УДК 539.173

VISCOSITY EFFECTS AT THE NUCLEAR DESCENT FROM THE FISSION BARRIER

S. V. Radionov, F. A. Ivanyuk, V. M. Kolomietz, A. G. Magner

We evaluate the temperature T_{scis} at the scission point and the descent time τ_{sc} from the saddle to the scission of heated nuclei within the liquid-drop model. We use the classical Lagrange-like equations of motion. The nuclear surface is parametrized by the two-parametric family of the Lawrence shapes. Conservative forces are defined through the free energy of the nucleus. We use the friction tensor derived from the boundary conditions on the nuclear surface and from exact solution of the continuity equation for incompressible and irrotational flow. The scission line is determined from the instability condition of the nuclear shape with respect to the variations of the neck radius. The numerical solution of the dynamical equations is carried out for the nucleus ^{236}U. We have defined the viscosity coefficient μ from the comparison of the experimental data for the kinetic energy of the fission fragments with computed one. We found a significiant deviation of μ obtained within our approach from the value of μ obtained within the standard hydrodynamical model.

1. Introduction

Many general features of nuclear dynamics can be described in macroscopic models in terms of collective variables. The few essential degrees of freedom are usually used to simplify a complex dynamic problem of large amplitude motion like nuclear fission. An available approach to such large amplitude motion problems is based on the standard liquid drop model LDM [1]. Up to now, the LDM is widely used for the description of the main macroscopic, i.e., averaged over many quantum states, characteristics of nuclear fission [1]. Within this model one starts with the classical equations of motion for the collective variables describing the shape of nuclear surface. The conservative forces and the mass coefficients are derived from the LDM assuming the irrotational motion and incompressibility of the nuclear liquid. The friction tensor is derived using the well known Rayleigh function in the same way as for infinite viscous matter.

In [2] the nuclear liquid drop model was modified by the combining the initial problem of the Navier-Stokes equation inside nucleus and the boundary conditions on its surface. The friction tensor takes into account the finite size of the liquid drop described by the boundary conditions. In the present work we solve the macroscopic equations of motion with friction parameters of [2] and we compare the results with those obtained within the standard hydrodynamic model. In the case of hot nuclei we derive the conservative forces in the classical Lagrange-like equations through the free energy.

2. Macroscopic equations of motion

The motion of incompressible, viscous and uniformly charged fluid can be described by the Navier-Stokes equation

$$m\rho_{eq}\frac{\partial}{\partial t}u_{\alpha} = -\frac{\partial}{\partial r_{\beta}}\prod{}_{\alpha\beta} - \rho_{p,eq}\nabla\varphi, \qquad (1)$$

where $\rho_{eq}(\vec{r})$ is the equilibrium particle density, $\rho_{p,eq}(\vec{r})$ is the equilibrium charge density, m is the nucleon mass, $\vec{u}(\vec{r},t)$ is the velocity field, and $\varphi(\vec{r},t) = \int \rho_{p,eq}(\vec{r}\,')\cdot|\vec{r}-\vec{r}\,'|^{-1}d\vec{r}\,'$ is the Coulomb potential (here φ is time dependent through the dependence on the time of the surface S which restricts the volume V of the system). In Eq. (1) and succeeding equations we use the convention that repeated indices are summed over. The momentum flux tensor $\prod_{\alpha\beta}(\vec{r},t)$ contains conservative and dissipative parts. Below we assume the potential motion of fluid. Taking into account Eq. (1) the momentum flux tensor takes the following form (see Sect.62 of [3]):

$$\prod_{\alpha\beta}=\left[-m\rho_{eq}\frac{\partial\chi}{\partial t}-\frac{Ze}{\mathrm{A}}\rho_{eq}\varphi\right]\delta_{\alpha\beta}-2\mu\frac{\partial^2\chi}{\partial r_\alpha\partial r_\beta}, \tag{2}$$

where $\chi(\vec{r},t)$ is the potential of the velocity field ($\vec{u}=\nabla\chi$), Z is the proton number, e is the electronic charge, A is the number of nucleons in the nucleus, and μ is the viscosity coefficient.

