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STATIONARY SELF-FOCUSING OF WHISTLER WAVES IN THE IONOSPHERE

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Possibility of the formation of two-dimensional coherent structures - whistler waveguides - during the active ionospheric experiments is investigated both analytically and numerically. It was shown that the lowest threshold for formation of such waveguides corresponds to wave frequency in the vicinity of the half electron cyclotron frequency. Thermal self-interaction (plasma displacement from the high pressure region and wave beam trapping into the formed channel) plays essential role for formation of the waveguides. To describe appearance and evolution of the stationary channels, the generalization of 2D nonlinear Schrödinger equation was proposed. Stability of two-dimensional structures was proved analytically and numerically.

1. Introduction

Plane whistler waves which propagate along the magnetic field $\vec{B}=(0,0,B_z)$ are fully electromagnetic right – hand polarized waves having $\vec{k}_\perp = 0,\ E_z = 0$. However, when the direction of propagation deviates from OZ axis, or when the wave beam is localized in transverse plane, these properties disappear. Together with nonzero $\vec{k}_\perp$ whistler wave obtains elliptic polarization (which can be assumed as a presence of left – hand polarized part of wave field) and the electrostatic wave component ($E_z \neq 0$). Here we consider propagation of beams of whistler waves with frequencies near the half of electron cyclotron frequency ($\omega \approx \omega_{Be}/2$) along the magnetic field in the ionosphere. The main nonlinear equation governing this propagation is a Generalized Nonlinear Schrödinger Equation (GNSE)

$$i\frac{\partial \psi}{\partial \varsigma} + D\Delta_\perp \psi + P\Delta_\perp^2 \psi + B\psi\,|\psi|^2 + K\psi\,|\psi|^4 = 0, \qquad (1)$$

where $\Delta_\perp = \partial^2/\partial\xi^2 + \partial^2/\partial\eta^2$, $\psi = E_z/F_0$, $F_0 = \sqrt{12\pi\, T n_0 \delta}$, $\delta \approx 2m/M$, $\varsigma = z\omega_0/c$, $\xi = x\omega_0/c$, $\eta = y\omega_0/c$, $\omega_0 \approx \omega_{Be}/2$, and the coefficients of equation are: $D = (1/(4u^2)-1)/(2\sqrt{v})$, $P = 1/(8v\sqrt{v})$, $B = -v^{3/2}$, $K = v^{5/2}\nu_e^2/\omega_0^2$, $u = \omega/\omega_{Be}$, $v = \omega_{Pe}^2/\omega_0^2$. Here ν_e is the frequency of electron collisions, n_0 is the density of electrons, T is the electron temperature. Equation (1) generalizes Eq. (1) of [1] into the case of 2D geometry and takes into account a nonlinearity saturation effects. The main nonlinear effect in the ionosphere is the plasma extrusion from the HF field region due to heating and pressure increasing. Near the point $\omega \approx \omega_{Be}/2$ the thresholds of modulation instabilities for whistler waves decrease dramatically, and one must account for the next terms in the nonlinearity expansion which are of the same order as the linear terms. It leads to an appearance of cubic – quintic saturable nonlinearity in the GNSE (1). In this paper we consider the case $u \le 1/2$, which means $\omega \le \omega_{Be}/2$ and the signs of GNSE coefficients are the following $D>0$, $P>0$, $B<0$, $K>0$. We are interested to find conditions for whistler waves propagation in stationary waveguides or channels localized in the plane perpendicular to direction of propagation – OZ axis, which are formed due to nonlinear self-interaction. Profiles of wave intensity and density perturbation in perpendicular plane in such waveguides do not depend on z.

