ALFVÉN INSTABILITIES CAUSED BY CIRCULATING ENERGETIC IONS IN OPTIMIZED STELLARATORS

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The work investigates instabilities of Alfvén eigenmodes that can be driven by circulating energetic ions in optimized stellarators of Wendelstein line (Helias). It is shown for the first time that several sideband resonances rather than the only one associated with toroidicity (and known from a theory relevant to tokamaks) may essentially contribute to the instability growth rate. New resonances enhance the instabilities and, moreover, they may result in instabilities in those cases when the conventional resonance is not efficient. Destabilization of the toroidicity-induced Alfvén eigenmodes and the eigenmodes existing due to both specific plasma shaping and Fourier harmonics of the magnetic field of a Helias is considered.

1. INTRODUCTION

Experiments on stellarators, in particular, on Wendelstein 7-AS, have demonstrated that neutral beam injection can lead to Alfvén instabilities and concomitant losses of energetic ions [1]. One can expect that similar instabilities driven by alpha particles may arise in a fusion reactor. On the other hand, analysis of Alfvén instabilities observed experimentally on stellarators is often relied on a theory developed for tokamaks. However, it is clear that because of the variety of recently discovered Alfvén eigenmodes [2–4] such an approach may be not justified to interprete the experimental data, and it cannot be used for a reliable prediction of the role of Alfvén instabilities in Wendelstein 7-X and a Helias reactor [5,6]. Moreover, as we will show in this work, wave-particle resonant interaction in stellarators has a number of peculiarities, so that description of even well-known instability of Global Alfvén Eigenmodes (GAE) may require a non-standard analysis.

In the present work we study the destabilization of various Alfvén eigenmodes by circulating energetic ions in optimized stellarators. Namely, we consider destabilization of Mirror-induced Alfvén Eigenmodes (MAEs), Helicity-induced Alfvén Eigenmodes (HAE_{11}, HAE_{21} and HAE_{22}, where the first and second subscipts denote the poloidal and toroidal mode coupling numbers, respectively [2,3]), Ellipticity-induced Alfvén Eigenmodes (EAE), Toroidicity-induced Alfvén Eigenmodes (TAE), and GAEs.

2. RESONANCES

Physical mechanism responsible for the destabilization of Alfvén eigenmodes by energetic ions is the resonance wave-particle interaction. In order to find the corresponding resonances in an arbitrary toroidal system we proceed from the following expression describing the energy exchange between a particle and a wave in the guiding center approximation:

$$\dot{\mathcal{E}} = e(v_\parallel \tilde{E}_\parallel + \mathbf{v}_D \cdot \tilde{\mathbf{E}}_\perp) + \mu_p \frac{\partial \tilde{B}}{\partial t}, \tag{1}$$

where $\mathcal{E}$ is the particle energy, μ_p is the particle magnetic moment, $\mathbf{v}_D$ is the drift velocity caused by the curvature and inhomogeneity of the magnetic field, $\tilde{\mathbf{E}}$ is the wave electric field, $\tilde{\mathbf{B}}$ is the wave magnetic field, the subscripts "$\perp$" and "$\|$" label the vector components across and along the magnetic field, respectively. The field $\tilde{E}_\parallel$ is small for Alfvén perturbations. Therefore, when the ion energy is sufficiently high, a term proportional to the drift velocity dominates for well circulating particles, i.e., $\partial\mathcal{E}/\partial t \approx e v_{Dr} \tilde{E}_r \sim (\partial B_0/\partial\vartheta)\tilde{E}_r$, where B_0 is the equilibrium magnetic field, $r = r(\psi)$ with ψ the toroidal magnetic flux and ϑ are the radial and poloidal coordinates, respectively (we use the flux coordinates ψ, ϑ, φ [7,8]). Let us take a perturbation in a form

$$\tilde{X} = \sum_{m,n} X_{m,n}(r) \exp(im\vartheta - in\varphi - i\omega t), \tag{2}$$

where m and n are the poloidal and toroidal wavenumbers, respectively, and expand B_0 in a Fourier series:

$$B_0 = \bar{B}[1 + \Sigma_{\mu,\nu}\epsilon^{(\mu\nu)} \cos(\mu\vartheta - \nu N\varphi)], \tag{3}$$