The potential $\chi(\vec{r},t)$ can be obtained from the continuity equation

$$\Delta\chi=0 \tag{3}$$

The equations of motion (1) and (3) have to be completed by the boundary conditions. Assuming a sharp surface of the nucleus, the boundary conditions on the moving surface are given by

$$(\vec{n}\vec{u})\big|_S=(\vec{n}\nabla\chi)\big|_S=u_S, \tag{4}$$

$$\prod_{nn}(\vec{r},t)\,|_S=P_S, \tag{5}$$

where $\vec{n}$ is the unit vector which is normal to the surface S, $\prod_{nn}(\vec{r},t)$ is the normal-normal component of the pressure tensor $\prod_{\alpha\beta}$, u_S is the velocity of the surface S, and P_S is the surface tension pressure.

Let us introduce the displacement field $\vec{\omega}(\vec{r},t)$ given by $\vec{u}(\vec{r},t)=\partial\vec{\omega}(\vec{r},t)/\partial t$. It is convenient to rewrite the boundary condition (5) in the form of the variational problem (see Sect. 4, p. 195 of [8])

$$\int_{t_0}^{t_1}dt\oint_S ds\,\delta\omega_n\left[\prod_{nn}-P_S\right]=0, \tag{6}$$

where $\delta\omega_n(\vec{r},t_0)=\delta\omega_n(\vec{r},t_1)=0$ for any variations of the normal component ω_n of the displacement field $\vec{\omega}(\vec{r},t)$. The variational problem (6) can be simplified by parametrizing the liquid drop surface S in terms of the collective variables $q(t)=q_1...q_N$ by writing [2]

$$\delta\omega_n\,|_S=\overline{u}_i\delta q_i,\qquad u_S=\overline{u}_i\dot{q}_i,\qquad \chi=\overline{\chi}_j\dot{q}_i. \tag{7}$$

We will assume that the nucleus is defined by rotation of the certain profile function $\rho=Y(z,t)$ around the z axis. The quantity $\overline{u}_i$ in Eq. (7) is given then as [2]

$$\overline{u}_i=\frac{\partial Y}{\partial q_i}/\Lambda,\qquad \Lambda=\sqrt{1+\left(\frac{\partial Y}{\partial z}\right)^2}. \tag{8}$$

In order to define the quantity $\overline{\chi}_i$ in Eq. (7) we substitute Eq. (7) into Eq. (3) and (4) and use Eq. (8). So we have

$$\Delta\overline{\chi}_i = 0, \qquad (\vec{n}\nabla\overline{\chi}_i)\,|_S = \frac{1\partial Y}{\Lambda\partial q_i}. \tag{9}$$

After substitution of Eqs. (2) and (7) into Eq. (6) we obtain macroscopic equations of motion for the collective variables $q_i(t)$

$$M_{ij}\ddot{q}_j + \left[\frac{\partial M_{ij}}{\partial q_k} - \frac{1}{2}\frac{\partial M_{jk}}{\partial q_i}\right]\dot{q}_j\dot{q}_k = K_i - Z_{ij}\dot{q}_j, \qquad i = 1,2...N, \tag{10}$$

where

$$M_{ij} = m\rho_{eq}\oint ds\overline{u}_i\overline{\chi}_j, \tag{11}$$

is the inertia tensor. The inertia tensor M_{ij} determines the collective kinetic energy T_{kin}. Namely,

$$T_{kin} = \frac{1}{2}m\rho_{eq}\int_V u^2 dV = \frac{1}{2}m\rho_{eq}\int_S \chi(\vec{n}\cdot\nabla\chi)dS = \frac{1}{2}M_{ij}(q)\dot{q}_i\dot{q}_j. \tag{12}$$

In Eq. (10) Z_{ij} is the friction tensor [2]

$$Z_{ij} = 2\mu\oint ds\overline{u}_i\frac{\partial^2\overline{\chi}_j}{\partial r_\alpha\partial r_\beta}n_\alpha n_\beta. \tag{13}$$

We point out that the friction tensor (13) differs from the friction tensor of the standard hydrodynamic model [1] (see Appendix A). In the case of heated system the conservative forces K_i in Eq. (10) relates to the free energy F as (see Appendix B)

$$K_i = -\left(\frac{\partial F}{\partial q_i}\right)_{V,T,\dot{q}}, \tag{14}$$

where T is the temperature of the system and $\dot{q} \equiv \dot{q}_1, \dot{q}_2 ....\dot{q}_N$ denotes the N collective velocities.

The free energy of the nucleus at finite temperature is given by

$$F = T_{kin} + F_S + F_C + F^* \tag{15}$$

here F_S and F_C are the free surface and Coulomb energy respectively and F^* is the internal free energy.