2. General properties of nonlinear whistler waveguides

Equation (1) has integrals:
number of quanta

$$N = \iint |\psi|^2\, d^2\vec{r}, \qquad (2)$$

momentum

$$\vec{I} = -\frac{i}{2}\iint\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)d^2\vec{r}, \qquad (3)$$

angular momentum

$$\vec{M} = -\frac{i}{2}\iint\left(\psi^*[\vec{r}\times\vec{\nabla}\psi] - \psi[\vec{r}\times\vec{\nabla}\psi^*]\right)d^2\vec{r}, \qquad (4)$$

and Hamiltonian

$$H = \iint\left(D|\vec{\nabla}\psi|^2 - P|\Delta_\perp\psi|^2 - \frac{B}{2}|\psi|^4 - \frac{K}{3}|\psi|^6\right)d^2\vec{r}. \qquad (5)$$

Stationary (along OZ axis) waveguides in the framework of (1) has a form $\psi(\vec{r})\exp(i\lambda\varsigma)$, where complex function $\psi(\vec{r})$ obeys partial differential equation

$$-\lambda\psi + D\Delta_\perp\psi + P\Delta_\perp^2\psi + B\psi|\psi|^2 + K\psi|\psi|^4 = 0. \qquad (6)$$

Multiplying (6) by ψ^* and integrating, an integral relation

$$\lambda N = P\iint|\Delta_\perp\psi|^2\, d^2\vec{r} - D\iint|\nabla_\perp\psi|^2\, d^2\vec{r} + B\iint|\psi|^4\, d^2\vec{r} + K\iint|\psi|^6\, d^2\vec{r} \qquad (7)$$

is obtained. Multiplying (6) by $r^2\, d\psi^*/dr$, integrating and adding the complex conjugate, another integral identity is found

$$\lambda N = -P\iint|\Delta_\perp\psi|^2\, d^2\vec{r} + \frac{B}{2}\iint|\psi|^4\, d^2\vec{r} + \frac{K}{3}\iint|\psi|^6\, d^2\vec{r}. \qquad (8)$$

The restriction on parameter λ: $\lambda < -D^2/4P$ can be found from the linear asymptotical behavior of any localized solution. After excluding λ from (7) – (8) one finds simplified expression for a Hamiltonian of solitary solutions

$$H = P\iint D|\Delta_\perp\psi|^2\, d^2\vec{r} + \frac{K}{3}\iint|\psi|^6 d^2\vec{r} = \frac{D}{2}\iint|\vec{\nabla}\psi|^2 d^2\vec{r} - \frac{B}{4}\iint|\psi|^4 d^2\vec{r}. \qquad (9)$$

From (9) it follows immediately that for $D > 0$, $P > 0$, $B < 0$, $K > 0$ which is the case under consideration Hamiltonian functional is always positive. Using the integral inequalities

$$\int|\nabla\psi|^2\, d^2\vec{r} < N^{1/2}\left(\int|\Delta\psi|^2\, d^2\vec{r}\right)^{1/2}, \qquad (10)$$

$$\int|\psi|^4\, d^2\vec{r} < N^{1/2}\left(\int|\psi|^6\, d^2\vec{r}\right)^{1/2}, \qquad (11)$$

we found the following estimation for Hamiltonian of solitary states

$$0 < H < DN^{1/2}\left(\int|\Delta\psi|^2\, d^2\vec{r}\right)^{1/2} - P\int|\Delta\psi|^2\, d^2\vec{r} + \frac{|B|}{2}N^{1/2}\left(\int|\psi|^6\, d^2\vec{r}\right)^{1/2} - $$

$$-\frac{K}{3}\int|\psi|^6\, d^2\vec{r} < \frac{N}{4}\left[\frac{D^2}{P} + \frac{3}{4}\frac{B^2}{K}\right]. \qquad (12)$$

It indicates that for fixed value of number of quanta N, a Hamiltonian functional is bounded from below and above. For every N it guarantees the existence of at least one stable solitary solution which corresponds to Hamiltonian's extremum.

Using integral inequalities (10) and (11) and the identity (7) it is easy to show that the waveguide parameter λ is bounded from below and to estimate a range of accessible λ – values:

$$-\frac{D^2}{4P}-\frac{B^2}{4K}<\lambda<-\frac{D^2}{4P}. \qquad (13)$$

From the virial relation for waveguide's effective width

$$r_{eff}^2 = N^{-1}\int_0^\infty \int_0^{2\pi} r^2 |\psi|^2 \, r dr d\theta ,$$

namely, from

$$N\frac{d^2 r_{eff}^2}{d\varsigma^2} = 8\left[D^2\int_0^\infty \left|\frac{d\psi}{dr}\right|^2 rdr - 4DP\int_0^\infty |\Delta_r\psi|^2 rdr + 4P^2\int_0^\infty \left|\frac{d}{dr}\Delta_r\psi\right|^2 rdr\right] -$$
$$-\frac{BD}{2}\int_0^\infty |\psi|^4 rdr - \frac{KD}{3}\int_0^\infty |\psi|^6 rdr - 3BP\int_0^\infty r\left|\frac{d\psi}{dr}\right|^2\left(\frac{d}{dr}|\psi|^2\right) rdr - \qquad (14)$$
$$-PK\int_0^\infty r\left(\frac{d}{dr}|\psi|^2\right)\left[6|\psi|^2\left|\frac{d\psi}{dr}\right|^2 + \left(\frac{d}{dr}|\psi|^2\right)^2\right] rdr ,$$

one finds that: (i) When $P = K = 0$, GNSE (1) with $B\,D < 0$ has no localized solutions at all, any wave packet will spread out; in the case $P = K = 0$, $BD > 0$ virial relation gives

$$N\frac{d^2 r_{eff}^2}{d\varsigma^2} = 8DH$$

and predicts collapsing of any wave packet with $DH < 0$. (ii) The sum of all linear terms in the virial relation acts defocusing, thus any linear wave packet described by the linear GNSE will also always spreads out. (iii) In the case of GNSE (1) with $D > 0$, $B < 0$, $P > 0$, $K > 0$ virial relation includes focusing (proportional to $KD > 0$ and to $BP < 0$) and defocusing (proportional to $BD < 0$ and $PK > 0$) nonlinear terms, different linear and nonlinear terms come into play at different spatial scales and on different values of field amplitude, thus it is natural to expect that several different stationary nonlinear structures may coexist in the framework of GNSE (1).

3. Variational analysis

In the framework of the variational approach [2] the GNSE (1) is formulated as a variational problem for functional L

$$\delta\int L dz = 0 , \qquad (15)$$

where $L = \int l d^2\vec{r}$ and l is the Lagrangian density of (1). In the cylindrical coordinates (r,θ,z) l may be written as

$$l = \frac{i}{2}\left[\psi\frac{\partial\psi^*}{\partial z} - \psi^*\frac{\partial\psi}{\partial z}\right] + D\left[\left|\frac{\partial\psi}{\partial r}\right|^2 + \frac{1}{r^2}\left|\frac{\partial\psi}{\partial\theta}\right|^2\right] - P\left|\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}\right|^2 - \frac{B}{2}|\psi|^4 - \frac{K}{3}|\psi|^6 . \qquad (16)$$

Here we use trial function in the form:

$$\psi = \frac{\mu\sqrt{N}}{\sqrt{2I_H^{(l)}}} f_l(\mu r)\exp[i\gamma \ln\cosh \mu r + il\theta + i\varphi(z)], \tag{17}$$

where $I_H^{(l)} = \int_0^\infty \xi \ f_l^2(\xi)d\xi$, $\mu = \mu(z)$, $\gamma = \gamma(z)$, and phase $\varphi(z)$ are adjustable functions to be found from variational procedure, and real function $f_l(\xi)$ determines the envelope profile

$$f_l(\xi) = \frac{\xi^{|l|}}{\cosh(\xi)}. \tag{18}$$

This function describes soliton – like modes at $l = 0$ or vortex – like azimuthal modes at $l \neq 0$. The nonlinear phase dependence on r – coordinate for this function is fixed to be $\sim i\gamma \ln\cosh \mu r$, as it was in the case of 1D exact chirped soliton solution [3]. After performing of a Ritz optimization procedure, the waveguide parameters μ and $\beta = \gamma \cdot \mu$ are obtained from the canonical Hamiltonian set of equations

$$\begin{cases} \dfrac{d\beta}{d\eta} = \dfrac{\partial H}{\partial \mu} \\ \dfrac{d\mu}{d\eta} = -\dfrac{\partial H}{\partial \beta}, \end{cases} \tag{19}$$

where

$$\eta = \frac{1}{N}\int_0^z \mu^2(z')dz',$$

and Hamiltonian

$$H = \frac{N}{2\pi I_H^{(l)}}\left[D\left(I_{d1}^{(l)}\mu^2 + I_{d2}^{(l)}\beta^2\right) - \frac{BI_b^{(l)}}{2}N\mu^2 - \frac{KI_K^{(l)}}{3}N^2\mu^4 - P\left(I_{p1}^{(l)}\mu^4 + I_{p2}^{(l)}\mu^2\beta^2 + I_{p3}^{(l)}\beta^4\right)\right]. \tag{20}$$

