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NONLINEAR EVOLUTION OF 3D DRIFT-ION-SOUND STANDING WAVES

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Drift-ion-sound standing waves are considered in a magnetized inhomogeneous plasma. Effects of three dimensionality, dispersion and vortex nonlinearity are taken into account. Perturbation theory solution is obtained by the multiple-time-scale formalism. It is shown that no secular terms are present up to the second order in amplitude and that second order corrections are homogeneous in the drift direction.

1. Introduction

Drift vortex structures play an important role in transport processes in plasmas. Detailed investigation of their stability must include an account of their connection with ion-sound waves, so the problem becomes three-dimensional [1]. General properties of the model [1] were studied in [2], stability analysis on the basis of this model was done in [3], but linear drift dispersion effects were neglected. The incorrectness of the neglect of the linear dispersion effects in the investigations of the stability of vortex structures of any amplitude was emphasized in [4].

In the present work the temporal evolution of spatially periodic 3D standing waves is studied. All physical effects contained in the model [1] are taken into account, namely the vortex nonlinearity and dispersion effects due to the emission of coupled drift and ion-sound waves. The model is shortly reviewed in the Section 2. For the waves of small but finite amplitude the perturbation theory based on multiple-time-scale formalism is built and second order solutions are found for the standing waves in the Section 3. Conclusions are made in the Section 4.

2. Model

Let us consider the inhomogeneous plasma slab in the external homogeneous magnetic field. Electrons, but no ions, are magnetized, smoothing an electrostatic potential Φ along the magnetic field lines. In this case 3D generalization [1, 2] of the Hasegawa-Mima model equations holds:

$$\partial\Psi/\partial t + J(\Phi,\Psi) = \partial\Phi/\partial y - \partial v/\partial z;$$

$$\partial v/\partial t + J(\Phi,v) = - \partial\Phi/\partial z; \qquad (1)$$

$$\Psi = \Phi - \Delta_\perp \Phi.$$

where v is velocity component along the magnetic field direction Oz, Ψ is only nonzero z-component of vorticity, $J(F,G) \equiv \partial F/\partial x\cdot\partial G/\partial y - \partial G/\partial x\cdot\partial F/\partial y$ is the Jacobian nonlinear operator and $\Delta_\perp \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$. The system (1) is written in dimensionless variables

$$\varepsilon\omega_B t,\ x/r_B,\ y/r_B,\ \varepsilon z/r_B,\ e\Phi/T_e\varepsilon,$$

where ion cyclotron frequency $\omega_B = eB/Mc$ and ion sound speed $c_s = (T_e/M)^{1/2}$ determine characteristic dispersion length $r_B = c_s / \omega_B$, L is the characterictic inhomogeneity length of plasma density and the small parameter ε is equal to the ratio r_B / L.

For perturbations proportional to $\exp(- i\omega t + ik_1x + ik_2y + ik_3z)$, we obtain two modes describing coupled drift waves and ion-acoustic ones:

$$\omega = 0.5\ (- k_2 \pm (k_2^2 + 4\ (1 + k_1^2 +k_2^2)\ k_3^2)^{1/2})/(1 + k_1^2 + k_2^2). \qquad (2)$$

Note that due to the very particular character of the vortex nonlinearity J(F,G) single monochromatic waves obeying this dispersion relation, but no sums of such waves, satisfy the nonlinear system (1). If we choose a finite sum of linear waves as an initial condition, we can built up the perturbation theory for waves of small but finite amplitude α, as it was done in [4] in the 2D case, by the use of multiple-time-scale formalism.

3. Perturbation theory

Let us consider the waves of small but finite amplitude α and use the parameter α to build the perturbation theory:

$$\partial/\partial t = \partial/\partial t_0 + \alpha\partial/\partial t_1 + \ldots, \qquad t_n = \alpha^n t;$$

$$\Psi(t, r) = \alpha\,(\Psi_0(t_0, t_1, t_2, \ldots| r) + \alpha\,\Psi_1(t_0, t_1, t_2, \ldots| r) + \ldots);$$

$$\Phi(t, r) = \alpha\,(\Phi_0(t_0, t_1, t_2, \ldots| r) + \alpha\,\Phi_1(t_0, t_1, t_2, \ldots| r) + \ldots);$$

$$v\,(t, r) = \alpha\,(v_0(t_0, t_1, t_2, \ldots| r) + \alpha\,v_1(t_0, t_1, t_2, \ldots| r) + \ldots).$$