For the energies F_S and F_C we use the following expressions [4, 10]:

$$F_C(q,T) = \alpha(T)B_C(q)\frac{Z^2}{A^{1/3}}, \quad F_S(q,T) = \sigma(T)B_S(q)\gamma A^{2/3}, \quad \gamma = 1 - 1.7826\left[\frac{\mathcal{N} - Z}{A}\right]^2, \tag{16}$$

where N is the neutron number.

In the low-temperature limit $T << T_C$ the temperature dependence of the surface tension $\sigma(T)$ and of the Coulomb force parameter $\alpha(T)$ are given by [11]

$$\sigma(T) = 17.94\left(1 - \frac{5T^2}{2T_C^2}\right) \quad MeV, \qquad \alpha(T) = 0.7(1 - x_C) \quad MeV \tag{17}$$

and the internal free energy has the form

$$F^* = -a_V T^2, \tag{18}$$

where the parameter x_C was chosen as $x_C = 0.76 \cdot 10^{-3}\ MeV^{-2}$ [10], $T_C = 18\ MeV$ is taken as the critical temperature for infinite nuclear Fermi-liquid [11], $a_V = \pi^2 A/4\varepsilon_F$ is the volume contribution to the level-density parameter and $\varepsilon_F = 40\ MeV$ is the Fermi energy.

Using Eq. (17) and (18) the free energy F (15) can be rewritten in the form

$$F = T_{kin} + U(q) - aT^2, \tag{19}$$

where $U(q)$ is the potential energy of the cold nucleus

$$U(q) = 17.94 \cdot B_S(q)\gamma A^{2/3} + 0.7 \cdot B_C(q) Z^2 / A^{1/3} \quad MeV \tag{20}$$

and the level-density parameter a is given by

$$a(q) = a_V + a_S + a_C \tag{21}$$

here

$$a_S(q) = 17.94 \cdot \frac{5}{2T_C^2} B_S(q)\gamma A^{2/3}, \qquad a_C(q) = 0.7 \cdot x_C B_C(q) \frac{Z^2}{A^{1/3}}. \tag{22}$$

The system of macroscopic equations (10) with the conservative forces K_i (14) should be completed by the condition for the determination of the temperature T along the dynamical trajectory. In order to obtain this condition let us consider the entropy S of the system

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,q,\dot{q}} = 2a(q)T. \tag{23}$$

The entropy S is changed due to a work of the friction forces

$$TdS = Z_{ij}(q)\dot{q}_i\dot{q}_j \cdot dt. \tag{24}$$

Using Eq. (23) we have

$$dS = 2 \cdot \frac{da(q)}{dq_i} dq_i T + 2a(q) \cdot dT . \tag{25}$$

From Eq. (25) one obtains the equation for the temperature:

$$\dot{T} = \frac{1}{2a}\left[Z_{ij}(q)\dot{q}_i\dot{q}_j - 2\frac{da(q)}{dq_i} \cdot \dot{q}_i \cdot T \right] \tag{26}$$

Also the temperature can be defined from the conservation law of the total energy E of the system.

Finally we have a closed system of equations for the variables q, T

$$M_{ij}\ddot{q}_j + \left[\frac{\partial M_{ij}}{\partial q_k} - \frac{1}{2}\frac{\partial M_{jk}}{\partial q_1} \right]\dot{q}_j\dot{q}_k = -\left(\frac{\partial F}{\partial q_i}\right)_{V,T,\dot{q}} - Z_{ij}\dot{q}_j, \qquad i = 1,2...N \tag{27}$$

$$\dot{T} = \frac{1}{2a}\left[Z_{ij}(q)\dot{q}_i\dot{q}_j - 2\frac{da(q)}{dq_i} \cdot \dot{q}_i \cdot T \right]. \tag{28}$$

3. Numerical results

We consider the symmetric fission of ^{236}U. We neglect the q-dependence of the level-density parameter assuming that the a corresponds to the value for the spherical nucleus $(B_S(q) = B_C(q) = 1)$ and is given by

$$a(q) = \frac{\pi^2 A}{4\varepsilon_F} + 17.94 \cdot \frac{5}{2T_C^2}\gamma A^{2/3} + 0.7 \cdot x_C \frac{Z^2}{A^{1/3}}. \tag{29}$$

Below we will use the empirical value of the level-density parameter $a(q) = A/8\ MeV^{-1}$.