Integrals $I_\sigma^{(l)}$ are defined by a choice of trial function

$$I_{d1}^{(l)} = \int_0^\infty \xi \left(\frac{\partial f_l(\xi)}{\partial \xi}\right)^2 d\xi; \quad I_{d2}^{(l)} = \int_0^\infty \xi f_l^2(\xi)\tanh^2(\xi)d\xi; \quad I_{d1}^{(0)} = I_{d2}^{(0)} \equiv I_d^{(0)};$$

$$I_b^{(l)} = \frac{1}{2\pi I_H^{(l)}}\int_0^\infty \xi \ f_l^4(\xi)d\xi; \quad I_k^{(l)} = \frac{1}{\left(2\pi I_H^{(l)}\right)^2}\int_0^\infty \xi f_l^6(\xi)d\xi;$$

$$I_{p1}^{(l)} = \int_0^\infty \xi f_l^2(\xi)\left(1 - \frac{2}{\cosh^2\xi} - \frac{(2l+1)\tanh\xi}{\xi}\right)^2 d\xi; \quad I_{p3}^{(l)} = \int_0^\infty \xi \tanh^4\xi \ f_l^2(\xi)d\xi;$$

$$I_{p2}^{(l)} = I_{p1}^{(l)} + I_{p3}^{(l)}.$$

Stationary points of the system (19) coincide with the Hamiltonian (5) extrema

$$\left.\frac{\partial H}{\partial \beta}\right|_{\mu=\mu_0,\,\beta=\beta_0} = 0; \quad \left.\frac{\partial H}{\partial \mu}\right|_{\mu=\mu_0,\,\beta=\beta_0} = 0. \tag{21}$$

These points determine the waveguide parameters μ_0 and β_0 for a given number of quanta *N*. Introducing parameters

$$N_1 = \frac{DI_{dp}^{(l)}(I_{p1}^{(l)} - I_{p3}^{(l)})}{|B| I_b^{(l)} I_{p3}^{(l)}};\; N_2^2 = \frac{3P(I_{p1}^{(l)} - I_{p3}^{(l)})^2}{4KI_k^{(l)} I_{p3}^{(l)}}; \tag{22}$$

$$\sigma = \frac{4(DI_{dp}^{(l)})^2 KI_k^{(l)}}{3(BI_b^{(l)})^2 PI_{p3}^{(l)}} = \left(\frac{N_1}{N_2}\right)^2;\; I_{dp}^{(l)} = \frac{I_{p1}^{(l)} - I_{p3}^{(l)}(2I_{d1}^{(l)}/I_{d2}^{(l)} - 1)}{I_{p1}^{(l)} - I_{p3}^{(l)}}; \tag{23}$$

one gets for the waveguides with $\beta = 0$ the following relation connecting μ^2 and N:

$$\mu_0^2 = \frac{DI_{d1}^{(l)} + |B| I_b^{(l)} N/2}{2(PI_{p1}^{(l)} + KI_k^{(l)} N^2/3)} < \mu_{0\max}^2; \tag{24}$$

where

$$\mu_{0\max}^2 = \frac{DI_{d1}^{(l)}}{4PI_{p1}^{(l)}}\left(1 + \sqrt{1 + \frac{3(BI_b^{(l)})^2 PI_{p3}^{(l)}}{4(DI_{d1}^{(l)})^2 KI_k^{(l)}}}\right). \tag{25}$$

For the stationary solutions with $\beta_0 \neq 0$, $l = 0$ another relation is obtained

$$\mu_c^2 = \frac{DI_{d1}^{(0)}}{P(I_{p1}^{(0)} - I_{p3}^{(0)})} \frac{1 - N/N_1}{1 - (N/N_2)^2}, \tag{26}$$

$$\mu_c^2(N=0) = \frac{DI_d^{(0)}}{P(I_{p1}^{(0)} - I_{p3}^{(0)})} > \frac{DI_d^{(0)}}{PI_{p2}^{(0)}} > \frac{DI_d^{(0)}}{2PI_{p1}^{(0)}} = \mu_0^2(N=0). \tag{27}$$