where $\epsilon^{(\mu\nu)}$ are the Fourier harmonics, N is the number of the field periods. Then using Eq. (3) and the equations $\dot{\vartheta} = \omega_\vartheta$, $\dot{\varphi} = \omega_\varphi$, where $\omega_\vartheta \approx const$ and $\omega_\varphi \approx const$ are the frequencies of the particle poloidal and toroidal motion, respectively, ($\omega_\vartheta = \iota\omega_\varphi$ with ι the rotational transform) we obtain:

$$\dot{\mathcal{E}} \sim \Sigma_{\mu,\nu,m,n,j}\epsilon^{(\mu\nu)}\mu exp\{-i[\omega - (m + j\mu)\omega_\vartheta + (n + j\nu N)\omega_\varphi]t\}, \tag{4}$$

where $j = 1, -1$. It follows from Eq. (4) that an energetic ion can transfer a considerable part of its energy for $\Delta t \gg max\{\omega_\varphi^{-1}, \omega_\vartheta^{-1}\}$ when the following resonance occurs:

$$\omega - (m + j\mu)\omega_\vartheta + (n + j\nu N)\omega_\varphi = 0. \tag{5}$$

In the case of tokamaks, $\mu = 1$, $\nu = 0$, therefore, the only possible resonance is $\omega = (k_\parallel + j\iota R^{-1})v_\parallel$, where $k_\parallel = (m\iota - n)/R_0$, R_0 the large radius of the torus. The situation changes when the magnetic configuration is not axisymmetric. Then there are many resonances coresponding to various Fourier harmonics of the magnetic field strength. In particular, the mirror harmonic ($\mu = 0, \nu = 1$), the helical harmonic ($\mu = 1, \nu = 1$), the diamagnetic harmonic ($\mu = \nu = 0$), and the toroidal harmonic $\mu = 1, \nu = 0$ are dominant in the Helias configurations.

This simple analysis indicates that new important resonances through which the energetic ions may drive Alfvén instabilities in stellarators may appear because of the absence of the axial symmetry. Straightforward calculations of the instability growth rate are required to find which resonances are really important.

3. INSTABILITIES IN THE HELIAS CONFIGURATIONS

The longitudinal components of the electric field and the magnetic field of Alfvén perturbations are small. Therefore, we may take $\tilde{\mathbf{A}}_\perp = 0$, where $\mathbf{A}$ is the vector potential of the electromagnetic field. Then $\tilde{\mathbf{E}} = -\nabla\tilde{\Phi}$ with $\tilde{\Phi}$ the scalar electric potential, and the growth rate of the instability can be expressed in terms of $\tilde{\Phi}$. We find:

$$\gamma_\alpha = \frac{2\pi}{c^2} \frac{Re \sum_{m,n} \int d^3x\, \mathbf{j}^\alpha_{\perp m,n} \cdot \nabla_\perp \Phi^*_{m,n}}{\sum_{m,n} \int d^3x\, v_A^{-2} \left[\left|\Phi'_{m,n}\right|^2 + (m^2/r^2)\left|\Phi_{m,n}\right|^2\right]}, \tag{6}$$

where $\mathbf{j}^\alpha_{mn}$ is the resonance current of the energetic particles, $\nabla\Phi_{m,n} \equiv \Phi'_{m,n}\mathbf{e}^1 + im\Phi_{m,n}\mathbf{e}^2 - in\Phi_{m,n}\mathbf{e}^3$, $\mathbf{e}^1 = \nabla r$, $\mathbf{e}^2 = \nabla\vartheta$, $\mathbf{e}^3 = \nabla\varphi$ are the basis vectors, $\Phi'_{m,n} \equiv \partial\Phi_{m,n}/\partial r$, v_A is the Alfvén velocity. In order to find the $\mathbf{j}^\alpha_{mn}$ in Eq. (6), we write $\tilde{\mathbf{j}}_\alpha = e \int d^3v (\mathbf{v}_\parallel + \mathbf{v}_D)\tilde{f}_{res}$. Here, the perturbed resonance part of the distribution function, $\tilde{f}_{res}$, obtained from the linearized kinetic equation, can be written as follows:

$$\tilde{f}_{res} = \frac{e}{M}\hat{\Pi} f_0 \int_0^\infty dt'\, \mathbf{v}_D(\tau) \cdot \nabla\tilde{\Phi}(\tau)|_{t-t'}. \tag{7}$$

where

$$\frac{1}{M}\hat{\Pi} = \frac{\partial}{\partial \mathcal{E}} + \frac{cn}{e\omega}\frac{\partial}{\partial J}, \tag{8}$$

