CORE-LOCALIZED ALFVÉN EIGENMODES IN STELLARATORS

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The work deals with discrete Alfvén eigenmodes in optimized stellarators of the Wendelstein line. It is shown that core-localized Alfvén eigenmodes do exist in these systems. In particular, mirror-induced Alfvén eigenmodes (MAE) and helicity-induced Alfvén eigenmodes (HAE_{21}) localized in the plasma core of the four-period Helias reactor are found. The results are obtained by solving numerically an eigenmode equation derived in *Ya. Kolesnichenko et al., Phys. Plasmas, 8, 491 (2001).*

1. INTRODUCTION

Recently it was predicted that there exist various kinds of Alfvén eigenmodes (AE) in optimized stellarators of the Wendelstein line [1–4]. If destabilized, these modes can considerably deteriorate the confinement of the energetic ions in a Helias reactor [5,6]. On the other hand, plasma inhomogeneity in Helias configurations strongly affects the AEs of global character, tending to prevent their existence, which is associated with low magnetic shear in these systems [2]. This has raised a question on the existence of core-localized AEs in Helias configurations.

Note that the problem of the existence of core-localized AEs, namely, TAE modes, is of interest for tokamaks, too, because such modes can be destabilized most easily. Therefore, it was studied some years ago in Refs. [7,8], where it was found that that there are odd and even core-localized TAE modes located near the maximum and minimum of the TAE gap, respectively. The question on the core-localized Alfvén eigenmodes in stellarators is not investigated yet. The only step in this direction was done in Ref. [4], where, using the ideal MHD CAS3D code, core-localized sound modes were found (but sound waves are known to be strongly damped in plasmas with $v_A < v_e$, where v_A is the Alfvén velocity, v_e is the electron thermal velocity).

The weak magnetic shear of a Wendelstein-like stellarator leads to a difference in features of AEs, including toroidicity-induced Alfvén eigenmodes (TAE), in these systems and in tokamaks where, in general, $\hat{s} \sim 1$ ($\hat{s}$ is the shear). The difference between the small-shear case, $\hat{s} \ll 1$, and the case of $\hat{s} \sim 1$ can be understood from the following consideration relevant to TAE modes. When shear is high, the longitudinal wave number, $k_{\parallel}(r)$, vanishes in many points along the radial coordinate r because the rotational transform $\iota(r)$ considerably varies with the radius (this can be seen from the equation $k_{\parallel} = (m\iota - n)/R$, where R is the major radius of the torus, m and n are the poloidal and toroidal wave numbers, respectively). Due to this fact, as r varies, the branches of Alfvén frequency $\omega_A(r) = k_{\parallel} v_A$ intersect for many pairs of the poloidal mode numbers and given n, i.e., for successive pairs m_i and $m_i + 1$, where $m_i = m_1 + i - 1$ with $i = 1, 2, ...i_{max}$, $i_{max} \gg 1$, and m_1 is the minimal poloidal mode number for which $k_{\parallel}$ can vanish for given $\iota(r)$. This implies that the characteristic length of plasma inhomogeneity increases by a factor of $i_{\max}$ because the envelop of the gap (along r) is formed by many pairs of branches, and each pair "feels" the change of plasma density at the distance about $a/i_{\max}$, where a is the plasma radius. In contrast to the large-shear case, i_{max} is about unity when $\hat{s} \ll 1$, which leads to strong influence of the inhomogeneity on the gaps.

The calculations show that the gaps in the Alfvén continuum in Helias configurations are closed for any realistic density profiles, i.e., any frequency lying in the gap at the plasma center meets the continuum at the periphery, where the density decreases [2]. As a result, calculated modes of global character exhibit signs of strong interaction with the continuum at the plasma periphery (large spikes, which are numerical approximations of singularities at the points where the frequency crosses the continuum). Therefore, when a gap is closed, the local Alfvén resonance takes place for the destabilized global modes located in the gap rather then for the other modes coupled with the destabilized modes due to toroidicity and other coupling factors, which is the case when $\hat{s} \sim 1$. This strongly increases the role of the continuum damping, which, in fact, "kills" most of global eigenmodes in the small-shear configurations. Two sketches of the gaps in Alfvén continuum in a small-shear system and in a large-shear system are given in Figs. 1,2.