We solved Eqs. (27) – (28) numerically with the two-parametric shape family in the Lawrence parametrization [6]

$$Y^2(z) = \frac{(\zeta_0^2 - z^2)(\zeta_2^2 + z^2)}{\zeta_0^3(\zeta_0^2/5 + \zeta_2^2)}. \tag{30}$$

Here and below all quantities of the length dimension are expressed in units of the radius R_0 of the equivalent spherical shape. The parameter ζ_0 in (30) determines the general elongation of the figure and ζ_2 is related to the neck radius.

We derived the scission line from the condition of the instability of the nuclear shape with respect to the variations of the neck radius

$$\frac{\partial^2 U(q)}{\partial \rho_{neck}^2} = 0, \tag{31}$$

where $\rho_{neck} = \zeta_2 / \sqrt{\zeta_0(\zeta_0^2/5 + \zeta_2^2}$ is the neck radius [6].

The macroscopic equations of motion (27) – (28) were solved with the initial conditions corresponding to the saddle point deformation, initial kinetic energy of 1 MeV (initial neck velocity $\dot{\zeta}_2 = 0$), and initial temperature $T_0 = 2\,MeV$. To solve the Neumann problem (9) for the velocity field potential we have used the method based on the theory of the potential [5]. For comparison, we have solved the dynamical equations (27) – (28) with the friction tensor of the standard hydrodynamic model computed within the Werner-Wheeler method [1] (see Appendix A, Eq. (40)).

Dynamical trajectories for the different values of the viscosity coefficient μ (in units of the $\mu_0 = 10^{-23}\ MeV \cdot s / fm^3$) are presented in Fig. 1.

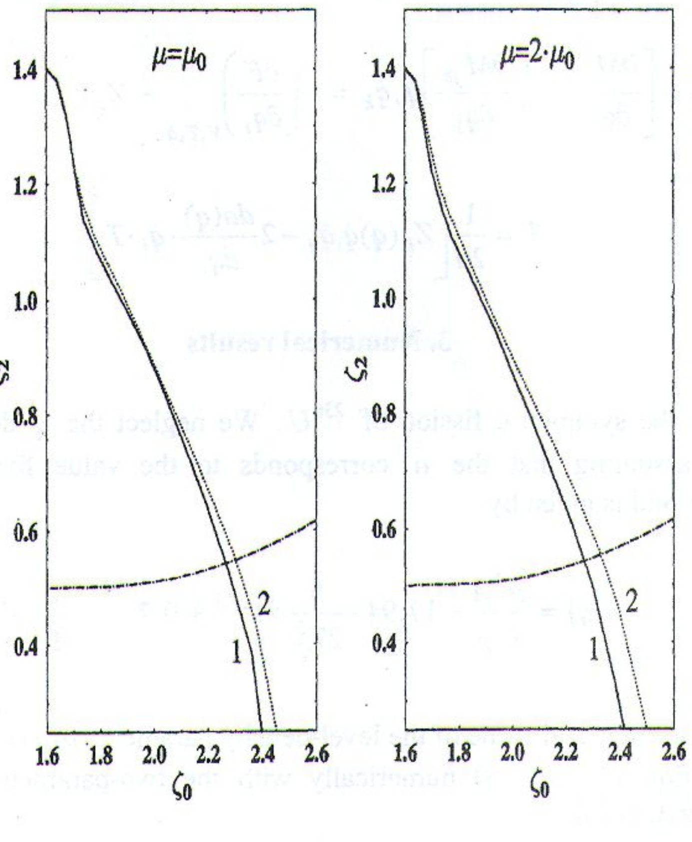

Fig. 1. Dynamical trajectories in the $\zeta_0 - \zeta_2$ plane. Curve *1* corresponds to the calculation with the friction tensor (36) computed with the help of the theory of potential. Curve *2* correspond to the calculations with the friction tensor of the standard hydrodynamical model (40) obtained within the Werner-Wheeler. Dot-dashed line is the scission line from the condition (31).

In Fig. 1 we show the dependence of the neck parameter ζ_2 on the elongation ζ_0 of the nucleus for dynamical trajectories calculated with various amounts of the viscosity coefficient μ. For the zero viscosity curves *1* and *2* coincide. We define the scission point as the intersection point of the dynamical path with the scission line (31). Fig. 1 illustrates that the viscosity hinders neck formation and leads to a more elongated scission configuration. With the growing of the viscosity coefficient μ difference between curves *1* and *2* increases. It speaks about the significant sensitivity of the dynamical calculations to the choice of the friction parameters.