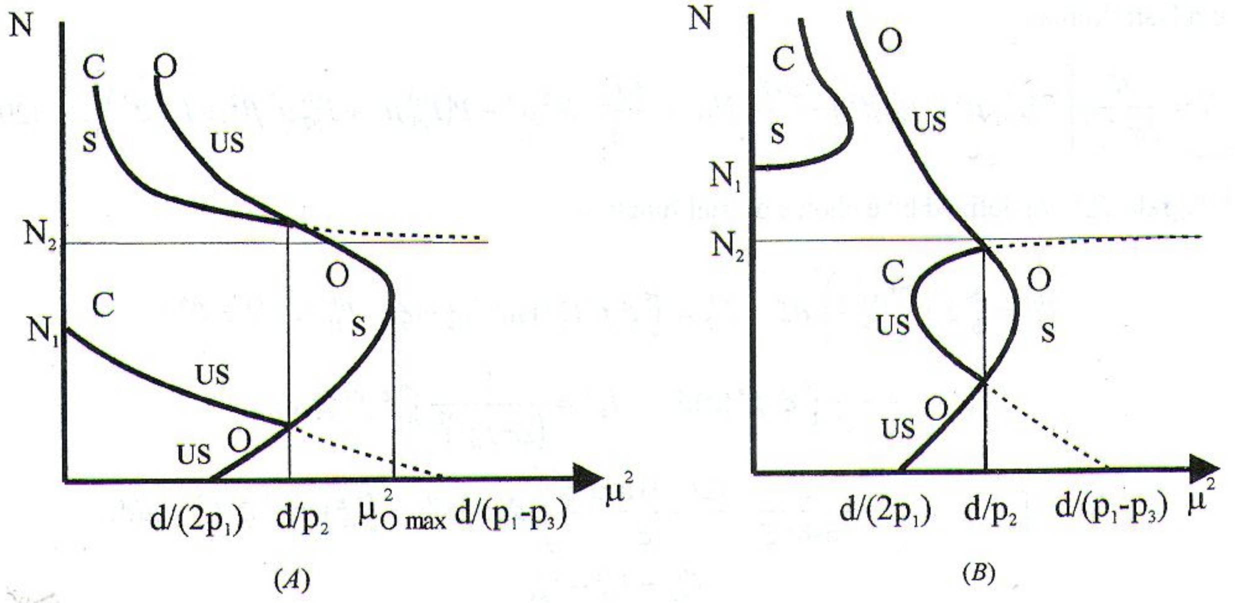

Fig. 1. N versus μ^2, $d = I_{d1}^{(l)}$, $p_\sigma = I_{p\sigma}^{(l)}$.
(A): for the case $N_1 < N_2$, ($\sigma < 1$); (B): for the case $N_1 > N_2$, ($\sigma > 1$).

A stationary solution is stable under the variation of two parameters – β and μ – if it gives maximum or minimum to Hamiltonian and it is unstable when corresponds to the Hamiltonian saddle point. The stability criterion in the framework of our variational approach is

$$h = \frac{\partial^2 H}{\partial \beta^2}\frac{\partial^2 H}{\partial \mu^2} - \left(\frac{\partial^2 H}{\partial \beta \partial \mu}\right)^2 \Bigg|_{\beta=\beta_0,\, \mu=\mu_0} > 0 . \tag{28}$$

The qualitative dependencies of N on μ_0^2 for waveguides with $\beta_0 = 0$ (marked "O") and for $\beta_0 \neq 0$, $l = 0$ (marked "C") are plotted in the Fig. 1. Dashed line indicates the region where $\beta^2 < 0$ and waveguides with $\beta_0 \neq 0$ can not exist. Stable and unstable branches are marked in the figure by "S" and "US" respectively. It is seen that variational approach with a trial function (17) predicts a bistability phenomenon for stationary waveguides propagation states – the coexistence of two waveguides with the same λ but with different numbers of quanta and amplitudes (see Fig. 1 (*A*) in the region (d/p_2, $\mu^2_{0\max}$). The bistability conclusion also will be confirmed by the direct numerical solution of stationary and nonstationary equations in the next section.