$f_0 = f_0(\mathcal{E}, \mu_p, J)$ with $J = \psi_p - v_\| B_3/\omega_B$ the canonical angular momentum, ψ_p is the poloidal magnetic flux, B_3 is a covariant component of $\mathbf{B}$. When obtaining Eq. (7) we used the fact (which can be easily seen) that J of the well circulating particles is approximately conserved even in the absence of the axial symmetry. Let us expand $\mathbf{v}_D$ into a Fourier series,

$$\mathbf{v}_D = \sum_{p,s} \mathbf{u}_{ps}(r) e^{ip\vartheta + is\varphi}, \tag{9}$$

where p and s are integers. Substituting this expression into Eq. (7), and then calculating $\mathbf{j}^{\alpha}_{mn}$, we obtain from Eq. (6):

$$\gamma_\alpha = \frac{2\pi^2 e^2}{c^2 M} \frac{\sum_{m,n} \int dr\, r \int d^3v\, \hat{\Pi} f_0 \sum_{p,s} \left|\mathbf{u}_{ps}\cdot\nabla\Phi_{m,n}\right|^2 \delta(\Omega^{ps}_{mn})}{\sum_{m,n} \int dr\, r v_A^{-2} \left[\left|\Phi'_{m,n}\right|^2 + (m^2/r^2)\left|\Phi_{m,n}\right|^2\right]}, \tag{10}$$

where $\Omega^{ps}_{mn} = \omega - (p+m)\omega_\vartheta - (s-n)\omega_\varphi$.

The obtained expressions for γ are applicable to any stellarators. We use them for the Helias configarations, in which case

$$\begin{aligned}\left|\mathbf{u}_{ps}\cdot\nabla\Phi_{m,n}\right|^2 = \sum_{\mu\cdot\nu\neq 0} \frac{\left(|v|^2 + v_\|^2\right)^2}{16 r^2 \omega_B^2} \\ \left|p\epsilon^{(\mu\nu)}\Phi'_{m,n} - m\epsilon^{(\mu\nu)\prime}\Phi_{m,n}\right|^2 \left(\delta_{p,\mu}\delta_{s,-\nu N} + \delta_{-p,\mu}\delta_{-s,-\nu N}\right) \\ + \frac{m^2 v_\perp^4 \epsilon_0'^2}{4r^2\omega_B^2}\left|\Phi_{m,n}\right|^2 \delta_{p,0}\delta_{s,0}.\end{aligned} \tag{11}$$

It follows from Eq. (11) that the summation over p, s in Eq. (10) is essentially the summation over μ, ν, which labels Fourier harmonics of the magnetic field strength. This implies that the growth rate contains the resonance condition given by Eq. (5), which was obtained from the energy consideration.

In order to demonstrate new physics coming from the violation of the axial simmetry we restrict the analysis below with the case of the well-localized modes with the frequency $\omega = |k_\||v_A$. Then Eq. (10) yields:

$$\gamma_\alpha = \frac{\pi^2 M_\alpha v_{A*}^2}{8\omega \bar{B}^2 r^2} \sum_{\nu,j} |\epsilon^{(1\nu)}|^2 |v_\|^r| \int d^3v\, \delta(v_\| - v_\|^r)(|v|^2 + v_\|^2)^2 \hat{\Pi} f_0, \tag{12}$$

where $j = \pm 1$, $\nu = 0, 1$, $v_\|^r$ is the longitudinal resonance velocity, $v_{A*} = v_A(r_*)$, r_* is the mode location radius. We observe that two Fourier harmonics of the magnetic field, $\epsilon^{(10)}$ and $\epsilon^{(11)}$, contribute to γ_α. The resonance velocity equals to

$$v_\|^r = v_{A*}\left(1 + 2j\frac{\iota_* - \nu N}{\mu_0\iota_* - \nu_0 N}\right)^{-1} \mathrm{Sgn} k_\|, \quad \text{and} \quad v_\|^r = v_{A*}\left(1 \pm \frac{(\mu\iota_l - N\nu)r_l}{2m\hat{s}_l L_{\rho_l}\iota_l}\right)^{-1}, \tag{13}$$