This work is aimed at the search of core-localized AEs in Wendelstein line stellarators. The analysis in the work is restricted with the case of high-frequency AEs modes.

2. CORE-LOCALIZED MIRROR-INDUCED ALFVÉN EIGENMODERS AND HELICITY-INDUCED ALFVÉN EIGENMODES

We proceed from the following set of equations:

$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{\bar{v}_A^2}-k_{m,n}^2\right)\frac{dE_{m,n}}{dr}\right]-\left[r(1-m^2)\left(\frac{\omega^2}{\bar{v}_A^2}-k_{m,n}^2\right)+r^2\left(\frac{\omega^2}{\bar{v}_A^2}\right)'\right]E_{m,n}$$
$$+\sum_{\mu,\nu}\frac{d}{dr}\left\{r^3\left[\frac{\omega^2}{\bar{v}_A^2}\left(\frac{\epsilon_g^{(\mu\nu)}}{2}-2\epsilon_B^{(\mu\nu)}\right)-k_{m,n}k_{m+\mu,n+\nu N}\frac{\epsilon_g^{(\mu\nu)}}{2}\right]\frac{dE_{m+\mu,n+\nu N}}{dr}\right\}=0, \quad (1)$$

where $E_{m,n}$ is the Fourier harmonic of $E=\Phi/r$ with Φ the scalar potential of the perturbed electric field, m and n are the poloidal and toroidal wave numbers, respectively; r is the average flux-surface radius defined by the relation $\psi=B_0r^2/2$ with ψ the toroidal magnetic flux; N is the number of the magnetic field periods; prime denotes d/dr; $k_{m,n}\equiv(m\iota-n)/R_0$. The coupling parameters $\epsilon_B^{(\mu\nu)}$ and $\epsilon_g^{(\mu\nu)}$ are the Fourier coefficients of the metric tensor component $g^{\psi\psi}$ and the magnetic field strength according to

$$g^{\psi\psi}=\bar{g}^{\psi\psi}\left[1+\frac{1}{2}\sum_{\mu,\nu=-\infty}^{\infty}\epsilon_g^{(\mu\nu)}\exp(i\mu\theta-i\nu N\phi)\right], \quad (2)$$

$$B=\bar{B}\left[1+\frac{1}{2}\sum_{\mu,\nu=-\infty}^{\infty}\epsilon_B^{(\mu\nu)}\exp(i\mu\theta-i\nu N\phi)\right], \quad (3)$$

where θ and ϕ are the poloidal and toroidal angles, respectively; $\bar{B}$ and $\bar{g}^{\psi\psi}$ are the toroidal averages of the corresponding quantities at the magnetic axis. One can see that $\epsilon_g^{(-\mu,-\nu)}=\epsilon_g^{(\mu\nu)*}$, $\epsilon_B^{(-\mu,-\nu)}=\epsilon_B^{(\mu\nu)*}$, where the superscript "*" denotes complex conjugate.

Equation (1) coincides with Eq. (28) of Ref. [2] up to some notations. It describes a set of coupled harmonics with different poloidal and toroidal wave numbers, m and n. Choosing appropriate sets of harmonics, one can study eigenmodes of various kinds. Here we deal with two kinds of modes. First, we study helicity-induced Alfvén eigenmodes (HAE_{21}), which result from the interaction of a pair of harmonics with the wave numbers (m,n) and $(m+2,n+N)$ due to the rotation of the elongation of the plasma cross section. Second, we study mirror-induced Alfvén eigenmodes (MAE) modes discovered in Ref. [2]. These modes arise from the interaction of harmonics with the wave numbers (m,n) and $(m,n+N)$, which is caused by the toroidal variation of the elongation, the magnetic field strength, and the cross section radius. We consider a system of four harmonics, (m,n), $(m,n+N)$, $(m+2,n+N)$ and $(m-2,n)$, rather than two. The reason is that coupling via the rather large parameter $\epsilon_g^{(21)}$ exerts strong influence on the MAE gap, which cannot be disregarded.