Fig. 2 shows the effect of the viscosity on the descent time τ_{sc} and on the temperature of the nucleus at the scission point T_{scis}.

The viscosity coefficients μ_1 and μ_2 in Fig. 2 were obtained from the comparison of experimental data [7] for the kinetic energy $T_{kin}^{\inf}$ of the fission fragments with computed one within the approaches *1* and *2*.

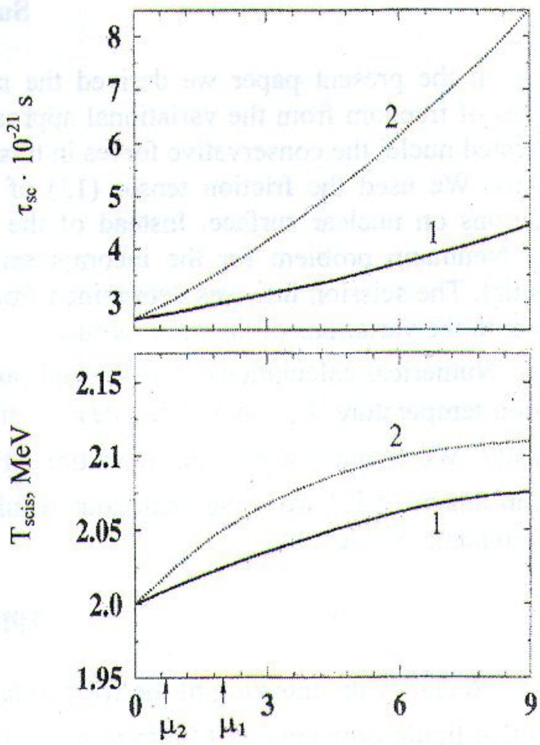

Fig. 2. The descent time τ_{sc} from the saddle to the scission point and the scission temperature T_{scis} as function of the viscosity coefficient μ (in units of the μ_0). Notations for the curves *1* and *2* are the same as in Fig. 1.

We approximated the value of $T_{kin}^{\inf}$ by the sum of the relative Coulomb energy V_{Coul} and the kinetic energy $T_{kin,c.m.}$ of the center-of-mass motion of the two equal spheroids at the scission point

$$T_{kin}^{\inf} = [V_{Coul}(d,c) + T_{kin,c.m.}(\dot{d})]^{scis}. \tag{32}$$

We assumed that the distance between the centers of mass d of two spheroids is equal to the distance between the two halves of the fissioning nucleus

$$d = \frac{5}{4}\zeta_0 \frac{\zeta_0^2 + 3\zeta_2^2}{\zeta_0^2 + 5\zeta_2^2}. \tag{33}$$

The corresponding velocity $\dot{d}$ was obtained by the differentiation of the (33) with respect to the time.

The elongation c of both separated spheroids is defined by the condition

$$2 \cdot c + d = 2 \cdot \zeta_0, \tag{34}$$

where ζ_0 is the elongation of the nucleus at the scission point.

The difference of our T_{scis} and τ_{sc} (see curve *1*) from the ones of (2) increase with growing viscosity. We point out that viscosity coefficient μ_1 derived with the friction tensor of [2] deviates significantly from the value μ_2 obtained with the standard hydrodynamic friction of [1].

Summary

In the present paper we derived the macroscopic equations of motion for the collective degrees of freedom from the variational approach to macroscopic nuclear dynamics. In the case of the heated nuclei the conservative forces in this equations are defined through the free energy of the nucleus. We used the friction tensor (13) of article [2] which takes into account the boundary conditions on nuclear surface. Instead of the traditional Werner-Wheeler method we solved the exact Neumann problem for the incompressible, irrotational velocity field using the theory of potential. The scission line was determined from the instability condition of the nuclear shape with respect to the variations of the neck radius.

Numerical calculations were carried out for the ^{236}U. We obtained the dependence of the scission temperature T_{scis} and of the descent time τ_{sc} from the saddle to the scission point on the viscosity. We found a significant deviation of the results for the T_{scis} and τ_{sc} obtained with the friction tensor of [2] from the analogous results obtained with the friction tensor of the standard hydrodynamic model [1].

