APPROACHES TO DESCRIPTION OF THE TOROIDAL PLASMA EQUILIBRIUM WITH ISLANDS

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Two approaches to descriptions of islands with small number of periods in toroidal systems are discussed. One of these uses the helicoidal poloidal magnetic flux and tokamak-like representation of the magnetic field. Another approach suggests the generalization of the parameterisation of the magnetic surfaces used for example, in VMEC equilibrium code, on the configuration with the magnetic islands.

1. Introduction

In contrast to the ideally axe-symmetric tokamak, the stellarator systems have no exactly nested system of the toroidal magnetic surfaces. The island structure could appear in stellarator with plasma even in stellarators with sufficiently good vacuum system of nested surfaces. Moreover, the island structure can appear even in the ideal tokamak systems due to the tearing instabilities.

In spite of existence of the known codes PIES [1] and HINT [2], which can describe plasma equilibrium in systems with magnetic islands, the development of the more simple approaches for such systems is desirable. Here two different approaches for describing the plasma equilibrium with islands are discussed.

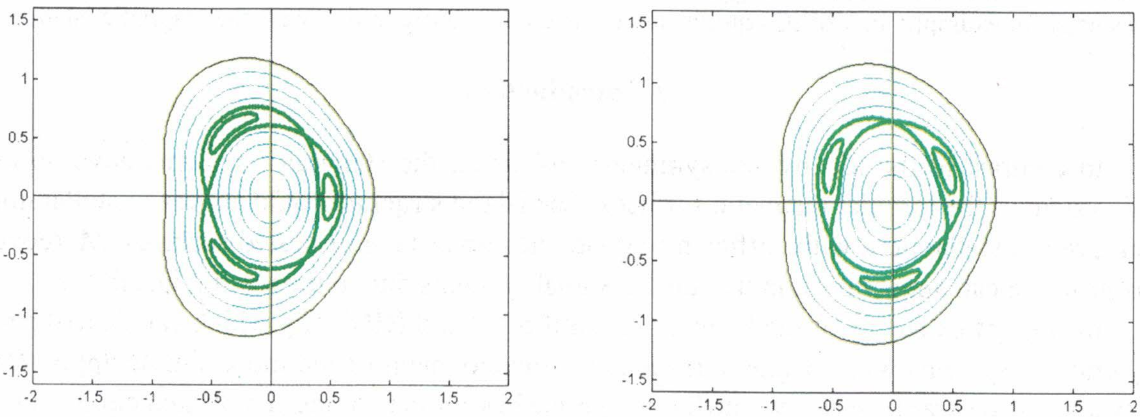
The difficulties with the description of equilibrium with islands are connected, mainly, with presence of regions with different topology of the magnetic surfaces, so that it is impossible to introduce unique topology of the poloidal coordinate and corresponding unique definition of the poloidal flux.

The main idea of the first approach is to subdivide the whole volume occupied by plasma with approximately nested toroidal surfaces on the tubular regions of two types. To the first type we refer the rather thick tubes whose magnetic surfaces could be considered as irrational ones. Between these basic tubes we stand out the thin tubes each of whose has the rational magnetic surface $\mu = n/m$ with small m,n. For brevity we shall call the first type of these regions by *irrational* tubes, while the tubes containing the rational magnetic surface as *rational* ones. The plasma equilibrium in the irrational tubes could be considered by using the usual flux coordinates (the magnetic surface label a, poloidal and toroidal angle variables, θ and ζ). The rational tubes are topologically unstable and they really will have the island structure. It is suggested that the topology of the islands in every considered irrational tube is known. In this case it is possible to introduce the corresponding helicoidal poloidal flux and to derive the tokamak-like equation for this flux. The solution should be joined with that in basic toroidal tubular regions.

In another approach it is suggested to expand the parameterisation of the magnetic surfaces in such a way that it could describe the magnetic configuration with islands.

2. Tokamak-like description of the 3D configurations

It is well known that tokamak with the magnetic islands can be described in the same manner as the configurations with nested magnetic surfaces, if the axial symmetry is not disturbed. Dublet-type system is the example of such configuration. One can note two character feature of the magnetic field representation for this case. Firstly, the poloidal angle variable is not used in $\boldsymbol{B}$ representation, and, secondly, the poloidal flux is measured through the surface, which edge has the same topology as the magnetic axes and the x-point of the separatrics. Thus, to try to use the analogous representation for the description of 3D configuration with one chain of the helical magnetic islands it is needed to use corresponding definition of the poloidal flux, when the edge of the surface through which the poloidal flux is measured should go in the same manner as the axes of the magnetic islands. It worth to remind that just the same definition is used in analytical approaches in order to have finite solution of the equation $\mathbf{B}\nabla\Psi = 0$ by method of perturbations, $\mathbf{B}_1\nabla\Psi_0 = -\mathbf{B}_0\nabla\Psi_1$. In this case the function Ψ_0 has extreme on the resonant surface, so that small perturbations can create the islands $\Psi = const$ in this region (for illustration, see Figure).