for the gap modes and GAEs, respectively. Here $\iota_* = (2n + \nu_0 N)/(2m + \mu_0)$, μ_0 and ν_0 are the mode coupling numbers, $\hat{s}$ is the magnetic shear, $L_\rho = -\partial\rho/\partial r$ with ρ the plasma mass density, the subscript l denotes the magnitudes at the radius around which GAEs are localized. The resonance velocities for the various gap modes in a Helias reactor [6] are shown in Fig. 1.

Let us analyze the destabilization of Alfvén eigenmodes by the energetic ions with the distribution function formed by Coulomb collisions,

$$f \propto \frac{\eta(v_\alpha - v)}{v^3 + v_c^3}, \tag{14}$$

where $\eta(x) = \int_{-\infty}^{x} dy\delta(y)$, $v_c = \sqrt{2\mathcal{E}_c/M_i}$ with $\mathcal{E}_c \sim (M_i/M_e)^{1/3}T$, T the plasma temperature. One can see that the second term in $\hat{\Pi}$ of Eq. (12) associated with the spatial inhomogeneity overrides the first term when $\omega_d > \omega_d^{cr} \equiv 1.5\omega$ [$\omega_d = -cn\mathcal{E}_\alpha/(e_\alpha B\iota r L_\alpha)$, $L_\alpha^{-1} = d\ln n_\alpha/dr$, n_α is the density of the energetic ions], which is essentially a necessary condition of the instability. In order to calculate the magnitude of the istability growth rate we have to specify the parameters of the system. We consider a Helias reactor with parameters given in Ref. [6]. The results of calculations of instabilities of various gap modes and GAEs are summarized in Tables I–II. We conclude from Tables I–II that (i) the helicity-induced resonance strongly enhances Alfvén instabilities except for TAE and HAE_{22} ones; but its effect on $(\omega_d/\omega)_{cr}$ is weak. (ii) MAE and TAE instabilities have largest growth rates; (iii) GAE instability can have a very large growth rate.

4. SUMMARY AND CONCLUSIONS

We have shown for the first time that because of the presence of several main Fourier harmonics of the magnetic field strength, several sideband resonances may essentially contribute to the instability growth rate. In particular, it is found that the resonance $\omega = [k_\parallel \pm (\iota - N)/R_0]v_\parallel$ associated with the helicity-induced drift motion plays a dominant role in the destabilization of the localized MAE and HAE_{21} modes by α-particles in a Helias reactor. On the other hand, new resonances have a minor influence on the well-localized TAEs in a Helias reactor because the corresponding resonance curves are situated well below the curve associated with the toroidicity. Note that in the optimized stellarators the dominant Fourier harmonic of the magnetic field, $\epsilon^{(01)}$, weakly affects the stability because only its derivative, which is small, contributes to γ_α. Non-conventional resonances play an important role for both gap modes and GAE modes, which is seen from Tables I–II.

An important practical consequence of the found new resonances is that the tangential neutral beam injection (NBI) can destabilize TAE modes not only when $v_b \geq v_A$ or $v_b \geq v_A/3$, where v_b is the velocity of injected particles, (which is the case in tokamaks) but also for lower beam velocities (energies), e.g., for $v_b \geq v_A|2N/\iota - 1)|^{-1}$ or $v_b \geq v_A|2N/\iota - 3)|^{-1}$ due to the helicity-induced resonance. On the other hand, other instabilities can be caused by particles with $v_b > v_A$ or $v_b < v_A/3$.

In conclusion, we note that although our analysis is relevant mainly to optimized stellarators, the drawn conclusion on the role of non-conventional resonances is valid also for other systems, e.g., for the partly optimized stellarator Wendelstein 7-AS (W7-AS). Furthermore, one can expect that in W7-AS the role of the non-conventional resonances is even larger because the Fourier spectrum of the magnetic field strength contains more harmonics.