Appendix A

To clarify the meaning of the friction tensor Z_{ij}(13) we will evaluate the change rate of the collective liquid drop energy. Multiplying Eq. (1) by $\vec{u}$, integrating it over the nuclear volume, and using Eqs. (4) and (5) we derive the change rate of the collective liquid drop energy $E_{coll} = T_{kin} + E_C + E_S$ as

$$\frac{\partial E_{coll}}{\partial t} = \frac{\partial}{\partial t}(T_{kin} + E_C + E_S) = -\sum_{ij} Z_{ij}\dot{q}_i\dot{q}_j \tag{35}$$

here E_C and E_S is the Coulomb and surface energy respectively. The friction tensor Z_{ij} in Eq. (35) is given by Eq. (13) and it can be rewritten as

$$Z_{ij} = \mu\oint ds \frac{\partial}{\partial n}(u_n^{(i)}u_n^{(j)}), \tag{36}$$

where $\vec{u}^{(i)} = \nabla\chi_i$.

Expression (36) for the friction tensor Z_{ij} can be also founded from the Rayleigh function [2]

$$R = -\frac{1}{2}\oint ds u_n \prod_{nn}^{(dis)} = -\frac{1}{2}\sum_{ij} Z_{ij}\dot{q}_i\dot{q}_j, \tag{37}$$

where $\prod_{\alpha\beta}^{(dis)}$ is the dissipative part of the pressure tensor (2)

$$\prod_{\alpha\beta}^{(diS)} = -2\mu\frac{\partial^2\chi}{\partial r_\alpha \partial r_\beta}. \tag{38}$$

We point out that the Rayleigh function R (37) is different from the analogous Rayleigh function $R^{(s\tan d)}$ of the standard hydrodynamic model [1, 2]

$$2R^{(s\tan d)} = \mu\int d^3r\Delta u^2 = -\oint dsu_n\prod_{nn}^{(dis)} - \int dsu_\tau\prod_{n\tau}^{(dis)}, \tag{39}$$

here index τ denotes the tangential component.
The friction tensor $Z_{ij}^{(s\tan d)}$ corresponding to the Rayleigh function $R^{(s\tan d)}$ is written as [2]

$$Z_{ij}^{(s\tan d)} = 2\mu\oint ds\sum_{\alpha\beta}\overline{u}_\alpha^{(i)}\frac{\partial^2\overline{\chi}_j}{\partial r_\alpha\partial r_\beta}n\beta = \mu\oint ds\frac{\partial(\overline{u}^{(i)}\overline{u}^{(j)})}{\partial n}, \tag{40}$$

where

$$u_\alpha = \sum_{i=1}^{N}\overline{u}_\alpha^{(i)}\dot{q}_i, \qquad \overline{u}_\alpha^{(i)} = \frac{\partial\overline{\chi}_i}{\partial r_\alpha}. \tag{41}$$

It should be noted that the Rayleigh function R differs from the Rayleigh function $R^{(s\tan d)}$ of the standard hydrodynamic model because to obtain Eq. (37) for the R the boundary conditions (4) and (5) were essentially applied. Both Rayleigh functions (37) and (39) and as well as the friction tensors (36) and (40) are not identical because, in general, the normal-tangential component of the pressure tensor $\prod_{\alpha\beta}^{(dis)}$ does not become zero on the nuclear surface.

Appendix B

Appendix B contains the derivation of the expression (14) for the conservative forces K_i in the case of heated system.

We start with the first law of thermodynamics. In addition to the volume and entropy, the generalized coordinates and velocities, q_i and $\dot{q}_i$, determine the thermodynamical state of the system. The first law of thermodynamics takes the form (see Sect. 15 of [9])

$$dE = TdS - PdV - K_i dq_i - Q_i d\dot{q}_i, \tag{42}$$

where $P = -(dE/\partial V)_{S,q,\dot{q}}$ is the pressure and

$$K_i = -\left(\frac{\partial E}{\partial q_i}\right)_{V,S,\dot{q}} \quad ; \quad Q_i = -\left(\frac{\partial E}{\partial \dot{q}_i}\right)_{V,S,q} \tag{43}$$

are the generalized forces and generalized momenta respectively. In many cases it is more convenient to use the temperature T instead of the entropy S. Using the thermodynamical definition of the free energy $F = E - TS$ we can write down the generalized forces K_i in the form (see Sect. 15 of [9])

$$K_i = -\left(\frac{\partial F}{\partial q_i}\right)_{V,T,\dot{q}} \tag{44}$$

i.e. the conservative forces K_i in equations of motion (27) should be defined through the derivation of the energy E or of the free energy F with respect to the shape parameters q_i at fixed entropy S or temperature T and collective velocities $\dot{q}$.

At the end of the Appendix B we will demonstrate a method which allow us to derive the macroscopic equations of motion (27) - (28) from the minimal-action principle.