Two cross-sections of the magnetic surfaces for the model configuration with islands.

Let's suppose that just described above definition of the poloidal flux is used and try to formulate the equilibrium problem using the tokamak-like representation of the magnetic field.

In systems with axial or helical symmetry a magnetic field is expressed only trough the poloidal magnetic flux Ψ and the basic vector **b.** In tokamak **b** is the gradient of toroidal angle φ, a label of partition across the magnetic tube,

$$2\pi\mathbf{B} = F(\Psi)\nabla\varphi + \nabla\Psi\times\nabla\varphi. \tag{1}$$

As was shown in [3], in general case of 3D configurations it is possible to have analogous representation for the magnetic field, but with two different vectors $\mathbf{b}_F$ and $\mathbf{b}_\Psi$:

$$2\pi\mathbf{B} = [\nabla\Psi\times\mathbf{b}_F] + F\mathbf{b}_\Psi\,. \tag{2}$$

In the Ref. [4] two scalar functions λ_1 and λ_2 were introduced instead of two vectors,

$$2\pi\mathbf{B} = F(\Psi)\nabla_t\lambda_1 + \nabla\Psi\times\nabla\lambda_2, \tag{3}$$

where

$$\nabla_t\lambda = \nabla\lambda - \chi\nabla\Psi, \qquad \chi = \frac{\nabla\lambda\cdot\nabla\Psi}{\left|\nabla\Psi^2\right|}. \tag{4}$$

Vector **B** in this representation satisfies to the condition $\mathbf{B}\cdot\nabla\Psi = 0$, while the condition $\nabla\cdot\mathbf{B} = 0$ leads to an equation

$$\mathrm{div}\nabla_t\lambda_1 = 0\,. \tag{5}$$

For the current density **j** we have:

$$2\pi\mu_0\mathbf{j} = F'(\Psi)[\nabla\Psi\nabla\lambda_1] + F[\nabla\Psi\nabla\chi] + \mathrm{rot}[\nabla\Psi\nabla\lambda_2]. \tag{6}$$

Thus, condition $\mu_0\mathbf{j}\cdot\nabla\psi = \nabla\times\mathbf{B}\cdot\nabla\psi = 0$ esquires the form:

$$\mathrm{div}[\nabla\Psi[\nabla\lambda_2\nabla\Psi]] = 0. \tag{7}$$

It can be represented in another form, too:

$$\mathrm{div}|\nabla\Psi|^2\nabla_t\lambda_2 = 0. \tag{8}$$

The equilibrium equation,

$$\nabla p = [\mathbf{jB}], \tag{9}$$

can be now rewritten in the form of the equation for the poloidal magnetic flux Ψ :

$$\mathrm{div}[\nabla\lambda_2[\nabla\Psi\nabla\lambda_2]] = -4\pi^2\mu_0 p'(\Psi) - FF'(\Psi)(\nabla_t\lambda_1)^2 - F'(\Psi)(\nabla\Psi[\nabla\lambda_1\nabla\lambda_2]) +$$

$$F^2(\nabla_t\lambda_1\cdot\nabla\chi) + F(\nabla\Psi\cdot[\nabla\chi\times\nabla\lambda_2]) - F\left(\frac{[\nabla\Psi\nabla\lambda_1]}{|\nabla\Psi|^2}\cdot\mathrm{rot}[\nabla\Psi\nabla\lambda_2]\right). \tag{10}$$

For vacuum configuration it acquires the form:

$$\mathrm{div}[\nabla\lambda_2[\nabla\Psi\nabla\lambda_2]] = F^2(\nabla_t\lambda_1\cdot\nabla\chi) + F(\nabla\Psi\cdot[\nabla\chi\times\nabla\lambda_2]) - F\left(\frac{[\nabla\Psi\nabla\lambda]}{|\nabla\Psi|^2}\cdot\mathrm{rot}[\nabla\Psi\nabla\lambda_2]\right). \tag{11}$$

Thus, equations (5), (8) and (10) describe the plasma equilibrium in 3D configuration with single chain of magnetic islands. In general case the configuration can possess few chains of islands. For the description of magnetic tube with single chain of islands one should now the topology of these islands. In the irrational magnetic tube it is possible to change the definition of the poloidal flux in such a way, that it becomes suitable for the description of the next chain of islands. The mathematical correction of this formulation of the equilibrium problem should be checked.