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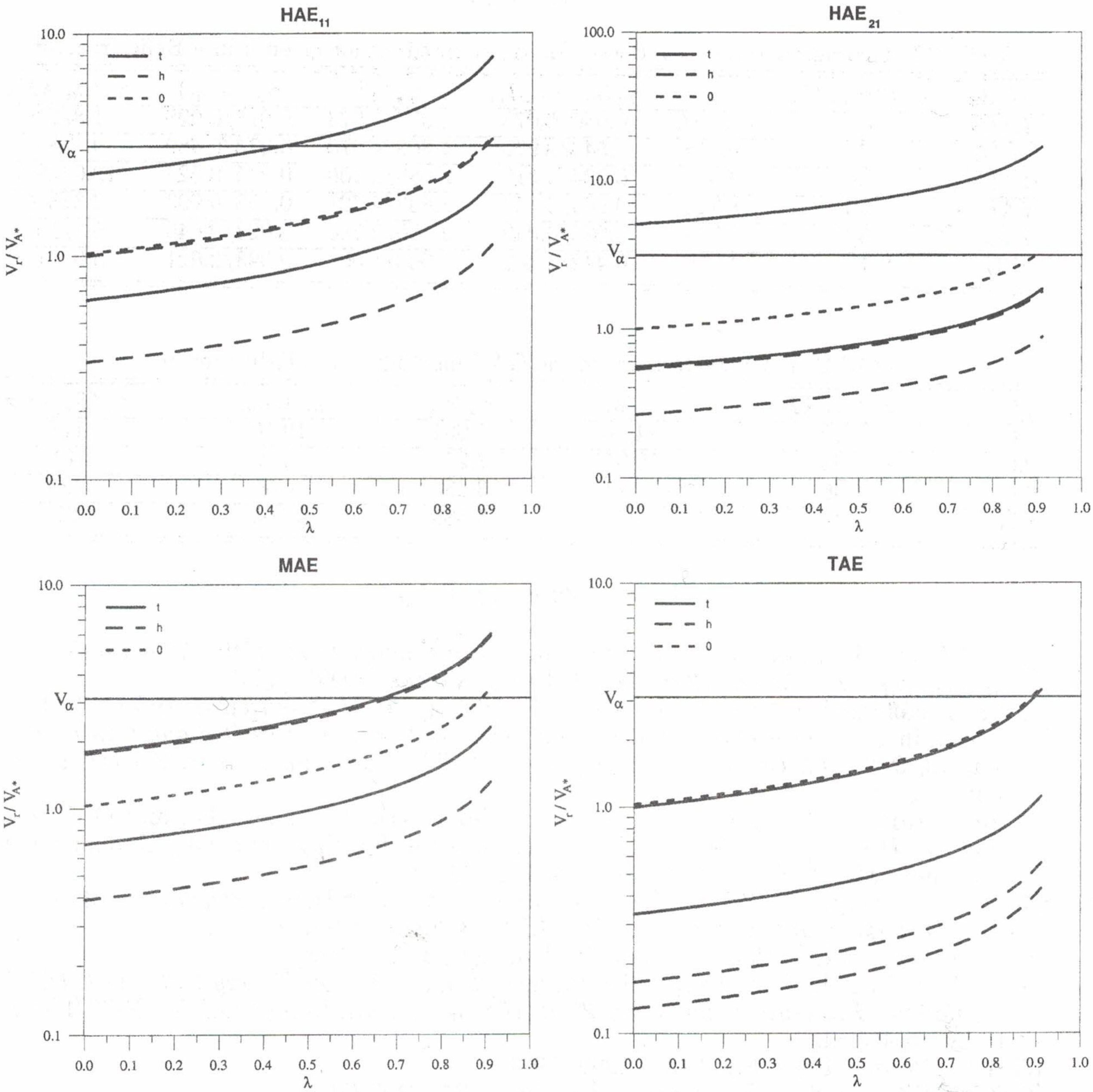

FIG. 1. Resonance velocity $v_r = |v_{\parallel}^r|/(1-\lambda)^{1/2}$ versus the pitch-angle parameter $\lambda = \mu_p \bar{B}/\mathcal{E}$ in a 4-period Helias reactor [6] for several kinds of Alfvén eigenmodes.