The calculations are carried out for a Helias reactor characterized by a 4-period magnetic configuration with strongly elongated cross section, the elongation, κ, varying along the torus between $\kappa_{min}=1.8$ and $\kappa_{max}=6.2$ (at $r/a=0.5$) [5]. To find the Fourier harmonics of the metric tensor, we employ the following equation valid in the vicinity of the magnetic axis, where the flux surfaces are well approximated by concentric ellipses:

$$g^{\psi\psi} = 2B_0(\phi)\{\delta(\phi) + \lambda(\phi)\cos[2\theta - 2\theta_0(\phi)]\}\psi, \tag{4}$$

where $\delta = (\kappa + \kappa^{-1})/2$, $\lambda = (\kappa^{-1} - \kappa)/2$, and $\theta_0(\phi)$ describes the rotation of the cross section. We assume that δ and λ vary along the torus as $\delta(\phi) = \delta_0 + \delta_1 \cos N\phi$, $\lambda(\phi) = \lambda_0 + \lambda_1 \cos N\phi$, and that the rotation of the cross section is uniform, $\theta_0(\phi) = (N\phi + \pi)/2$. Then we find the parameters δ_0, δ_1, λ_0, and λ_1 from κ_{min} and κ_{max} and substitute the obtained expressions to Eq. (4) (see Ref. [2] for details). We obtain that the coupling parameter associated with the elongation is rather large, $\epsilon_g^{(21)} = 0.84$. In addition, the toroidal modulation of the elongation results in the considerable (2,0) and (2,2) harmonics of the metric tensor. The latter, as was shown in Ref. [2], is of importance for the MAE modes. From Eq. (4) we obtain $\epsilon_g^{(20)} = \epsilon_g^{(22)} = 0.27$. The parameters $\epsilon_B^{(21)}$, $\epsilon_g^{(20)}$, and $\epsilon_g^{(22)}$ are small in the central part of the plasma and, therefore, are neglected here. The toroidal variation of the magnetic field is $\epsilon_B^{(01)} = 0.08$, whereas that of $g^{\psi\psi}$ due to the variation of the elongation and the radius is $\epsilon_g^{(01)} = 0.54$.

It is known that discrete eigenmodes appear near the flux surfaces where two cylindrical continuum branches would cross in the absence of coupling, which yields the following equation for the rotational transform at the intersection point [2]:

$$\iota = \frac{2n + \nu N}{2m + \mu}, \tag{5}$$

where (m, n) and $(m + \mu, n + \nu N)$ are the wave numbers of the interacting continuum branches, and the so-called coupling numbers (μ, ν) are $(2, 1)$ for HAE_{21} modes and $(2, 0)$ for MAE modes. As the Helias configuration considered here is characterized by small shear,

$$\iota = 0.84897 + 0.12591\frac{r}{a} + 0.00481\frac{r^2}{a^2} - 0.02035\frac{r^3}{a^3}, \tag{6}$$

so that

$$0.85 \le \iota \le 0.96, \tag{7}$$

there are only few numbers (m, n) satisfying Eq. (5). We have selected for our study the HAE_{21} modes with $(m, n) = (6, 4)$ and $(m, n) = (10, 8)$ and MAE modes with $(m, n) = (7, 4)$.