3. Numerical modeling

In order to perform a numerical modeling of GNSE waveguides, it is more convenient to use it in a fully 3D form in Cartesian coordinates (x, y, z). This equation has no peculiarity at the point $r = 0$. After the standart rescalings, the equation takes a form

$$i\frac{\partial \psi}{\partial \varsigma} + \Delta_\perp \psi + \Delta_\perp^2 \psi - \psi |\psi|^2 + K\psi |\psi|^4 = 0 , \tag{29}$$

where $K \to KD^2/PB^2$. Stationary (along OZ) solutions have a form $\psi(\xi,\eta,\varsigma) = \psi(\xi,\eta)\exp(i\lambda\varsigma)$ and obey partial differential equation

$$-\lambda\psi + \Delta_\perp \psi + \Delta_\perp^2 \psi - \psi |\psi|^2 + K\psi |\psi|^4 = 0 . \tag{30}$$

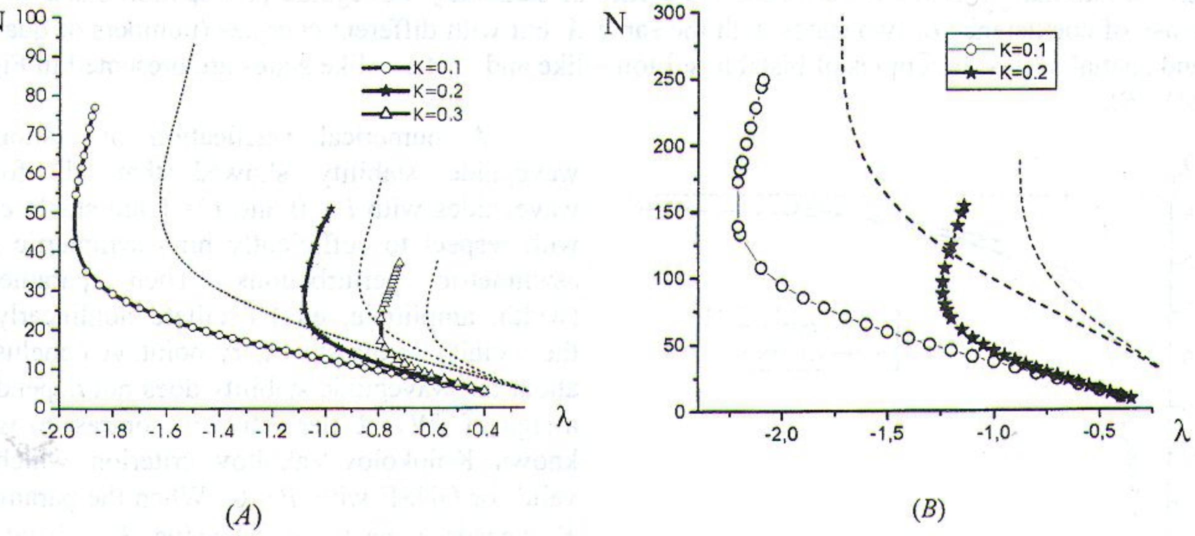

Fig. 2. An energy dispersion diagram for stationary soliton-like ($l = 0$, case (*A*)) and vortex – like (l=1, case (*B*)) solutions of GNSE. Numerical result. Dashed lines indicate $N(\lambda)$ dependencies predicted by the variational approach. Here $D = P = - B = 1$.

An equation (30) was integrated numerically using the relaxation process in the Fourier space which is a generalization of well – known stabilizing multiplier method [5]. To check the stability of the structures, the evolutionary equation (29) was also integrated numerically by a standart split step Fourier method [5].

Simulation results are presented in a form of energy dispersion diagrams (EDD) – dependencies $N(\lambda)$. EDD for soliton – like ($l = 0$) and vortex – like ($l = 1$) nonlinear structures for different values of K are plotted in Fig. 2 (*A*), (*B*).