Now we choose an initial condition as a standing wave which corresponds in two-dimensional case (k_3 -> 0) to the monochromatic standing wave (exact solution in 2D case):

$$\Phi = \alpha\sin(k_1x)\sin(k_2y)\cos(k_3z), \qquad \Psi = (1+k_1^2+k_2^2)\,\Phi;$$

$$v = -\,\alpha\, f(\mathbf{k})\sin(k_1x)\cos(k_2y)\sin(k_3z).$$

By the multiple-time-scale formalism we obtain the solution

$$\Phi = \alpha\sin(k_1x)\sin(k_2y + \omega_1t)\cos(k_3z) +$$
$$+\,\alpha^2 g(\mathbf{k})\sin(2k_1x)\cos(2k_3z)\,(1 - \cos(\omega_2t)) + O(\alpha^3);$$
$$\Psi = \alpha\,(1+k_1^2+k_2^2)\sin(k_1x)\sin(k_2y + \omega_1t)\cos(k_3z) +$$
$$+\,\alpha^2 g(\mathbf{k})\,(1 + (2k_1)^2)\sin(2k_1x)\cos(2k_3z)\,(1 - \cos(\omega_2t)) + O(\alpha^3); \qquad (3)$$
$$v = -\,\alpha\, f(\mathbf{k})\sin(k_1x)\cos(k_2y + \omega_1t)\sin(k_3z) -$$
$$-\,\alpha^2 g(\mathbf{k})\,(1 + (2k_1)^2)^{1/2}\sin(2k_1x)\sin(2k_3z)\sin(\omega_2t) + O(\alpha^3).$$

where frequencies are equal to

$$\omega_1 = (k_2 + (k_2^2 + 4\,k_3^2(1+k_1^2+k_2^2))^{1/2})\,/\,2\,(1+k_1^2+k_2^2)), \qquad \omega_2 = 2\,k_3\,/\,(1 + (2k_1)^2)^{1/2}.$$

and factors $f(\mathbf{k})$ and $g(\mathbf{k})$ are determined by

$$f(\mathbf{k}) = k_3/\omega_1, \qquad g(\mathbf{k}) = k_1\,k_2\,/\,8\omega_1.$$

Higher harmonics generation and no resonant terms appear in this approximation. Full nontrivial temporal evolution of the 3D standing wave (3) is determined by the coupling of drift waves to the ion-sound ones.

4. Conclusions

The temporal evolution of spatially periodic 3D drift-ion-sound standing waves was studied. All physical effects contained in the model [1] were taken into account, namely the additional vortex nonlinearity and dispersion effects connected to the coupling of drift and ion-sound waves. For the waves of small but finite amplitude the perturbation theory based on multiple-time-scale formalism was built. The solution obtained in this way for the 3D standing wave (3) shows that no resonant terms are present and nonlinear corrections are homogeneous in the drift direction, all that up to the second order in amplitude and for any values of the wave vector **k**. On the other hand, no signs of instability appear in this approximation.

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НЕЛІНІЙНА ЕВОЛЮЦІЯ ТРИВИМІРНИХ ДРЕЙФОВО-ІОННОЗВУКОВИХ СТОЯЧИХ ХВИЛЬ

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Розглянуто дрейфово-іоннозвукові стоячі хвилі у магнетизованій неоднорідній плазмі. Враховано тривимірні ефекти дисперсії та вихрової нелінійності. За допомогою теорії збурень, що базується на багаточасовому формалізмі, одержано розв'язки рівнянь моделі. Показано, що в другому порядку по амплітуді секулярні доданки відсутні, а збурення є однорідними в дрейфовому напрямку.

НЕЛИНЕЙНАЯ ЭВОЛЮЦИЯ ТРЕХМЕРНЫХ ДРЕЙФОВО-ИОННОЗВУКОВЫХ СТОЯЧИХ ВОЛН

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Рассмотрены дрейфово-ионнозвуковые стоячие волны в замагниченной неоднородной плазме. Учтены трехмерные эффекты дисперсии и вихревой нелинейности. С помощью теории возмущений, основанной на многовременном формализме, получены решения уравнений модели. Показано, что во втором порядке по амплитуде секулярные слагаемые отсутствуют, а возмущения однородны в дрейфовом направлении.