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# THE SCATTERING OF ELECTROMAGNETIC WAVES IN MAGNETIZED PLASMA WITH LOWER AND UPPER HYBRID PUMP

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The scattering of a transverse electromagnetic wave by turbulent density fluctuations in the homogeneous and inhomogeneous magnetoactive plasma with the lower and upper hybrid pump wave was investigated. We calculate the coefficient of the scattering for the plasma with density gradient in the presence of parametric instabilities of lower hybrid and upper hybrid waves.

**1.** When electromagnetic waves propagate through plasma a small fraction of the radiation is scattered. By launching a beam of radiation into plasma and collecting some of the scattered radiation, a lot of information about the properties of the plasma may be obtained from the spectral properties of the scattered radiation. It is well known that investigations of electromagnetic waves scattering by density fluctuations are important for studying such problems as plasma diagnostics, wave transformation mechanisms in plasmas, measurements of the efficiency of the HF pump power dissipation, definition of dispersion properties of plasma wave processes etc. This method of plasma diagnostics has proved to be very powerful both in the ionosphere and in laboratory plasmas.

The theory of scattering of electromagnetic waves in plasma was developed in [1 – 4]. Combined wave scattering and transformation in the plasma were predicted in [1].

The expressions for the fluctuations in density, current, fields and the velocity distributions, all for magnetized equilibrium plasmas, were derived in [5]. The fluctuation dissipation theorem was used in these derivations including the derivation of the fluctuations in the velocity distribution.

Authors of ref. [6] considered scattering in the presence of a pump wave in the lower hybrid range of frequencies. The fluctuations were determined by solving the kinetic equations with the field given in the electrostatic limit.

In previous works [7, 8] the wave scattering processes in the homogeneous nonequilibrium plasma have been investigated. In [7] the scattering of the electromagnetic waves by density fluctuations in the magnetized plasma is found to depend significantly upon the presence of parametric decay instabilities driven by a lower hybrid pump wave. This possibility offers interesting new approaches to plasma diagnostics.

The main objective of this work is to consider the nonlinear scattering processes in magnetized plasma with high-frequency pumps and to calculate the differential scattering cross-sections for various types of parametric instability which can take place in the plasma.

**2.** In present section we investigate the scattering of a transverse electromagnetic wave by turbulent density fluctuations in the inhomogeneous magnetoactive plasma with the lower hybrid parametric instability. The plasma inhomogeneity is defined by the exponential density gradient when the distribution function $f_{0e,i}$ is proportional to $\exp(\alpha' y)$, when $\alpha' = (1/n_0)(d\,n_0/dy)$ is the plasma inhomogeneity parameter. We suppose that such plasma subjected to the magnetic field $\vec{B}_0 = B_0\vec{z}$ and pump wave electric field $\vec{E}_0 = E_0\vec{y}\cos\omega_0 t$, where the frequency $\omega_0$ lies in the lower hybrid frequency region $\omega_0 \sim \omega_{lk} \approx \omega_{LH}(1+(m_i/m_e)\cos^2\theta)^{1/2}$, $\omega_{LH} \approx \omega_{pi}$ (where $\omega_{p\alpha}$ is the Langmuir frequency of the particles of type $\alpha$, $\alpha = e,i$ and $\theta$ is the angle between the direction of wave propagation and the magnetic field).

Let us study the propagation of transverse electromagnetic waves in the plasma. The waves scattering takes place due to interaction between the fluctuations and electromagnetic waves. Under the assumption that only the scattering by the electron density fluctuations occurs we can write the differential scattering cross section in the form [4]

$$d\Sigma = \frac{1}{2\pi}\left(\frac{e^2}{mc^2}\right)\frac{\omega'^2\omega''^2}{\omega^4_{pe}} R|\xi|^2 < \delta n_e^2 >_{\Delta\omega,\vec{q}} d\omega'' dO. \quad (1)$$

Here $\Delta\omega = \omega' - \omega''$, $\vec{q} = \vec{k}' - \vec{k}''$, $\omega', \omega''$ and $\vec{k}', \vec{k}''$ are the frequencies and the wave vectors of the incident and scattered waves, O is the space angle, $< \delta n_e^2 >_{\Delta\omega,\vec{q}}$ is the correlator of the electron density fluctuations at the combination frequency $\Delta\omega$ . The factor $R|\xi|^2$ depends on the incident and scattered wave's directions with respect to the magnetic field [4]. We note that the expression (1) is valid when frequencies of the incident and scattered waves are much greater than the electron Langmuir frequency, i.e. $\omega', \omega'' >> \omega_{pe}$ .