We solve Eq. (1) with the code BOA [2]. We begin with considering the HAE_{21} modes with $(m, n) = (6, 4)$. For these modes the crossing point of the cylindrical continua determined by Eqs. (5) and (6) is located close to the magnetic axis. Two radial profiles of the plasma mass density, ρ, are considered, $\rho(r) = \rho_0 = \text{const}$ and $\rho(r) = \rho_0\{1 + [r/(ax_n)]^{10}\}^{-1}$, where $x_n = 0.7$. The latter is expected to be the case in the considered reactor configuration. We have found the discrete eigenmodes presented in Figs. 3 and 4. We observe that the effect of the density profile on the eigenmodes and the eigenfunctions is weak. Although the HAE_{21} gap is closed for the realistic density profile, the wave functions have no signs of the presence of the continuum. The reason is that the wave functions are localized in the plasma core and fade away at the periphery, where their frequencies cross the continuum. Thus, the considered eigenmodes do not experience continuum damping.

Results of calculations for the HAE_{21} mode $(m, n) = (10, 8)$ are presented in Figs. 5 and 6. For this mode the crossing point of the cylindrical continua and the eigenmodes are located at $r \sim a/2$. In the case of the uniform density, the eigenmodes consist of two groups characterized by different phasing of the wave functions. The (m, n) and $(m + 2, n + N)$ components of the modes with the frequency near lower edge of the gap have the same phases, whereas those of the modes with the frequency near the the upper edge of the gap, have the opposite phases. A similar difference in phasing is observed for the core-localized TAE [7,8]. Another difference between the upper and lower groups of the eigenmodes consists in the different dependence of the spatial mode structure on the eigenfrequency: the wave functions of the upper and lower groups are characterized by the Sturmian and anti-Sturmian behavior (in the sense of Ref. [9]), respectively (i.e., the increase/decrease of the eigenvalue with the increase of the number of zeros of the eigenfunction). The wave functions of the Sturmian and anti-Sturmian sequences resemble those of successive levels of a quantum particle in a potential well. Note that anti-Sturmian sequences of gap eigenmodes were observed in calculations of MHD spectra in Wendelstein 7-AS (W7-AS) with the CAS3D code [4], but in our calculations Sturmian sequences rather than anti-Sturmian are more typical.

Each wave function in the same part of the spectrum obtained in the case of the non-uniform density consists of two distinct parts, a sharp spike at the point where the eigenfrequency crosses the continuum and a smooth part similar to wave functions in the uniform plasma. They seem to fall into two categories, the functions consisting of a large smooth part and a small spike and vice versa. The former are expected to experience weak continuum damping and can be excited by energetic particles.

Although MAEs are studied with a more complicated system of four equations, the obtained solutions look similar to the case of HAE_{21}. The calculated continuous spectrum for the $(7,4)$ MAE mode is shown in Figs. 7 and 8 [although the calculations took account of four harmonics, $(m,n) = (7,4)$, $(m,n+N) = (7,8)$, $(m+2,n+N) = (9,8)$ and $(m-2,n) = (5,4)$, only two branches of the continuum that correspond to the main harmonics of the mode, the coupled $(m,n) = (7,4)$ and $(m,n+N) = (7,8)$ harmonics, are drawn]. Like previously, the eigenmodes in the uniform plasma form the Sturmian and anti-Sturmian sequences. The non-uniformity of the plasma density makes the continuum gap closed. However, the eigenmodes do not disappear although small sharp spikes at the points where the eigenfrequency crosses the continuum branches are evidence of possible continuum damping. To answer the question of whether these modes can be destabilized by energetic ions, thorough analysis of all destabilizing and damping factors is required.

3. SUMMARY AND CONCLUSIONS

We have found that core-localized Alfvén eigenmodes, which can be most easily destabilized by energetic ions, do exist in Wendelstein-line stellarators. In particular, HAE_{21} and MAE discrete modes localized in the region $r/a \leq 0.5$ were found in the Helias Reactor HSR4/18. They form two *groups* of eigenmodes with different phasing in the lower and upper parts of the gap. Calculations were carried out with the use of the metric tensor obtained numerically and analytically, from which it follows that the latter provides a reasonable approximation.

