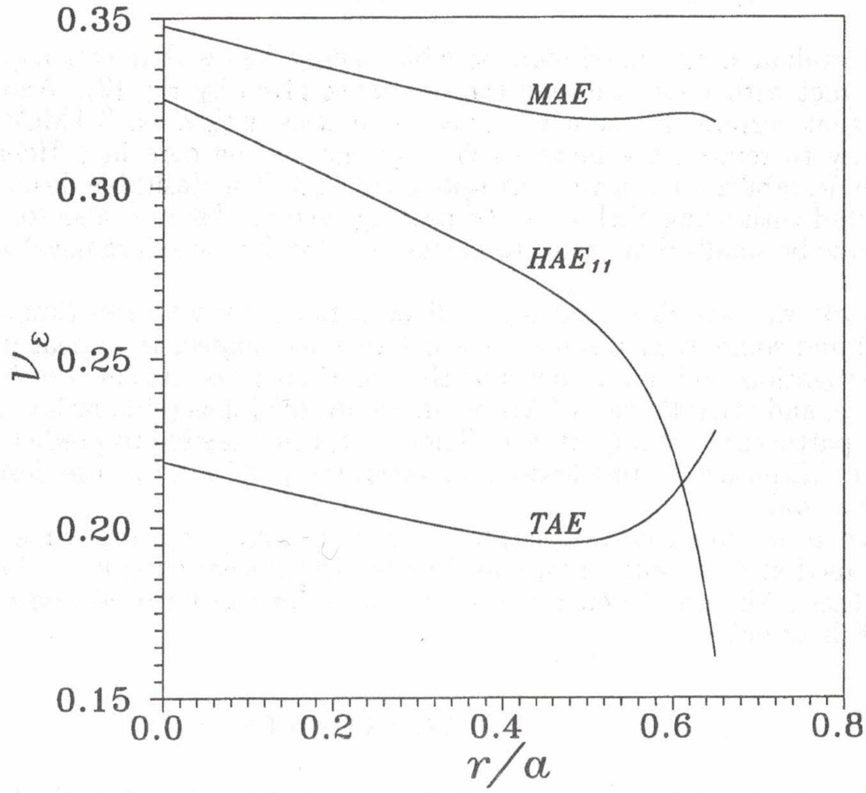

FIG. 3. Local energy losses versus r/a for $v_\alpha = 3.2 v_A(r=0)$ and $\epsilon_{eff} = \epsilon_m + \epsilon_h + \epsilon_t$.

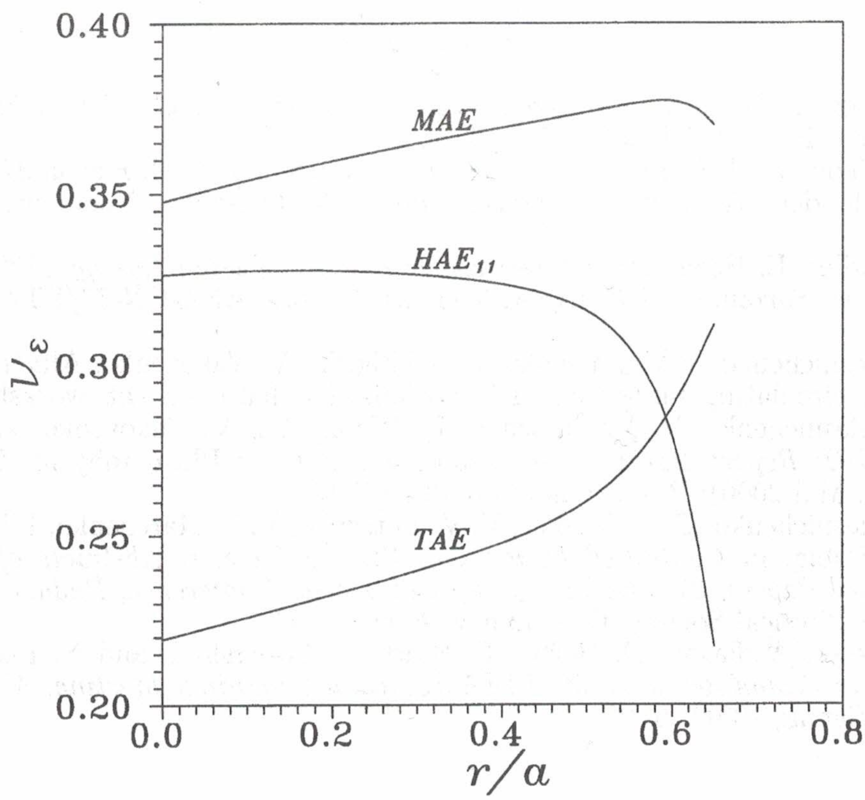

FIG. 4. The same as in Fig. 3 but for $\epsilon_{eff} = \epsilon_m - \epsilon_h - \epsilon_t$.

3. SUMMARY AND CONCLUSIONS

We have evaluated the maximum possible energy losses that can happen when α-particles interact with waves through the resonance given by Eq. (2). Another result of the work is that certain AIs influence only α-particles with $\mathcal{E} \ll 3.5\,\mathrm{MeV}$. This seems to open a way to remove the helium ash from the plasma core in a Helias reactor by means of the destabilization of the corresponding AEs. The desirable destabilization can be implemented with using NBI or an RF heating system. There is also some probability (which seems to be small) that the instabilities required for the ash removal will be excited by α-particles.

In this work, we have disregarded the effect of bounce resonances (known from tokamak studies) and some other resonances which one may expect to appear in stellarators. Further investigation taking into account the mentioned resonances, the finite width of the resonances, and the influence of AIs on all groups of alphas (circulating, transitioning, and trapped particles) is of importance. Then it will be possible to predict the complete picture of the AE-induced alpha losses and assess the possibility to use destabilized AEs for the ash removal.

In conclusion, we note that it would be of interest to carry out experiments on W7-AS and W7-X aimed at destabilizing various AEs by an external antenna and observing the influence of these AEs on the energetic ions (the latter can be produced either by NBI and/or ICRF heating).

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ВПЛИВ ДЕСТАБІЛІЗОВАНИХ АЛЬФВЕНІВСЬКИХ ВЛАСНИХ МОД НА АЛЬФА-ЧАСТИНКИ В ГЕЛІАСІ-РЕАКТОРІ

Я. І. Колесниченко, В. В. Луценко, Г. Вобіґ, Ю. В. Яковенко

Оцінено максимально можливі втрати пролітних альфа-частинок, пов'язані зі збудженням альфвенівських нестійкостей у Геліасі-реакторі з чотирма періодами поля. Знайдено, що деякі дестабілізовані альфвенівські власні моди впливають на утримання лише тих альфа-частинок, енергія яких є багато меншою, ніж 3.5 МеВ. Ймовірно, що це відкриває можливості видаляти гелієвий попіл із реактора шляхом збудження відповідних власних мод або енергійними іонами, або спеціальними антенами.