Let us write down the minimal-action principle in the following form [8]:

$$\delta \int_{t_1}^{t_2} \mathcal{F} dt = \delta \int_{t_1}^{t_2} [T_{kin} + \lambda (T_{kin} + U + E^* - E_0)] dt = 0, \tag{45}$$

where λ is the Lagrange multiplier which takes into account the conservation of the total energy $E = T_{kin} + U + E^* = E_0$, provided that in the initial time t_1

$$\delta q_i(t_1) = 0, \qquad \delta \dot{q}_i(t_1) = 0 \tag{46}$$

and in the final time t_2 the quantities δq_i and $\delta \dot{q}_i$ are arbitrary.

We will consider the entropy S as the collective variable which together with the q and $\dot{q}$ determines the dynamical behavior of the system. From Eq. (45) one obtains

$$\int_{t_1}^{t_2} \left(\frac{\partial \mathcal{F}}{\partial q_i} \delta q_i + \frac{\partial \mathcal{F}}{\partial \dot{q}_i} \delta \dot{q}_i + \frac{\partial \mathcal{F}}{dS} \delta S \right) dt + \mathcal{F} \big|_{t_2} \delta t_2 = 0. \tag{47}$$

Using the thermodynamic definition of the temperature $T = (\partial E / \partial S)_{V,q,\dot{q}}$ we have

$$\left(\frac{\partial \mathcal{F}}{dS} \right)_{V,q,\dot{q}} \delta S = \lambda \left(\frac{\partial E}{\partial S} \right)_{V,q,\dot{q}} \delta S = \lambda T \delta S, \tag{48}$$

where (see Eq. (24))

$$T \delta S = Z_{ij} \dot{q}_j \dot{q}_i \delta t = Z_{ij} \dot{q}_j \delta q_i. \tag{49}$$

After substituting Eqs. (48) and (49) into Eq. (47) we obtain

$$\int_{t_1}^{t_2} \left[\frac{\partial \mathcal{F}}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial \mathcal{F}}{\partial \dot{q}_i} \right) + \lambda Z_{ij} \dot{q}_i \right] \delta q_i dt + \left[-(2\lambda + 1) T_{kin} + \lambda (T_{kin} + U + E^* - E_0) \right] \delta t_2 = 0 \tag{50}$$

Using the condition $T_{kin} + U + E^* = E_0$

$$-(2\lambda + 1) T_{kin} + \lambda (T_{kin} + U + E^* - E_0) = -(2\lambda + 1) T_{kin}. \tag{51}$$

Therefore Eq. (50) takes the form

$$\int_{t_1}^{t_2}\left[\frac{\partial \mathcal{F}}{\partial q_i}-\frac{d}{dt}\left(\frac{\partial \mathcal{F}}{\partial \dot{q}_i}\right)+\lambda Z_{ij}\dot{q}_j\right]\delta q_i dt-(2\lambda+1)T_{kin}\delta t_2=0. \quad (52)$$

The variations δq_i and δt_2 are independent, so

$$\frac{\partial \mathcal{F}}{\partial q_i}-\frac{d}{dt}\left(\frac{\partial \mathcal{F}}{\partial \dot{q}_i}\right)+\lambda Z_{ij}\dot{q}_j=0, \qquad i=1,2\ldots N \quad (53)$$

$$-(2\lambda+1)T_{kin}=0, \quad (54)$$

therefore $\lambda=-1/2$ and

$$\mathcal{F}=\frac{1}{2}\left(T_{kin}-U-E^*\right) \quad (55)$$

and Eq. (53) leads to the Lagrange equations of motion in the following form:

$$\frac{\partial \mathcal{L}}{\partial q_i}-\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right)-Z_{ij}\dot{q}_j=0 \quad (56)$$

with the Lagrange function $\mathcal{L}=T_{kin}-U-E^*$.