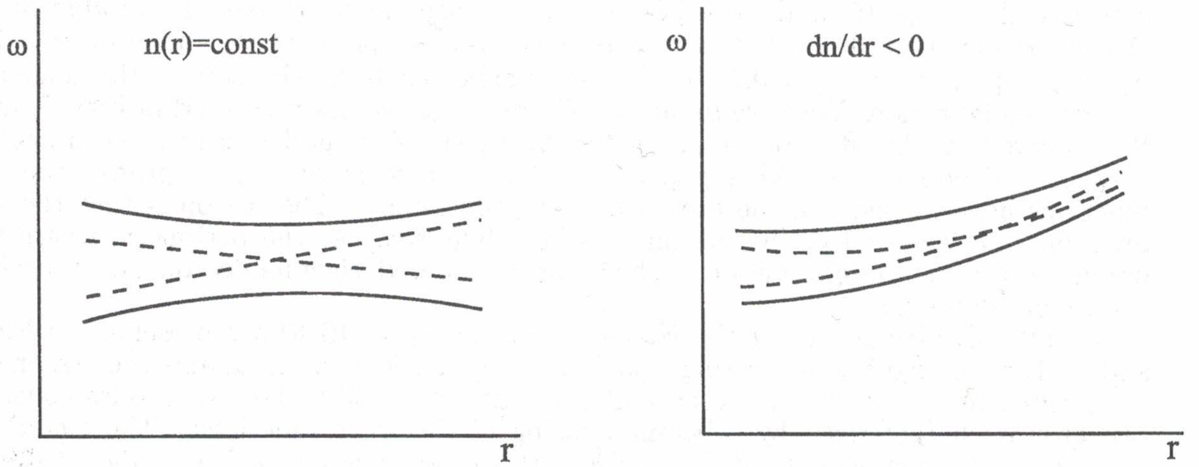

FIG. 1. Alfvén continuum for small shear $\hat{s} \ll 1$.

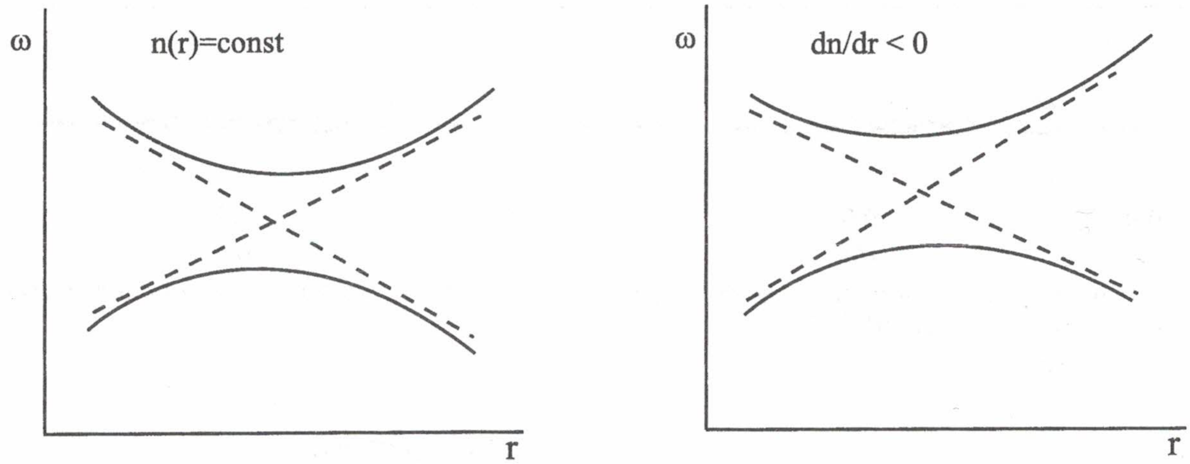

FIG. 2. Alfvén continuum for large shear $\hat{s} \sim 1$.