3. Parameterisation of the magnetic surfaces with islands

In this section another approach for the description of plasma equilibrium in 3D configuration with islands is suggested. In the well known equilibrium code VMEC [5] the parameterisation of the magnetic surfaces is used to describe the system with nested magnetic surfaces. Here the attempt is made to expand such approach to the system with the magnetic islands.

One of the possible ways of the nested magnetic surfaces parameterisation is the following one:

$$r = r_0(\zeta) + \rho(a,\theta,\zeta)\cos\theta \tag{12}$$

$$z = z_0(\zeta) + \rho(a,\theta,\zeta)\sin\theta \tag{13}$$

Here r, z, ζ are cylindrical coordinates, the function $\rho(a,\theta,\zeta)$ is unknown one and should be defined from equilibrium equations. For simplicity here the case of so called stellarator symmetry is considered. In the case of nested magnetic surfaces it can be parameterized as:

$$\rho = a + \Delta(a,\theta,\zeta), \tag{14}$$

with

$$\Delta(a,\theta,\zeta) = \sum_{mn} \delta_{mn}(a)\cos(m\theta - n\zeta) \tag{15}$$

The functions $\delta_{m,n}$ can be defined from equilibrium equations.

Let's suggest now that we have some solution with such parameterisation (i.e. the solution with nested magnetic surfaces). Let $a = a_s$ is the resonant magnetic surface with $\mu(a_s) = N/M$. In principle, the chain of islands with corresponding structure can exists near this resonant surface. To describe these islands, let's expand the parameterisation by the following way:

$$\rho(a,\theta,\zeta) = a_s + \Delta_s \pm \sqrt{(a + \Delta - a_s - \Delta_s)^2 + \gamma_{MN}(a)[1 - \cos(M\omega - N\zeta)]}, \tag{16}$$

where $\Delta_s = \Delta(a_s\theta,\zeta)$, $\omega = \theta + \sum_{m,n} \alpha_{m,n} \sin(m\theta - n\zeta)$, with ω being some poloidal angle variable of flux coordinates. Here the sign (+) corresponds to $a + \Delta > a_s + \Delta_s$, and the sign (-) corresponds to $a + \Delta < a_s + \Delta_s$.

The parameterisation (16) describe by unique radial variable a the whole plasma column except the internal part of the islands. The parameters $\delta_{m,n}, \gamma_{m,n}$ and $\alpha_{m,n}$ are defined by equilibrium equations and by the boundary conditions. To solve the problem completely, one needs, in addition, the solution inside the islands. Such a solution can be received by using the parameterisation with separate radial variable. In principle, the analogous approach can be suggested for the system with few different chains of the magnetic islands.

Figure illustrates the possibility to describe the configuration with islands by suggested parameterisation. Here the functions $\delta_{m,n}, \gamma_{m,n}$ and $\alpha_{m,n}$ are modeling ones, which are not the exact solutions of the equilibrium equations.

Conclusions

Two approaches to descriptions of magnetic islands in toroidal systems were presented. First of them uses the helicoidal poloidal magnetic flux and tokamak-like representation of the magnetic field. The corresponding system of equilibrium equations was represented. Second approach generalises the usual parameterisation of the nested magnetic surfaces on the configurations with the magnetic islands. Both of these approaches can be used for modification of modern 3D equilibrium codes as well as for designing of new numerical codes.

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REFERENCES

1. *Reiman A.H., Greenside H.* // Comput. Phys. Commun. – 1986. – Vol. 43. – P. 157.
2. *Hayashi T., Sato T., Takei A.* // Phys. Fluids. – 1990. – Vol. B 2. – P. 329.
3. *Degtyarev L.M., Drozdov V.V., Mikhailov M.I. et. al.* // Sov. J. Plasma Phys. – 1985. – Vol. 11. – P. 22.
4. *Shafranov V.D.* Plasma Physics and Controlled Fusion // 28th EPS Conference on Controlled Fusion and Plasma Physics, Portugal, 2001, to be published.
5. *Hirshman S.P., Lee D.K.* // Comput. Phys. Commun. – 1986. – Vol. 39. – P. 161.

ПІДХОДИ ДО ОПИСУ ТОРОЇДАЛЬНОЇ ПЛАЗМОВОЇ РІВНОВАГИ З ОСТРОВАМИ

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Обговорюються два підходи до опису островів із малим числом періодів у тороїдальних системах. Один з них використовує гвинтовий полоїдальний магнітний потік та токамакоподібне представлення магнітного поля. Інший підхід пропонує узагальнення параметризації магнітних поверхонь, яка використовується, наприклад, у коді рівноваги VMEC, на конфігурації з магнітними островами.