At first we consider the combination scattering of electromagnetic waves by the electron density fluctuations in the plasma, assuming that the pump wave can decay into lower hybrid and electron drift waves

$$\omega_0 = \omega_{lk} + \omega_{De} . \quad (2)$$

The frequency and the damping rate of electron drift wave are

$$\omega_{De} = -\frac{k_\perp \alpha' T_e}{m_e \Omega_e}, \quad (3)$$

$$\gamma_{De} = -\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \omega_{De}\left(-\frac{\omega_{De} k_x^2 \rho_i^2}{2k_{II}\upsilon_{Te}}\left(1+\frac{T_e}{T_i}\right)+\frac{T_e\omega_{De}}{T_i k_{II}\upsilon_{Ti}}\left(1+\frac{T_i}{T_e}\right)\exp\left(\frac{-\omega_{De}^2}{2k_{II}^2\upsilon_{Ti}^2}\right)\right), \quad (4)$$

where $\rho_i$ is the ion Larmor radius and $\upsilon_{Te,i}$ and $\Omega_{e,i}$ are the thermal velocity and the cyclotron frequency of particles, respectively.

According to (4) the interaction between the wave and the electrons is destabilizing and we have well known drift instability, which for a long time considered to be unavoidable in a finite size plasma. Note, however, that the ion Landau damping term will become important in a short device where $k_{II}$ has to take rather large values. Moreover, in a device with magnetic shear, $k_{II}$ can take small values only locally and damping is obtained by convection into regions with larger $k_{II}$.

We obtain the expression for the correlator of electron density fluctuations $< \delta n_e^2 >_{\Delta\omega,\vec{q}}$. We only note that this correlator is expressed in terms of the spectral correlation functions of non-interacting particles and these functions, in turn, depend on the correlator of source fluctuations. For the inhomogeneous plasma, the correlator of source fluctuations, in the general case, depends not only on the difference $\vec{r} - \vec{r}'$, but also on each of the coordinates $\vec{r}$ and $\vec{r}'$. Since we consider the case of weakly inhomogeneous plasma, it may assume that the dependence of the correlator of source fluctuations on $\vec{r} - \vec{r}'$ is more sharp than the dependence on the coordinate $\vec{r}$, because the former dependence is related to the wavelength of drift oscillations, whereas the latter is related to the characteristic scale length of plasma inhomogeneity.

In the plasma transparency region the main contribution to the correlator of the electron density is given by lower frequency drift oscillations and for the differential scattering cross section one obtains

$$\frac{d\Sigma}{dO} = \frac{d\Sigma_+}{dO} + \frac{d\Sigma_-}{dO}, \quad (5)$$

where

$$\frac{d\Sigma_\pm}{dO} \approx \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2(\omega' \pm \omega_{De})^2}{\omega_{pe}^4} R|\xi|^2 \Bigg\{ \int <\delta n_e^2>^0_{\Delta\omega,\vec{q}} d\omega'' +$$

$$+\left[\frac{\mu^2}{4}|\chi_e^0|^2|\chi_i^0|^2 \frac{1}{|\varepsilon_{\mp 1}|^2}\left(\frac{\mu^2}{4}<\delta n_e^2>^0_{\Delta\omega,\vec{q}} + \frac{q^2}{16\pi^2 e^2}<\delta\vec{E}^2>_{\Delta\omega\mp\omega_0,\vec{q}}\right) + |\chi_e^0|^2 <\delta n_i^2>^0_{\Delta\omega,\vec{q}}\right]\Bigg\} \frac{1}{\operatorname{Im}\varepsilon_E \dfrac{\partial \operatorname{Re}\varepsilon_E}{\partial \Delta\omega}}\Bigg|_{\Delta\omega=\omega_{De}} \quad (6)$$

In the equation (6) the nonlinear plasma permittivity is

$$\varepsilon_E(\omega,\vec{k}) = \varepsilon_0(\omega,\vec{k}) + \frac{\mu^2}{4}\chi_e^0\chi_i^0\left[\frac{1}{\varepsilon_1(\omega+\omega_0,\vec{k})} + \frac{1}{\varepsilon_{-1}(\omega-\omega_0,\vec{k})}\right], \quad (7)$$

where

$$\varepsilon_n(\omega + n\omega_0, \vec{k}) = 1 + \sum_\alpha \chi_\alpha^n(\omega + n\omega_0, \vec{k}), \quad n = 0, \pm 1, \quad (8)$$