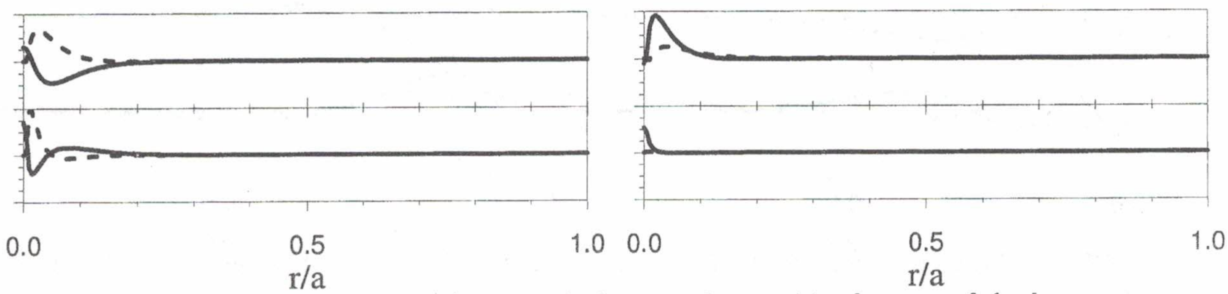

FIG. 3. Wave functions of the HAE_{21} mode for $m = 6$, $n = 4$ in the case of the homogeneous plasma.

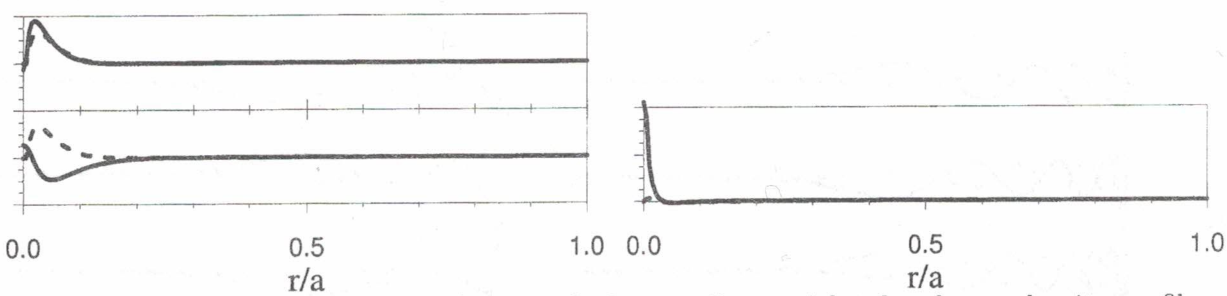

FIG. 4. Wave functions of the HAE_{21} mode for $m = 6$, $n = 4$ for the plasma density profile $\rho(r) = \rho_0\{1 + [r/(ax_n)]^{10}\}^{-1}$.

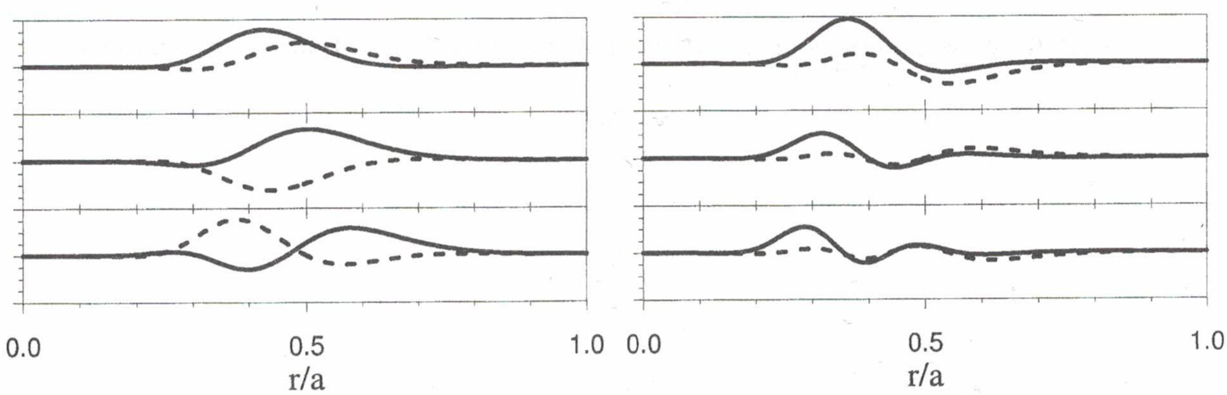

FIG. 5. Wave functions of the HAE_{21} mode for $m = 10$, $n = 8$ in the case of the homogeneous plasma.