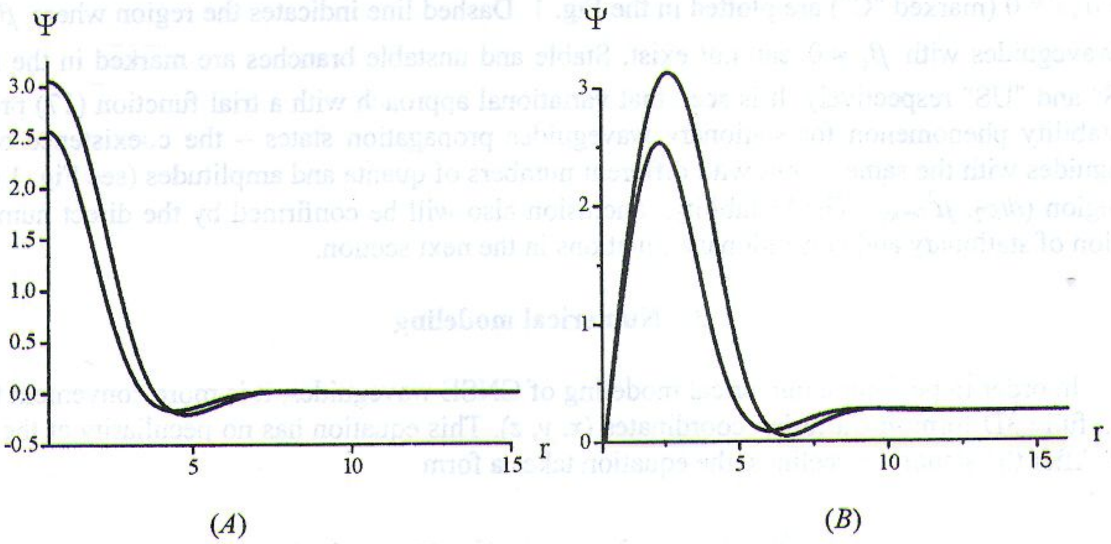

Fig. 3. Stable stationary radially – symmetric soliton – like ($l = 0$, case (*A*), $\lambda = -1.89$) and vortex – like ($l = 1$, case (*B*), $\lambda = -2.10$) solutions of GNSE with $D = P = -B = 1$, $K = 0.1$.

It is seen that soliton – like and vortex – like waveguides exist inside the restricted (both in N and in λ) area and the size of their existence domain decreases when K increases. At every solitary or vortex branch there exists a range of λ – values, where our numerical simulations confirm the variational conclusions about the bistability of stationary waveguide propagation states in the sense of coexistence of two states with the same λ but with different energies (numbers of quanta) and spatial scales. Examples of bistable soliton – like and vortex – like states are presented in Fig. 3 (*A*), (*B*).

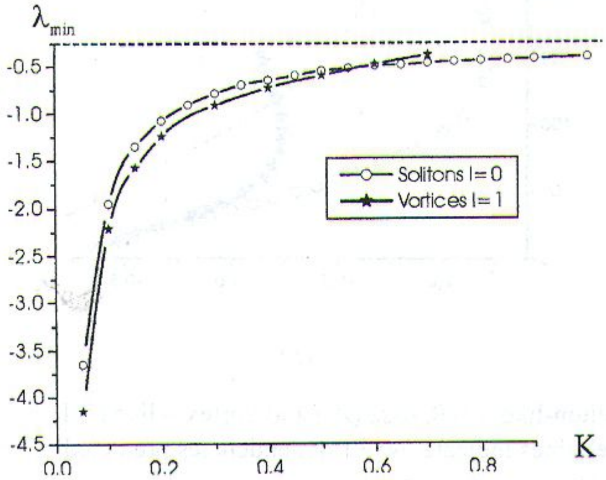

Fig. 4. The minimum possible value of λ where soliton – like ($l = 0$) and vortex – like ($l = 1$) waveguides exist versus K. Here $D = P = -B = 1$.

A numerical verification of stationary waveguide stability showed that all found waveguides with $l = 0$ and $l = 1$ are stable even with respect to sufficiently high symmetric and asymmetric perturbations. Their parameters (width, amplitude, etc.) oscillate nonlinearly in the vicinity of the stationary point. A conclusion about the waveguide stability does not depend on a sign of $\partial N / \partial \lambda$ derivative in contrast to well-known Kolokolov-Vakhitov criterion which is valid for GNSE with $P = 0$. When the parameter K exceeds some threshold value K_{cr}, localized solutions of Eq. (30) disappear.