$\mu = \dfrac{k_\perp E_0 c}{\omega_0 B_0} << 1$ and $<\delta n_e^2>^0_{\Delta\omega,\vec{q}}$ , $<\vec{E}^2>^0_{\Delta\omega,\vec{q}}$ are the common correlation spectral functions of the non-interacting particles in a magnetized plasma in the absence of the pump field [6]. Near the parametric instability threshold we may present the imaginary part of nonlinear dielectric permittivity in the form

$$\operatorname{Im}\varepsilon_E \sim \left(1 - \frac{E_0^2}{E_{th}^2}\right),$$

where $E_{th}^2 = 8\,\omega_0^2\,B_0^2\,\gamma_{lk}\,\gamma_{De}\dfrac{(kr_{De})^2}{(k_\perp c)^2}$ is the threshold electric field strength.

It follows from (6) that the scattering cross section increases anomalously when the pump wave amplitude tends to the threshold strength of the electric field (the case of critical opalescence). This fact indicates that the linear fluctuation theory is inadequate in this case.

Now consider the regime of fluctuations which have reached high level and produces a stationary turbulent state of plasmas. We introduce the redefined threshold value [9,10]

$$\tilde{E}_{th}^2 \approx E_{th}^2 \frac{\nu_{eff}^2}{\gamma_{lk}\gamma_{De}}, \quad (9)$$

where $\nu_{eff} \approx \gamma_{De}\dfrac{E_0^2}{E_{th}^2}$ is the effective collision frequency which defines the additional wave damping.

Using the fluctuation-dissipative theorem for calculating the correlators of non-interacting particles, substituting formula (9) into (6) and taking into account that for electron drift wave $|\chi_e^0| \approx |\chi_i^0| = \dfrac{1}{(qr_{De})^2}$ as a result we have the expression for the differential scattering cross section [11]

$$\frac{d\Sigma_\pm}{dO} \approx \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2(\omega' \pm \omega_{De})^2}{\omega_{pe}^4} R|\xi|^2 \frac{q^2 T_i}{e^2 (qr_{De})^2} \times$$

$$\times\left[1 + \frac{\mu^2}{8}\frac{T_e}{T_i}\frac{1}{(qr_{De})^2}\frac{\omega_{De}^2}{\gamma_{De}\gamma_{lk}} + \frac{\mu^4}{64}\frac{1}{(qr_{De})^4}\frac{\omega_{lk}}{\gamma_{lk}}\right]. \quad (10)$$

It can be seen from expression (10) that for typical parameters of hot plasma $\mu \approx 10^{-1}$, $qr_{De} \approx 10^{-1}$, $B_0$=50 kG, $T_e$=5 kev, n=$10^{14}$ $sm^{-3}$ the main contribution to the differential scattering cross section are given by terms depending on pump wave amplitude, which can exceed the terms due to the thermal noise scattering by several orders of value. We note also that in this case the cross section of electromagnetic wave scattering is comparitively sensitive to the density gradient ($d\Sigma_{\pm} \sim \alpha'$).

**3.** We study now the scattering processes of electromagnetic wave by density fluctuations when the frequency pump wave $\omega_0$ is close to the upper hybrid frequency $\omega_{UH}$. The important role of parametric instabilities in the region of upper hybrid resonance was pointed out in [11].

We then suppose that electron-ion homogeneous plasma imbedded the magnetic field $\vec{B}_0 = B_0\vec{z}$ and pump wave electric field $\vec{E}_0 = E_0\vec{y}\cos\omega_0 t$ excites ion-sound waves and the modified convective cells.

We consider the decay of the pump wave into upper hybrid wave $\omega_{UH1} \approx \left(\omega_{pe}^2 + \Omega_e^2\right)^{\frac{1}{2}}$ ($\omega_{pe} > \Omega_e$, we have the case of weakly magnetized plasma,) and modified convective cells [12]

$$\omega_0 = \omega_{UH1} + \omega_c. \tag{11}$$

Here $\omega_c =< m_i / m_e)^{1/2} \cos\theta \cdot \Omega_i$ is the real part of the frequency of modified convective cell and $\mathrm{Im}\,\omega \equiv \gamma_c \approx (1/2)\nu_{ei}$ ( $\nu_{ei}$ is the electron-ion collision frequency). Note that convective modes arise in the magnetized plasma with a small value of $\beta$ ($\beta$ is the ratio of the plasma pressure to the magnetic pressure) and can occur in the ionospheric plasma.