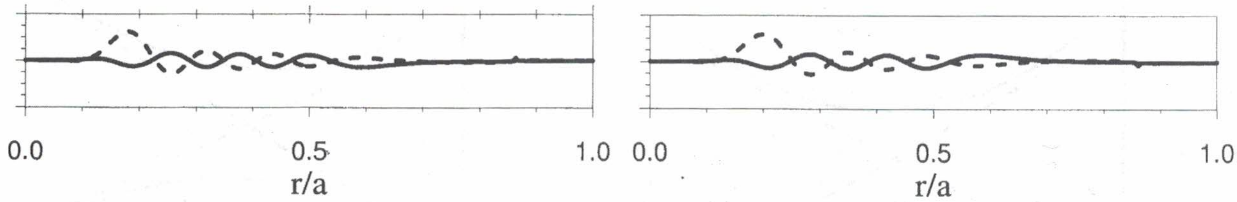

FIG. 6. Wave functions of the HAE_{21} mode for $m = 10$, $n = 8$ for the plasma density profile $\rho(r) = \rho_0\{1 + [r/(ax_n)]^{10}\}^{-1}$.

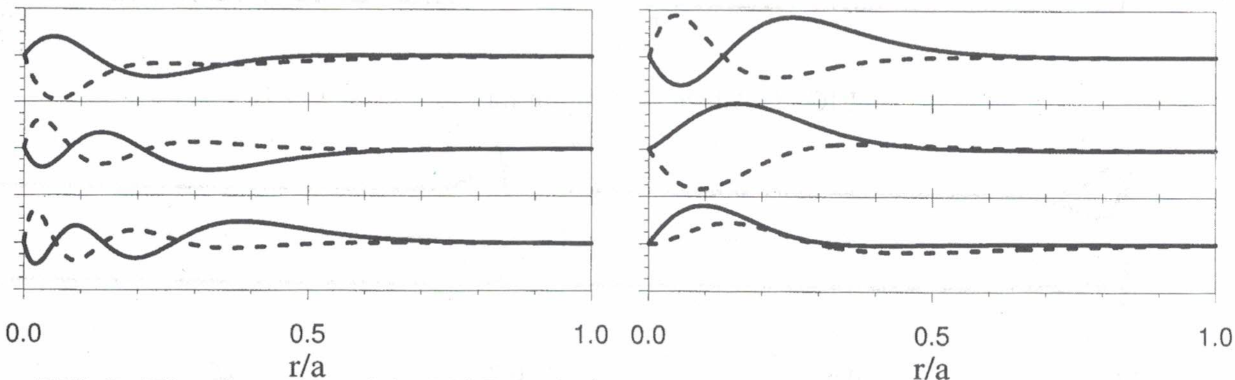

FIG. 7. Wave functions of the MAE mode for $m = 7$, $n = 4$ in the case of the homogeneous plasma.