It was also confirmed numerically that soliton parameter λ is bounded from below, for every K there exists some minimum value of $\lambda = \lambda_{\min}$ (see Fig. 4).

Conclusions

A propagation of whistler wave beams along the magnetic field lines with the frequencies near the half electron gyrofrequency ($\omega \approx \omega_{Be}/2$) in the ionosphere is described by the single non-linear Schrödinger equation (1) for the parallel electric field component. This equation includes term $\sim P\Delta_{\perp}^{2}E_{z}$ in it's linear part.

The main nonlinear effect in the ionosphere is the plasma extrusion from the HF field region due to heating and pressure increasing. Near the point $\omega \approx \omega_{Be}/2$ the thresholds of modulational instabilities for whistler wave decrease dramatically, and one must account for the next terms in the nonlinearity expansion which are of the same order as the linear terms. It leads to an appearance of cubic – quintic saturable nonlinearity in the GNSE (1).

In the considered case $D > 0$, $P > 0$, $B < 0$, $K > 0$, the Hamiltonian of GNSE (1) is bounded from below and above for every N, which indicates that there exists at least one stable solitary solution which corresponds to the Hamiltonioan's exact extremum.

The sum of all linear terms in the virial relation for waveguide's effective width acts defocusing, which indicates that any linear wave packet described by linear GNSE (1) always spreads out. At the same time nonlinear part of virial relation includes focusing, as well as defocusing terms. It may result in a coexistence of several stationary nonlinear structures with different spatial scales in the framework of Eq. (1).

Variational approach with the trial function (17) predicts a bistability phenomenon for the stationary waveguide propagation states: the coexistence of two stable solutions with different energies (numbers of quanta N) and spatial scales for the same value of nonlinear shift of wavenumber λ. It also predicts that the sign of derivative $\partial N/\partial\lambda$ may change within the region of stability, thus a commonly used Vakhitov-Kolokolov criterion (which works for GNSE with $P = 0$) in the considered case $P > 0$, $K > 0$ can not be applied.

Direct numerical integration of stationary (with respect to z) (Eq. (6)) and nonstationary (Eq. (1)) equations showed that there really exist soliton-like (with zero topological charge) and vortex-like (first azimuthal mode $l = 1$) stable nonlinear waveguides. Simulations found exactly the bistability regions for waveguides (see Fig. 2). All stationary solutions were shown to be stable even with respect to sufficiently high symmetric and asymmetric perturbations of initial state.

As it was pointed out in [6], some data conclusively show the formation of regular structures during ionospheric heating experiments. Our theoretical investigation gives possible explanation of such phenomena.

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СТАЦІОНАРНЕ САМОФОКУСУВАННЯ ВІСТЛЕРІВ В ІОНОСФЕРІ

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Аналітично та чисельно досліджено можливість утворення двовимірних когерентних структур – хвилеводних каналів вістлерівських хвиль, що спостерігались під час активних експериментів в іоносфері Землі. Мінімальному порогу утворення таких структур відповідає частота хвилі, що дорівнює половині електронної гірочастоти. Суттєву роль в їх утворенні грає теплова самовзаємодія – видавлювання частинок із області підвищеного тиску при розігріванні плазми пучком електромагнітних хвиль та захоплення пучка в канал, що утворюється. Показано, що утворення стаціонарних каналів описується двовимірним нелінійним рівнянням Шредінгера. Аналітично та чисельно доведено стійкість таких структур.

СТАЦИОНАРНАЯ САМОФОКУСИРОВКА ВИСТЛЕРОВ В ИОНОСФЕРЕ

Т. А. Давыдова, Ю. А.練Залізняк

Аналитически и численно исследована возможность образования двумерных когерентных структур – волноводных каналов вистлеровских волн, которые наблюдались во время активных экспериментах в ионосфере Земли. Минимальному порогу образования таких структур соответствует частота волны, равная половине электронной гирочастоты. Существенную роль в их образовании играет тепловое самовзаимодействие – выдавливание частиц из области повышенного давления при нагреве плазмы пучком электромагнитных волн и захват пучка в образовавшийся канал. Показано, что образование стационарных каналов описывается двумерным нелинейным уравнением Шредингера. Аналитически и численно доказана устойчивость таких структур.

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