The parametric instability threshold for this decay is the following [13]:

$$E_{th}^2 = 16\,\frac{\omega_0^2 B_0^2}{k^2 c^2}\,\frac{m_e \nu_{ei}^2 \omega_{UH1}}{m_i \Omega_e^2 \omega_c^2}. \tag{12}$$

In the region above the instability threshold ($E_0 > E_{th1}$) the plasma becomes turbulent and we use the nonlinear stabilization mechanism described in [9]. Thus the differential scattering cross section is mainly given by "pump field" term

$$\frac{d\Sigma_{\pm}}{dO} \approx \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2(\omega' \pm \omega_c)^2}{\omega_{pe}^4} R|\xi|^2 \frac{q^2 E_0^2 c^2}{\omega_0^2 B_0^2} \left(\frac{\omega_{pi}}{\Omega_i}\right)^8 \frac{\omega_c^4}{\omega_{UH1}^2 \nu_{ei}^2} \frac{q^2 T_e^2}{128\pi e^2} \frac{\Omega_e^2}{\omega_{pe}^2}. \tag{13}$$

For typical ionospheric plasma parameters in the $F$ layer at about 250 km, $n_0$= $10^6$ $sm^{-3}$, Te~Ti=0.1 ev, $B_0$=0,45 G, $\nu_{ei}$=$5 \cdot 10^2$ $sec^{-1}$ the differential cross section given by (13) exceeds the corresponding one due to thermal noise by several orders of magnitude.

Consider next our decay of pump wave into upper hybrid wave $\omega_{UH2} \approx \Omega_e\left(1 + \frac{\omega_{pe}^2 \sin^2\theta}{2\Omega_e^2}\right)$ (in this case we have the case of strongly magnetized plasma, i.e. $\omega_{pe} < \Omega_e$ and we assume again that $\gamma_{UH2} \approx \nu_{ei}$) and ion-sound wave $\omega_s = k\upsilon_s$, $\upsilon_s = (T_e / T_i)^{1/2}$ is the ion-sound velocity. This parametric decay is possible when the condition $\cos \geq 3(m_e / m_i)^{1/2}$ is satisfied.

The threshold field for such instability is defined by [13]

$$E_{th2}^2 \approx \frac{25}{2}\left(\frac{5\pi m_e}{2m_i}\right)^{1/2}\frac{\upsilon_{Te}^2}{c^2}B_0\frac{\omega_0^2\left(\omega_0^2-\Omega_e^2\right)\nu_{ei}}{\omega_{pe}^4\Omega_e^3}. \quad (14)$$

We calculate in similar manner the differential cross section in the region above threshold ($E_0 > E_{th2}$) and estimate that for typical parameters of hot plasma $n_0 = 10^{14}$ $sm^{-3}$, Te = 10 kev, $B_0$ = =50 kG, the pump field differential cross section is greater then usual thermal noise term by 2 –3 orders of value.

**4**. The nonlinear scattering of electromagnetic waves in an inhomogeneous magnetoactive plasma is considered. The differential cross-section for the scattering of electromagnetic waves near the lower and upper hybrid resonance frequencies are obtained. For typical laboratory parameters, the differential cross sections in the presence of electromagnetic waves are much higher than the corresponding cross sections when fluctuations are presented. The results of this paper should be useful for understanding the processes of electromagnetic wave scattering in the laboratory and ionospheric plasma with lower and upper hybrid pump and also for experimental investigations using scattering techniques for studying the influence of plasma in thermonuclear devices.

Note that the last investigations of electromagnetic wave scattering in the magnetized plasma have been carried out in [13 - 17].

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## РОЗСІЯННЯ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ В ЗАМАГНІЧЕНІЙ ПЛАЗМІ З НИЖНЬО- ТА ВЕРХНЬОГІБРИДНОЮ НАКАЧКОЮ

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Досліджено розсіяння поперечної електромагнітної хвилі на флуктуаціях густини плазми для однорідної та неоднорідної замагніченої плазми в присутності верхньо- та нижньогібридної хвилі накачки. Обчислено коефіцієнт розсіяння для плазми з градієнтом густини в присутності параметричних нестійкостей нижньо- та верхньогібридної хвиль.

## РАССЕЯНИЕ ЭЛЕКТРОМАГНИТНЫХ ВОЛН В ЗАМАГНИЧЕННОЙ ПЛАЗМЕ С НИЖНЕ- И ВЕРХНЕГИБРИДНОЙ НАКАЧКОЙ

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Исследовано рассеяние поперечной электромагнитной волны на флуктуациях плотности плазмы для однородной и неоднородной магнитоактивной плазмы в присутствии верхне- и нижнегибридной волны накачки. Вычислен коэффициент рассеяния для плазмы с градиентом плотности в присутствии параметрических неустойчивостей нижне- и верхнегибридной волн.