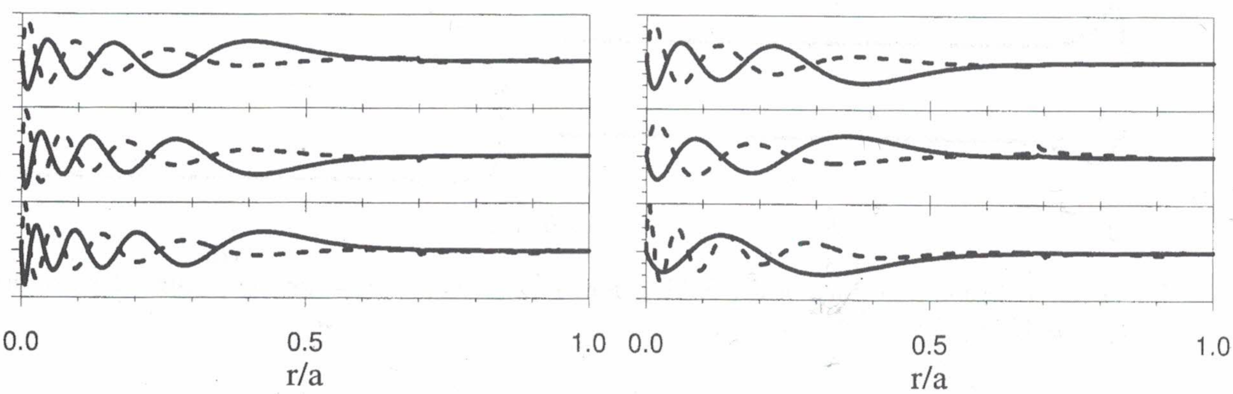

FIG. 8. Wave functions of the MAE mode for $m = 7$, $n = 4$ for the plasma density profile $\rho(r) = \rho_0\{1 + [r/(ax_n)]^{10}\}^{-1}$.

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References

[1] C. Nührenberg, Plasma Phys. Control. Fusion **41**, 1055 (1999).
[2] Ya. I. Kolesnichenko, V. V. Lutsenko, H. Wobig, Yu. V. Yakovenko, and O. P. Fesenyuk, Report IPP III/261, Mai 2000; Phys. Plasmas **8**, 491 (2001).
[3] Ya. I. Kolesnichenko, V. V. Lutsenko, H. Wobig, Yu. V. Yakovenko, and O. P. Fesenyuk, 27th EPS Conference on Contr. Fusion and Plasma Phys., Budapest (2000), Report P1.071.
[4] C. Nührenberg, in *ISSP-19 "Piero Caldirola", Theory of Fusion Plasmas, J. W. Connor, O. Sauter, and E. Sindoni (Eds.)* (SIF, Bologna, 2000), p. 313.
[5] C. D. Beidler, E. Harmeyer, F. Herrnegger *et al.*, in *Fusion Energy 2000, 18th IAEA Conference Proceedings, Sorrento, 2000* (IAEA, Vienna), Report IAEA-CN-77/FT/4, to be published.
[6] Ya. I. Kolesnichenko, V. V. Lutsenko, H. Wobig, Yu. V. Yakovenko, Effect of destrabilized Alfven eigenmodes on alpha particles in a Helias reactor, paper No. Th.8, this workshop.
[7] G. Y. Fu, Phys. Plasmas **2**, 1029 (1995).
[8] H. L. Berk, J. W. Van Dam, D. Borba, J. Candy, G. T. A. Huysmans, and S. Sharapov, Phys. Plasmas **2**, 3401 (1995).
[9] J.P. Goedbloed, Phys. Fluids. **18**, 1258 (1975).

АЛЬФВЕНІВСЬКІ ВЛАСНІ МОДИ, ЩО ЛОКАЛІЗОВАНІ В ЦЕНТРІ ПЛАЗМИ СТЕЛАРАТОРІВ

Я. І. Колесниченко, В. В. Луценко, Г. Вобіг, Ю. В. Яковенко

У роботі досліджено питання про дискретні альфвенівські власні моди в оптимізованих стелараторах серії Вендельштайн. Показано, що локалізовані в центрі плазми альфвенівські власні моди існують у цих системах. Зокрема, знайдено дзеркальні альфвенівські власні моди (MAE) та гвинтові альфвенівські власні моди (HAE_{21}), що локалізовані в центрі плазми чотириперіодного Геліас-реактора. Результати отримано шляхом чисельного розв'язання рівняння власних мод, що його виведено в роботі Ya. Kolesnichenko et al., Phys. Plasmas, 8 491 (2001).