TABLES

TABLE I. Characteristics of α-induced Alfvén instabilities for given n in a Helias reactor

$Mode$	$\omega/(2\pi), kHz$	n	$\gamma_\alpha R_0/(v_{A*}\beta_\alpha)$	$\gamma_\alpha/\gamma_\alpha^0$	$\gamma_\alpha/(\omega\beta_\alpha)$	$\gamma_\alpha/\beta_\alpha, kHz$
HAE_{22}	112	-11/-16	0.050/0.078	1.074/1.073	0.016/0.025	1.8/2.8
MAE	72	-9/-15	0.546/0.976	4.368/4.370	0.273/0.489	19.7/35.2
HAE_{11}	56	-6/-13	0.334/0.819	2.089/2.100	0.215/0.527	12.0/29.5
HAE_{21}	40	-9/-14	0.064/0.103	4.833/4.721	0.058/0.093	2.3/3.7
EAE	32	-7/-13	0.020/0.040	2.150/2.180	0.023/0.046	0.7/1.5
TAE	16	-4/-10	0.442/1.166	1.000/1.000	0.993/2.621	15.9/41.9

TABLE II. Characteristics of the GAE instability in a Helias reactor

r_l/a	$\omega/(2\pi), kHz$	n/m	$\gamma_\alpha/\gamma_\alpha^0$	$\gamma_\alpha/(\omega\beta_\alpha)$	$\gamma_\alpha/\beta_\alpha, kHz$
0.6	12	-4/-4	1.001	10.10	121
0.6	31	-10/-10	7.277	1.34	42
0.6	46	-15/-15	3.083	4.78	220
0.44	198	-10/-5	5.585	0.105	21

References

[1] A. Weller, M. Anton, R. Brakel et al., in *Fusion Energy 1998, 17th IAEA Conference Proceedings, Yokohama, 1998, Vol. 1* (IAEA, Vienna, 1999), p.295.

[2] Ya. I. Kolesnichenko, V. V. Lutsenko, H. Wobig, Yu. V. Yakovenko, and O. P. Fesenyuk, in *Controlled Fusion and Plasma Physics, Abstracts of Invited and Contributed Papers, 27th EPS Conference, Budapest, 2000* (The European Physical Society, Petit-Lancy, 2000), p. 88.

[3] Ya. I. Kolesnichenko, V. V. Lutsenko, H. Wobig, Yu. V. Yakovenko, and O. P. Fesenyuk, *IPP Report III/261* (Max-Planck-Institut für Plasmaphysik, Garching bei München, Mai 2000); Phys. Plasmas **8**, 491 (2001).

[4] C. Nührenberg, in *ISSP-19 "Piero Caldirola", Theory of Fusion Plasmas, J. W. Connor, O. Sauter, and E. Sindoni (Eds.)* (SIF, Bologna, 2000), p. 313.

[5] F. Wagner, Transactions of Fusion Technology **33**, 67 (1998).

[6] C. D. Beidler, E. Harmeyer, F. Herrnegger *et al.*, in *Fusion Energy 2000, 18th IAEA Conference Proceedings, Sorrento, 2000* (IAEA, Vienna), Report IAEA-CN-77/FT/4, to be published.

[7] A. H. Boozer, Phys. Fluids **24**, 1999 (1981).

[8] R. B. White and M. S. Chance, Phys. Fluids **27**, 2455 (1984).

АЛЬФВЕНІВСЬКІ НЕСТІЙКОСТІ, ЩО ЗБУДЖУЮТЬСЯ ПРОЛІТНИМИ ЕНЕРГІЙНИМИ ІОНАМИ В ОПТИМІЗОВАНИХ СТЕЛАРАТОРАХ

Я. І. Колесниченко, В. В. Луценко, Г. Вобіґ, Ю. В. Яковенко

Досліджено нестійкості, що можуть збуджуватися пролітними енергійними іонами в оптимізованих стелараторах серії Вендельштайн (Геліас). Вперше показано, що декілька сайдбенд-резонансів, а не лише один, пов'язаний з тороїдальністю (й відомий з теорії токамаків) можуть давати істотний внесок у інкремент нестійкості. Нові резонанси підсилюють нестійкості й, більше того, можуть призводити до нестійкості в тих випадках, коли раніше відомий резонанс не є ефективним. Розглянуто дестабілізацію тороїдальних альфвенівських власних мод, а також інших власних мод, що існують завдяки специфічній формі магнітних поверхонь та Фур'є-гармонікам магнітного поля Геліас-конфігурацій.