The Lagrange equations (56) can be rewritten as

$$M_{ij}\ddot{q}_j+\left[\frac{\partial M_{ij}}{\partial q_k}-\frac{1}{2}\frac{\partial M_{jk}(q)}{\partial q_i}\right]\dot{q}_j\dot{q}_k=K_i-Z_{ij}\dot{q}_j, \qquad i=1,2\ldots N, \quad (57)$$

where

$$K_i=-\left(\frac{\partial E}{\partial q_i}\right)_{V,S,\dot{q}}=-\left(\frac{\partial F}{\partial q_i}\right)_{V,T,\dot{q}}. \quad (58)$$

The change of the temperature T with the time is defined from the equation (see Eqs. (25) and (26)):

$$\dot{T}=\frac{1}{2a}\left[Z_{ij}(q)\dot{q}_i\dot{q}_j-2\frac{da(q)}{dq_i}\cdot\dot{q}_i\,T\right] \qquad i=1,2\ldots N. \quad (59)$$

REFERENCES

1. *Davies K. T. R., Sierk A. J., Nix J. R.* // Phys. Rev. - 1976. - Vol. C13. - P. 2385.
2. *Ivanyuk F. A., Kolomietz V. M., Magner A.G.* // Phys. Rev. - 1995. - Vol. C52. - P. 678.
3. *Landau L. D. and Lifshitz E. M.* Fluid Mechanics. - Addison-Wesley: Reading, Mass. - 1959.
4. *Myers W.D, Swiatecki W.J.* // Ark. Fys. - 1967. - Vol. 36. - P. 343.

5. *Jeffreys H., Swirles B.* Methods of Mathematical Physics. – Cambridge: Cambridge University Press, 1966.
6. *Hasse R.W., Myers W.D.* Geometrical Relationships of Macroscopic Nuclear Physics. – Berlin: Springer-Verlag, 1988.
7. *Chubarian G. G. et al.* // Progress in Research (Apr. 1, 1995 - Mar. 31, 1996.). – Texas: A&M University, Cyclotron Institute, 1996. - P. 58.
8. *Berdichevsky V. L.* Variational Principle for Mechanics of Continua. – Moskow: Nauka, 1983.
9. *Landau L. D., Lifshitz E. M.* Statistical physics. – NY: Pergamon, 1958.
10. *Guet C., Strumberger E., Brack M.* // Phys. Lett. – 1988. – Vol. B205. – P. 427.
11. *Ravenhall D. G., Pethick C. J., Lattimer J. M.* // Nucl. Phys. – 1983. – Vol. A407. – P. 571.

ВПЛИВ В'ЯЗКОСТІ НА СПУСК ЯДРА З БАР'ЄРА ПОДІЛУ

С. В. Радіонов, Ф. О. Іванюк, В. М. Коломієць, О. Г. Магнер

Досліджується температура T_{scis} у точці розриву та час спуску з бар'єра до точки розриву τ_{sc} для поділу нагрітих ядер. Для цього використовуються класичні рівняння руху типу Лагранжа в рамках моделі рідкої краплі. Поверхня ядра параметризується двопараметричним класом лоренцевих фігур. Консервативні сили визначаються через вільну енергію ядра. Ми використовуємо тензор тертя, отриманий із граничних умов на ядерній поверхні та з точного розв'язку рівняння неперервності для нестискуваного й безвихрового потоку рідини. Лінія розриву визначалася з умови нестійкості форми ядра по відношенню до варіацій радіуса шийки. Чисельний розв'язок динамічних рівнянь було виконано для ядра ^{236}U. Коефіцієнт в'язкості μ було отримано з порівняння експериментальних даних для кінетичної енергії осколків поділу з розрахованим значенням цієї величини. Було знайдено значну відмінність μ, отриманого в рамках нашого підходу від μ стандартної гідродинамічної моделі.

ВЛИЯНИЕ ВЯЗКОСТИ НА СПУСК ЯДРА С БАРЬЕРА ДЕЛЕНИЯ

С. В. Радионов, Ф. А. Иванюк, В. М. Коломиец, А. Г. Магнер

Исследуется температура T_{scis} в точке разрыва и время спуска с барьера до точки разрыва τ_{sc} для деления нагретых ядер. Для этого используются классические уравнения движения типа Лагранжа в рамках модели жидкой капли. Поверхность ядра параметризуется двухпараметрическим семейством лоренцевых фигур. Консервативные силы определяются через свободную энергию ядра. Мы используем тензор трения, полученный из граничных условий на ядерной поверхности и из точного решения уравнения непрерывности для несжимаемого и безвихревого потока жидкости. Линия разрыва определялась из условия нестабильности формы ядра по отношению к вариациям радиуса шейки. Численное решение динамических уравнений выполнено для ядра ^{236}U. Коэффициент вязкости μ был получен из сравнения экспериментальных данных для кинетической энергии осколков деления с вычисленным значением этой величины. Было найдено значительное отличие μ, полученного в рамках нашего подхода от μ в стандартной гидродинамической модели.